LATIN SERIES · VOLUME 5

The Fundamentals of Music

Boethius

De Institutione Musica

The Fundamentals of Music

Five Books on Ratio, Interval, and Consonance

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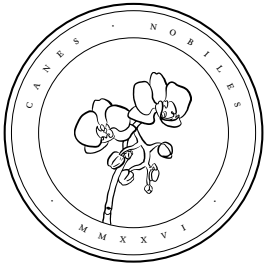
with a word-for-word interlinear English translation

preserving the original word order



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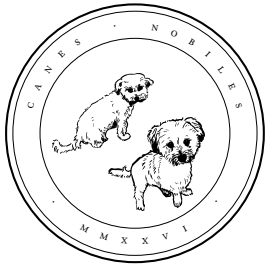

Volume 5 of the Canes Nobiles Latin interlinear texts

The Latin text follows Migne's Patrologia Latina,
which reprints Glarean's Basel edition of 1570

Set from Open Greek and Latin's Patrologia Latina corpus, with corrections

Translated and composed with AI, edited by Christopher Hench
Artwork by Esther Steves

Migne prints v for consonantal u and j for consonantal i. His plates, and the notation signs he prints as images, are not reproduced, nor are Glarean's own additions to the text; where Boethius cites a sign as a label, the Greek letter he names for it stands in its place, and in IV.6 the three interval signs are set as T, S, and D. The Laconian decree in Book I is printed in Greek and glossed like any other word.

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BOOK I

Musica naturaliter nobis esse conjunctam, et mores vel honestare vel evertere

I.1 · MUSIC SHAPES CHARACTER

Boethius opens with the claim that music is bound to us by nature: every age answers to it, and a mode can steady a mind or unstring it. Plato is his witness that a state must guard its music. He quotes the Spartan decree against Timotheus of Miletus, driven out for adding a string, and gathers stories of music's power: Pythagoras calming a drunken youth, songs that cured the sick, the trumpet that kindles men in battle. Since we cannot be without music, we must learn its proportions, not merely enjoy it.

Omnium quidem perceptio sensuum, ita sponte ac naturaliter
of-all indeed the-perception of-the-senses so unbidden and naturally
quibusdam viventibus adest, ut sine his animal non possit
to-certain living-creatures is-present that without these an-animal not could
intelligi. Sed non aeque eorumdem cognitio ac firma
to-be-conceived but not equally of-those-same-(senses) the-knowledge and firm
perceptio animi investigatione colligitur. Illaboratum est enim
grasp of-the-mind by-the-investigation is-gathered unlabored is for
quod sensum percipiendis sensibilibus rebus adhibemus.
(the-fact)-that the-sense for-perceiving sensible things we-apply
Quae vero sit ipsorum sensuum secundum quos agimus
what but may-be of-the-very senses according-to which we-act
natura, et quae rerum sensibilium proprietas; id non obvium
the-nature and what of-things sensible (is)-the-property this not obvious
neque cuilibet explicabile esse potest, nisi quem conveniens
nor to-anyone explicable to-be is-able unless whom a-fitting
investigatio veritatis contemplatione direxerit. Adest enim
search of-truth by-the-contemplation shall-have-guided is-present for

cunctis mortalibus visus, qui utrum venientibus ad visum
to-all mortals sight which whether by-coming to the-seeing
figuris, an ad sensibilia radiis emissis efficiatur, inter doctos
shapes or to the-sensibles by-rays sent-out is-effected among the-learned
quidem dubitabile est, vulgus quoque ipsa dubitatio praeterit.
indeed a-doubtful-(point) is the-crowd even the-very doubt passes-by
Rursus cum quis triangulum respicit, vel quadratum, facile
again when someone a-triangle beholds or a-square easily
id quod oculis invenitur agnoscit. Sed quaenam trianguli
that which by-the-eyes is-found he-recognizes but what-precisely of-the-triangle
vel quadrati sit natura, a mathematica necesse est petat.
or of-the-square may-be the-nature from mathematics necessary it-is that-he-seek
Idemque de caeteris sensibus dici potest, maximeque de
and-the-same about the-other senses to-be-said can and-most-of-all about
arbitrio aurium, quarum vis ita sonos captat, ut non modo
the-judgment of-the-ears whose power so sounds catches-at that not only
de his judicium capiat, differentiasque cognoscat, verum etiam
of these judgment it-takes and-the-differences discerns but also
delectetur saepius, si dulces coaptatique modi sunt, angatur
it-is-delighted very-often if sweet and-well-fitted the-measures are it-is-pained
vero, si dissipati atque incohaerentes feriant sensum.
but if broken and incoherent (measures)-strike the-sense
Unde fit, ut cum sint quatuor matheseos disciplinae,
whence it-comes that since there-are four of-mathematics disciplines
caeterae quidem ad investigationem veritatis laborent; musica
the-others indeed toward the-search of-truth labor music
vero non modo speculationi, verum etiam moralitati conjuncta
but not only to-speculation but also to-morality joined
sit. Nihil est enim tam proprium humanitati, quam remitti
is nothing is for so proper to-human-nature as to-be-relaxed
dulcibus modis astringique contrariis. Idque non modo sese
by-sweet measures and-to-be-braced by-their-contraries and-this not only itself

in singulis vel studiis vel aetatibus tenet, verum per cuncta
in single either pursuits or ages holds but through all
diffunditur studia, et infantes ac juvenes, necnon etiam senes,
is-spread pursuits and infants and the-young and-also even the-old
ita naturaliter affectu quodam spontaneo modis musicis
so naturally by-a-feeling certain unbidden to-measures musical
adjunguntur, ut nulla omnino sit aetas quae a cantilenae
are-attached that no at-all there-is age which from of-song
dulcis delectatione sejuncta sit. Hinc etiam internosci potest,
sweet the-delight severed is hence also to-be-discerned it-can
quod non frustra a Platone dictum est, mundi animam,
that not idly by Plato said it-was of-the-world (that)-the-soul
musica convenientia fuisse conjunctam. Cum enim ex eo
by-musical accordance to-have-been joined when for by that
quod in nobis est junctum convenienterque coaptatum, illud
which in us is joined and-fittingly framed that
excipimus, quod in sonis apte convenienterque conjunctum
we-take-in which in sounds aptly and-fittingly joined
est, eoque delectamur, nos quoque ipsos eadem similitudine
is and-by-it are-delighted we too ourselves by-the-same likeness
compactos esse cognoscimus. Amica est enim similitudo.
put-together to-be we-recognize friendly is for likeness
Dissimilitudo vero odiosa atque contraria. Hinc etiam morum
unlikeness but (is)-hateful and contrary hence also of-morals
quoque maxime permutationes fiunt. Lascivus quippe
too most-of-all the-changes come-about a-wanton for
animus, vel ipse lascivioribus delectatur modis, vel saepe
mind either itself by-more-wanton is-delighted measures or often
eosdem audiens cito emollitur, ac frangitur. Rursus asperior
the-same hearing quickly is-softened and is-broken again a-harsher
mens vel incitatioribus gaudet, vel incitatioribus
mind either in-more-vehement-(measures) rejoices or by-more-vehement

asperatur. Hinc est etiam quod modi musici gentium
is-roughened hence is also (the-fact)-that measures musical of-nations
vocabulo designati sunt, ut Lydius modus, et Phrygius. Quo
by-the-name marked are as the-Lydian mode and the-Phrygian in-what
enim quasi unaquaeque gens gaudet, eodem modus ipse
for as-it-were each nation rejoices by-that-same the-mode itself
vocabulo nuncupatur. Gaudet enim gens modis morum
name is-called rejoices for a-nation in-measures of-(its)-morals
similitudine. Neque enim fieri potest ut mollia duris,
by-the-likeness nor for to-come-to-pass it-can that soft-things to-hard
dura mollioribus adnectantur aut gaudeant. Sed amorem
hard-things to-softer be-tied or take-joy but love
delectationemque (ut dictum est) similitudo conciliat. Unde
and-delight as said it-was likeness wins whence
Plato etiam maxime cavendum existimat, ne de bene
Plato also most-of-all to-be-guarded-against thinks lest in well
morata musica aliquid permutetur. Negat enim esse ullam
mannered music anything be-altered he-denies for there-to-be any
tantam morum in republica labem, quam paulatim de
so-great of-morals in a-commonwealth ruin as little-by-little from
pudenti ac modesta musica invertere. Statim enim idem
decent and sober music to-turn-(it)-aside at-once for the-same
quoque audientium animos pati, paulatimque discedere,
too of-the-hearers (that)-the-minds to-suffer and-little-by-little to-fall-away
nullumque honesti ac recti retinere vestigium, si
and-no of-the-honorable and of-the-right to-retain trace if
vel per lasciviores modos inverecundum aliquid, vel per
either through more-wanton measures shameless something or through
asperiores ferox atque immane mentibus illabatur.
harsher (something)-fierce and monstrous into-(their)-minds glides

Nulla enim magis ad animum disciplinis via, quam auribus
no for more to the-mind for-teachings way than through-the-ears
patet. Cum ergo per eas rhythmi modique ad animum
lies-open when therefore through them rhythms and-measures to the-mind
usque descenderint, dubitari non potest quin aequo modo
even-down shall-have-descended to-be-doubted not it-can but-that in-like manner
mentem atque ipsa sunt efficiant atque conforment. Id
the-mind as themselves they-are they-fashion and shape this
vero etiam intelligi in gentibus potest. Nam quae
indeed also to-be-conceived among nations can for (those)-who
asperiores sunt Getarum, durioribus delectantur modis. Quae
harsher are of-the-Getae in-harder are-delighted measures (those)-who
vero mansuetae, mediocribus, quanquam id hoc tempore
but (are)-gentle in-middling-(ones) although this in-this time
pene nullum est. Quod vero lascivum ac molle est genus
almost nothing is what but wanton and soft is the-race
humanum, id totum scenicis ac theatralibus modis tenetur.
human that wholly by-stage and theater measures is-held
Fuit vero pudens ac modesta musica, dum simplicioribus
was indeed decent and sober music while on-simpler
organis ageretur. Ubi vero varie permisteque tractata
instruments it-was-performed when but variously and-promiscuously handled
est, amisit gravitatis atque virtutis modum, et pene in
it-was it-lost of-gravity and of-worth the-measure and almost into
turpitudinem prolapsa, minimum antiquam speciem servat.
baseness fallen least (its)-ancient semblance keeps
Unde Plato praecipit minime oportere pueros ad omnes
whence Plato enjoins by-no-means that-it-behooves boys in all
modos erudiri, sed potius ad valentes ac simplices. Atque
measures to-be-trained but rather in strong and simple-(ones) and
hic maxime illud est retinendum, quod si quoquo modo
here most-of-all this is to-be-held-fast that if in-whatever manner

per parvissimas mutationes hinc aliquid permutaretur,
through the-smallest changes herein anything were-altered
recens quidem minime sentiri, post vero magnam facere
at-first indeed scarcely to-be-felt afterward but great to-make
differentiam, et per aures ad animum usque delabi.
difference and through the-ears to the-mind even-down to-glide
Idcirco magnam esse custodiam reipub. Plato arbitratur
therefore a-great to-be safeguard of-the-commonwealth Plato judges
musicam optime moratam pudenterque conjunctam, ita ut sit
(that)-music best mannered and-decently joined so that it-be
modesta ac simplex, et mascula, nec effeminata, nec fera, nec
sober and simple and manly neither effeminate nor savage nor
varia. Quod Lacedaemonii maxima ope servavere, dum apud
shifting which the-Lacedaemonians with-greatest effort preserved while among
eos Taletas Cretensis Gortinus magno pretio accitus, pueros
them Thaletas the-Cretan of-Gortyn at-a-great price summoned the-boys
disciplina musicae artis imbueret. Fuit enim id antiquis in
in-the-training of-the-musical art was-steeping was for this for-the-ancients as
morem; diuque permansit. Quoniam vero eis Timotheus
a-custom and-long it-endured since but among-them Timotheus
Milesius super eas quas ante repererat unum addidit nervum,
the-Milesian beyond those which before he-had-found one added string
ac multipliciorem musicam fecit, exegere de Laconica.
and more-manifold music made they-drove-(him) out-of Laconia
Consultumque de eo factum est. Quod quoniam insigne
and-a-decree about him made was which since a-mark
est Spartiatarum lingua S litteram in R vertentium, ipsum
is of-the-Spartans in-tongue S the-letter into R turning itself
de eo consultum eisdem verbis Graecis apposui.
about him the-decree in-the-same words Greek I-have-set-down

Ἐπειδὴ Τιμόθεορ ὁ Μιλήσιορ, ἐλθὼν ἐττὰν ἁμετέραν πόλιν,
since Timotheus the Milesian having-come to-the our city
τὰν παλαιὰν μολπὴν ἀτιμάδει, καὶ τὰν διὰ τᾶν ἑπτὰ χορδᾶν
the ancient song dishonors and the through the seven strings
κιθάριξιν ἀποστρεφόμενορ, πολυφωνίαν εἰσάγων λυμαίνεται
lyre-playing turning-from many-voicedness bringing-in he-corrupts
τὰρ ἀκοὰρ τῶν νέων, διά τε τᾶρ πολυχορδίαρ καὶ τᾶρ
the ears of-the young through both the many-stringedness and the
καινότητορ τῶ μέλεορ, ἀγεννῆ καὶ ποικίλαν ἀντὶ ἁπλόαρ
newness of-the song ignoble and motley instead-of simple
καὶ τεταγμέναρ ἀμφιέννυται τὰν μοῦσαν, ἐπὶ χρώματορ
and well-ordered he-clothes the muse upon the-chroma
συνιστάμενορ τὰν τῶ μέλεορ διασκευάν, ἀντὶ τᾶρ ἐναρμονίω
composing the of-the song fashioning instead-of the enharmonic
ποιῶν ἀντίστροφον ἀμοιβάν· παρακλαθεὶς δὲ καὶ ἐττὸν ἀγῶνα
making an-answering exchange invited and also to-the contest
τᾶρ Ἐλευσινίαρ Δάματρορ ἀπρεπῆ διεσκευάσατο τὰν τῶ
of-the Eleusinian Demeter unseemly he-arranged the of-the
μύθω διασκευάν, τὰρ γὰρ τᾶρ Σεμέλαρ ὠδῖναρ οὐκ ἔνδικα
myth fashioning the for of-the of-Semele birth-pangs not rightly
τὼρ νέωρ διδάσκει. Δέδοκται ὑπὲρ τούτων τὼρ
the young he-teaches it-has-been-resolved concerning these-things (that)-the
βασιλέωρ καὶ τὼρ ἐφόρωρ μέμψασθαι Τιμόθεον, κ. τ. λ.
kings and the ephors to-censure Timotheus and the rest

Quod consultum id scilicet continet, idcirco Timotheo
this decree this namely contains (that)-therefore with-Timotheus
Milesio Spartiatas succensuisse, quod multiplicem musicam
the-Milesian the-Spartans to-have-been-angry because manifold music
reddens puerorum animis quos acceperat erudiendos,
rendering-(it) of-the-boys to-the-minds whom he-had-received to-be-trained
officeret, et a virtutis modestia praepediret, et quod
he-hindered and from of-virtue the-restraint held-(them)-back and because

harmoniam, quam modestam susceperat, in genus
the-harmony which sober he-had-received into the-genus
chromaticum, quod est mollius, invertisset. Tanta igitur fuit
chromatic which is softer he-had-turned so-great therefore was
apud eos musicae diligentia, ut eam animos quoque obtinere
among them for-music the-care that it minds even to-possess
arbitrarentur. Vulgatum quippe est, quam saepe iracundias
they-judged commonly-told indeed it-is how often angers
cantilena represserit, quam multa vel in corporum, vel
a-song has-restrained how many-things either in of-bodies or
in animorum affectionibus miranda perfecerit. Cui enim
in of-minds the-conditions wondrous it-has-wrought to-whom for
est illud ignotum, quod Pythagoras ebrium adolescentem
is that unknown that Pythagoras a-drunken youth
Taurominitanum sub Phrygii modi sono incitatum,
of-Tauromenium under of-the-Phrygian mode the-sound inflamed
spondeo succinente reddiderit mitiorem et sui
with-a-spondee sounding-beneath rendered-(him) gentler and of-himself
compotem. Nam cum scortum in rivalis domo esset clausum,
master for when a-harlot in a-rival's house was shut-up
atque ille furens domum vellet amburere, cumque Pythagoras
and he raging the-house wished to-burn and-while Pythagoras
stellarum cursus (ut ei mos nocturnus erat) inspiceret,
of-the-stars the-courses as for-him the-custom nightly was was-observing
ubi intellexit sono Phrygii modi incitatum, multis
when he-perceived by-the-sound of-the-Phrygian mode (him)-inflamed by-many
amicorum admonitionibus a facinore noluisse desistere,
of-(his)-friends warnings from the-crime to-have-refused to-desist
mutari modum praecepit atque ita furentis animum
to-be-changed the-mode he-ordered and so of-the-raging the-mind
adolescentis ad statum mentis pacatissimae temperavit.
of-the-youth to a-state of-mind most-peaceful he-tempered

Quod scilicet Marcus Tullius commemorat in eo libro quem
which namely Marcus Tullius recounts in that book which
de consiliis suis composuit, aliter quidem, sed hoc modo.
about counsels his-own he-composed otherwise indeed but in-this manner
Sed ut aliqua similitudine adductus, maximis minima
but that by-some likeness led with-the-greatest the-smallest
conferam, ut cum vinolenti adolescentes tibiarum etiam
I-may-compare as when wine-flushed youths of-pipes even
cantu (ut fit) instincti, mulieris pudicae fores frangerent,
by-the-song as happens goaded of-a-woman chaste the-doors were-breaking
admonuisse tibicinam ut spondeum caneret Pythagoras dicitur;
to-have-counseled the-pipe-girl that a-spondee she-play Pythagoras is-said
quod cum illa fecisset, tarditate modorum et gravitate
which when she had-done by-the-slowness of-the-measures and by-the-gravity
canentis, illorum furentem petulantiam consedasse. Sed ut
of-the-player their raging wantonness to-have-settled but that
similia breviter exempla conquiram. Terpander atque Arion
like briefly examples I-may-gather Terpander and Arion
Methymnaeus Lesbios atque Iones, gravissimis morbis, cantus
of-Methymna the-Lesbians and the-Ionians from-gravest sicknesses of-song
eripuere praesidio. Ismenias vero Thebanus Boeotorum pluribus,
rescued by-the-aid Ismenias but the-Theban of-the-Boeotians for-many
quos sciatici doloris tormenta vexabant, modis fertur cunctas
whom of-sciatic pain the-torments racked with-measures is-said all
abstersisse molestias. Sed et Empedocles cum ejus hospitem
to-have-wiped-away the-troubles but also Empedocles when his host
quidam gladio furibundus invaderet, quod ejus patrem ille
a-certain-(man) with-a-sword frenzied was-assailing because his father he
accusatione damnasset, inflexisse dicitur modum canendi, itaque
by-accusation had-condemned to-have-bent is-said the-mode of-singing and-so
adolescentis iracundiam temperasse. In tantum vero priscae
the-youth's wrath to-have-tempered in so-much indeed of-ancient

philosophiae studiis vis musicae artis innotuit, ut
philosophy in-the-pursuits the-power of-the-musical art became-known that
Pythagorici, cum diurnas in somno resolverent curas, quibusdam
the-Pythagoreans when daily in sleep they-loosed cares certain
cantilenis uterentur, ut eis lenis et quietus sopor irreperet.
songs used that upon-them gentle and quiet slumber might-steal
Itaque experrecti, aliis quibusdam modis stuporem somni
and-so awakened with-other certain measures the-dullness of-sleep
confusionemque purgabant. Id nimirum scientes, quod tota
and-confusion they-purged this doubtless knowing that the-whole
nostrae animae corporisque compago musica coaptatione
of-our soul and-body frame by-musical fitting-together
conjuncta sit. Nam ut sese corporis habet affectus, ita etiam
joined is for as itself of-the-body holds the-condition so also
pulsus cordis motibus incitantur. Quod scilicet Democritus
the-beats of-the-heart by-(its)-motions are-stirred which namely Democritus
Hippocrati medico tradidisse fertur, cum eum quasi insanum
to-Hippocrates the-physician to-have-imparted is-said when him as-if mad
cunctis Democriti civibus id opinantibus, in custodia medendi
(though)-all of-Democritus the-citizens this supposing in confinement of-healing
causa viseret. Sed quorsum haec? Quia non potest dubitari
for-the-sake he-visited but to-what-end these because not it-can to-be-doubted
quin nostrae animae et corporis status, eisdem quodammodo
but-that of-our soul and body the-state by-the-same in-a-manner
proportionibus videatur esse compositus, quibus harmonicas
proportions seems to-be composed by-which harmonic
modulationes posterior disputatio conjungi copularique
modulations the-later discussion to-be-joined and-coupled
monstrabit. Inde est enim quod infantes quoque cantilena
will-show thence is for (the-fact)-that infants even song
dulcis oblectat. Aliquid vero asperum atque immite, ab
sweet delights something but harsh and savage from

audiendi voluptate suspendit. Nimirum id etiam omnis aetas
of-hearing the-pleasure holds-(them)-back doubtless this too every age
patitur, omnisque sexus. Quae licet suis actibus distributa
undergoes and-every sex which though in-their-own doings parted
sint, una tamen musicae delectatione conjuncta sunt. Quid
be one nevertheless of-music by-the-delight joined they-are what
enim fit, cum in fletibus luctus ipsos modulantur dolentes?
for happens when in weepings griefs the-very set-to-measure the-grieving
Quod maxime muliebre est, ut cum cantico quodam dulcior
which most womanly is that with a-chant certain sweeter
fiat causa flendi. Id vero etiam fuit antiquis in morem,
becomes the-cause of-weeping this indeed also was for-the-ancients as a-custom
ut cantus tibia luctibus praeiret. Testis est Papinius Statius
that song on-the-pipe the-mournings went-before witness is Papinius Statius
hoc versu: Cornu grave mugit adunco, Tibia cui teneros
in-this verse with-horn deep moans curved the-pipe whose tender
suetum producere manes. Et qui suaviter canere non potest,
(is)-wont to-lead-forth ghosts and (he)-who sweetly to-sing not can
sibi tamen aliquid canit, non quod eum aliqua voluptate
to-himself nevertheless something sings not because him with-any pleasure
id quod canit afficiat, sed quod quamdam insitam dulcedinem
that which he-sings touches but because a-certain inborn sweetness
ex animo proferentis quoquomodo proferant, delectantur.
out-of the-mind of-the-utterer in-whatever-way they-bring-forth (men)-are-delighted
Nonne illud etiam manifestum est, in bellum pugnantium
is-it-not that also manifest is in war of-the-fighters
animos tubarum carmine accendi? Quod si verisimile
(that)-the-minds of-the-trumpets by-the-song to-be-kindled which if likely
est, ab animi pacato statu, quemquam ad furorem atque
is from of-the-mind peaceful the-state anyone to frenzy and
iracundiam posse proferri. Non est dubium, quod conturbatae
wrath to-be-able to-be-carried not is doubt that of-a-troubled

mentis iracundiam, vel nimiam cupiditatem modestior modus
mind the-wrath or excessive desire a-more-sober measure
possit astringere. Quid quod cum aliquis cantilenam
can to-restrain what of-(the-fact)-that when someone a-song
libentius auribus atque animo capit, ad illud etiam non
more-gladly with-ears and mind takes-in to this also not
sponte convertitur, ut motum quoque aliquem similem
of-(his)-will is-turned that a-motion too some like
auditae cantilenae corpus effingat, et quod omnino
to-the-heard song the-body shapes and (the-fact)-that altogether
aliquod melos auditum sibi memor animus ipse decerpat?
some tune heard for-itself mindful the-mind itself culls
Ut ex his omnibus perspicue nec dubitanter appareat,
so-that from these all clearly nor doubtfully it-may-appear
ita quidem nobis musicam naturaliter esse conjunctam ut
so indeed to-us (that)-music naturally to-be joined that
ea ne si velimus quidem carere possimus. Quocirca
it not-even if we-should-wish indeed to-be-without could-we wherefore
intendenda vis mentis est, ut id quod natura est
to-be-strained the-power of-mind is that that which by-nature is
insitum, scientia quoque possit comprehensum teneri. Sicut
inborn by-knowledge also may grasped to-be-held just-as
enim in visu quoque non sufficit eruditis colores formasque
for in sight too not it-suffices for-the-learned colors and-shapes
conspicere, nisi etiam quae sit horum proprietas investigaverint:
to-behold unless also what is of-these the-property they-have-searched-out
sic non sufficit cantilenis musicis delectari, nisi etiam quali
so not it-suffices by-songs musical to-be-delighted unless also by-what
inter se conjunctae sint vocum proportione discatur.
among themselves joined are of-voices proportion it-be-learned

Tres esse musicas, in quibus de vi musicae narratur

I.2 · THE THREE KINDS OF MUSIC

Boethius distinguishes three kinds of music: that of the world, that of man, and that established in instruments. The world's music is seen in the heavens, whose swift motion cannot be soundless though we do not hear it, in the frame of the elements, and in the round of the seasons; human music is the tempering that joins soul to body and the soul's parts to each other. Instrumental music works by tension, by breath, or by striking, and the present work discusses it first.

Principio igitur de musica disserenti, illud interim
at-the-beginning therefore about music to-(one)-discoursing this meanwhile
dicendum videtur, quot musicae genera ab ejus studiosis
to-be-said it-seems how-many of-music kinds by its students
comprehensa esse noverimus. Sunt autem tria. Et prima
comprehended to-be we-know there-are moreover three and the-first
quidem mundana est; secunda vero humana; tertia quae in
indeed of-the-world is the-second but the-human the-third which in
quibusdam constituta est instrumentis, ut in cithara vel in
certain established is instruments as in the-cithara or in
tibiis, caeterisque quae cantilenae famulantur. Et primum ea
pipes and-the-rest which to-song do-service and first that
quae est mundana in his maxime perspicienda est quae in
which is of-the-world in these most to-be-observed is which in
ipso coelo, vel compage elementorum, vel temporum varietate
the-very heaven or in-the-frame of-the-elements or of-the-seasons in-the-variety
visuntur. Qui enim fieri potest, ut tam velox coeli
are-seen how for to-come-about can-it that so swift of-heaven
machina tacito silentique cursu moveatur? Et si ad nostras
the-machine with-mute and-silent course should-be-moved and if to our
aures sonus ille non pervenit, quod multis fieri de causis
ears sound that not arrives which for-many to-come-about from causes

necesse est, non poterit tamen motus tam velocissimus
necessary is not it-will-be-able nevertheless the-motion so most-swift
ita magnorum corporum, nullos omnino sonos ciere,
so great of-bodies no at-all sounds to-stir
cum praesertim tanta sint stellarum cursus coaptatione
since especially by-so-great are of-the-stars the-courses by-a-fitting-together
conjuncti, ut nihil aeque compaginatum, nihil ita commixtum
joined that nothing equally framed-together nothing so commingled
possit intelligi. Namque alii excelsiores, alii inferiores
could to-be-conceived for-indeed some higher others lower
feruntur, atque ita omnes aequali incitatione volvuntur, ut
are-borne and so all with-equal impulse are-rolled that
per dispares inaequalitates ratus cursuum ordo ducatur.
through unlike inequalities a-settled of-the-courses the-order may-be-led
Unde non potest ab hac coelesti vertigine ratus ordo
whence not can from this heavenly whirling the-settled order
modulationis absistere. Jam vero quatuor elementorum
of-modulation to-stand-apart now but of-the-four elements
diversitates contrariasque potentias, nisi quaedam harmonica
the-diversities and-contrary powers unless a-certain harmonic-(concord)
conjungeret, qui fieri posset, ut in unum corpus ac
should-join how to-come-about could-it that into one body and
machinam convenirent? Sed haec omnis diversitas ita
machine they-should-come-together but this all diversity so
et temporum varietatem parit et fructuum, ut tamen
both of-the-seasons the-variety brings-forth and of-fruits that nevertheless
unum anni corpus efficiat. Unde si quid horum, quae
one of-the-year body it-may-effect whence if any of-these which
tantam varietatem rebus ministrant, animo et cogitatione
so-great-(a) variety to-things supply in-mind and thought
discerpas, cuncta pereant, nec (ut ita dicam) quidquam
you-should-tear-apart all-(things) would-perish nor as thus I-may-say anything

consonum servent. Et ut in gravibus chordis is vocis est
consonant they-would-keep and as in low strings such of-the-voice is
modus, ut non ad taciturnitatem gravitas usque descendat,
the-measure that not to muteness the-lowness all-the-way may-descend
atque in acutis ille custoditur acuminis modus ne
and in the-high-(strings) that is-guarded of-highness the-measure lest
nervi nimium tensi vocis tenuitate rumpantur, sed
the-strings too-much stretched of-the-voice by-the-thinness be-broken but
totum sibi sit consentaneum atque conveniens; ita etiam in
the-whole to-itself may-be agreeing and accordant so also in
mundi musica pervidemus, nihil ita nimium esse posse ut
of-the-world the-music we-see-clearly nothing so excessive to-be to-be-able that
alterum propria nimietate dissolvat. Verum quidquid illud est,
another by-its-own excess it-should-dissolve but whatever that is
aut suos affert fructus, aut aliis auxiliatur ut afferant.
either its-own it-brings fruits or to-others it-gives-aid that they-may-bring
Nam quod constringit hiems, ver laxat, torret aestas,
for what binds-fast winter spring loosens scorches summer
maturat autumnus, temporaque vicissim vel ipsa suos
ripens autumn and-the-seasons in-turn either themselves their-own
afferunt fructus, vel aliis ut afferant subministrant. De
bring fruits or to-others that they-may-bring they-supply about
quibus posterius studiosius disputandum est. Humanam
which-(things) later more-carefully to-be-discussed it-is the-human
vero musicam, quisquis in sese ipsum descendit, intelligit.
but music whoever into himself his-very-self descends understands
Quid est enim quod illam incorpoream rationis vivacitatem
what is for (that)-which that bodiless of-reason liveliness
corpori misceat, nisi quaedam coaptatio, et veluti
with-the-body may-mingle unless a-certain fitting-together and as-it-were
gravium leviumque vocum, quasi unam consonantiam efficiens,
of-low and-of-high voices as-if one consonance effecting

temperatio? Quid est autem aliud quod ipsius inter
a-tempering what is moreover other (that)-which of-the-very among
se partes animae conjungat, quae (ut Aristoteli placet)
themselves the-parts of-the-soul may-join which as to-Aristotle it-pleases
ex rationabili irrationabilique conjuncta est? Quid vero
from the-rational and-the-irrational joined it-is what but
quod corporis elementa permisceat, aut partes sibimet
(that)-which of-the-body the-elements may-mix-together or the-parts to-themselves
rata coaptatione contineat? Sed de hac quoque posterius
by-a-settled fitting-together may-hold but about this too later
dicam. Tertia est musica, quae in quibusdam consistere
I-shall-speak the-third is music which in certain to-consist
dicitur instrumentis. Haec vero administratur, aut intentione,
is-said instruments this but is-managed either by-tension
ut nervis, aut spiritu, ut tibiis, vel his quae ad aquam
as in-strings or by-breath as in-pipes or in-those which by water
moventur, aut percussione quadam, ut in his quae in
are-moved or by-striking a-certain as in those which in
concava quaedam virga aerea feriuntur, atque inde diversi
hollow certain by-a-rod brazen are-struck and thence diverse
efficiuntur soni. De hac igitur instrumentorum musica
are-produced sounds about this therefore of-instruments the-music
primum hoc opere disputandum videtur. Sed prooemii satis
first in-this work to-be-discussed it-seems but of-preface enough
est. Nunc de ipsis musicae elementis est disserendum.
is now about the-very of-music elements it-is to-be-discoursed

De vocibus ac de musicae elementis

I.3 · VOICES AND THE ELEMENTS OF MUSIC

Boethius traces consonance back to motion: consonance needs sound, sound needs a striking, and a striking needs motion, so sound is defined as a striking of air unbroken all the way to hearing. Slow, sparse motions make low sounds and swift, dense ones high — a trembling string strikes the air many times, but the blows fuse into one sound, as a spinning top's red stripe seems to dye the whole. Pitch therefore rests on number, and consonance is a concord of unlike voices brought into one.

Consonantia, quae omnem musicae modulationem regit, praeter
consonance which all of-music modulation rules apart-from
sonum fieri non potest. Sonus vero praeter quemdam
sound to-come-about not can sound but apart-from a-certain
pulsum percussionemque non redditur. Pulsus vero atque
pulse and-striking not is-rendered a-pulse but and
percussio nullo modo esse potest, nisi praecesserit motus. Si
striking in-no way to-be can unless shall-have-preceded motion if
enim cuncta sint immobilia, non poterit alterum alteri
for all-(things) be motionless not will-be-able one with-another
concurrere, ut alterum impellatur ab altero. Sed cunctis
to-run-together that the-one be-driven by the-other but all-(things)
stantibus motuque carentibus, nullum fieri necesse est
standing and-of-motion being-devoid no to-come-about necessary it-is
sonum. Idcirco definitur sonus: Aeris percussio indissoluta
sound therefore is-defined sound of-air a-striking unbroken
usque ad auditum. Motuum vero alii sunt velociores,
all-the-way to hearing of-motions but some are swifter
alii tardiores, eorumdemque motuum alii rariores sunt alii
others slower and-of-those-same motions some sparser are others
spissiores. Nam si quis in continuum motum respiciat, ibi
denser for if anyone upon a-continuous motion should-look there

aut velocitatem aut tarditatem necesse est comprehendat.
either swiftness or slowness necessary it-is that-he-grasp
Sin vero quis moveat manum, aut frequenti eam motu
but-if indeed anyone should-move his-hand either with-frequent it motion
movebit aut raro. Et si tardus quidem fuerit ac
he-will-move or with-sparse and if slow indeed it-shall-have-been and
rarior motus graves necesse est sonos effici ipsa
sparser the-motion low necessary it-is sounds to-be-produced by-the-very
tarditate et raritate pellendi. Sin vero motus sint celeres
slowness and sparseness of-striking but-if indeed the-motions be swift
ac spissi, acutos reddi necesse est sonos.
and dense high to-be-rendered necessary it-is sounds
Idcirco enim idem nervus si intendatur amplius, acutum
therefore for the-same string if it-be-stretched further a-high-(sound)
sonat; si remittatur, grave. Quando enim tensior est,
it-sounds if it-be-slackened a-low-(sound) when for tenser it-is
velociorem pulsum reddit, celeriusque revertitur, et frequentius
a-swifter pulse it-renders and-more-quickly returns and more-frequently
ac spissius aerem ferit. Qui vero laxior est, solutos ac
and more-densely the-air strikes (the-one)-which but slacker is loose and
tardos pulsus effert, rarusque ipsa imbecillitate feriendi,
slow pulses brings-forth and-sparse by-the-very feebleness of-striking
nec diutius tremit. Neque enim quotiens chorda pellitur, unus
nor longer trembles not for as-often-as a-string is-struck one
edi tantum putandus est sonus, aut unam in his esse
to-be-uttered only to-be-thought is sound or one in these to-be
percussionem, sed totiens aer feritur quotiens eum chorda
striking but so-many-times the-air is-struck as-often-as it the-string
tremebunda percusserit. Sed quoniam junctae sunt velocitates
trembling shall-have-struck but since joined are the-swiftnesses
sonorum, nulla intercapedo sentitur auribus. Et unus sonus
of-the-sounds no gap is-felt by-the-ears and one sound

sensum pellit vel gravis, vel acutus, quamvis uterque ex
the-sense strikes either low or high although each from
pluribus constet, gravis quidem ex tardioribus, et rarioribus,
many consists the-low indeed from slower and sparser-(motions)
acutus vero ex celeribus ac spissis, veluti si conum, quem
the-high but from swift and dense as-if if a-cone which
turbonem vocant, quis diligenter exornet, eique unam
a-top they-call someone carefully should-deck-out and-on-it one
virgulam coloris rubri vel alterius ducat, et eum qua
small-stripe of-color red or of-another should-draw and it with-what
potest celeritate convertat, tunc totus conus rubro colore
it-can speed should-spin then the-whole cone with-red color
videtur infectus, non quod totus ita sit, sed quod partes puras
seems dyed not because whole so it-be but because the-parts pure
rubrae virgae velocitas comprehendat, et apparere non sinat.
of-the-red stripe the-swiftness may-embrace and to-appear not may-allow
Sed de his posterius. Igitur quoniam acutae voces spissioribus
but about these later therefore since high voices by-denser
et velocioribus motibus incitantur, graves vero tardioribus
and swifter motions are-stirred the-low but by-slower
ac raris, liquet additione quadam motuum ex gravitate
and sparse-(ones) it-is-clear by-addition a-certain of-motions from lowness
acumen intendi, detractione vero motuum laxari ex
highness to-be-stretched-up by-withdrawal but of-motions to-be-slackened from
acumine gravitatem. Ex pluribus enim motibus acumen, quam
highness lowness from more for motions highness than
gravitas constat. In quibus autem pluralitas differentiam facit,
lowness consists in which-(things) moreover plurality the-difference makes
eam necesse est in quadam numerositate consistere. Omnis vero
it necessary is in a-certain numerousness to-consist every but
paucitas ad pluralitatem ita sese habet, ut numerus ad numerum
fewness to plurality so itself holds as number to number

comparatus. Eorum vero quae secundum numerum conferuntur,
compared of-those but which according-to number are-brought-together
partim sibi sunt aequalia, partim inaequalia. Quocirca soni
partly to-themselves are equal partly unequal wherefore sounds
quoque partim sunt aequales, partim vero sunt inaequalitate
too partly are equal partly but are by-inequality
distantes. Sed in his vocibus, quae nulla inaequalitate
standing-apart but in these voices which by-no inequality
discordant, nulla omnino consonantia est, etenim consonantia
disagree no at-all consonance there-is for-indeed consonance
est dissimilium inter se vocum in unum redacta concordia.
is of-unlike among themselves voices into one brought a-concord

De speciebus inaequalitatum

I.4 · THE SPECIES OF INEQUALITY

Boethius lists the five species of inequality. A multiple contains the lesser number whole two, three, or more times; a superparticular adds one part of it, as 3:2 and 4:3; a superpartient adds several parts, as 5:3; the last two kinds compound a multiple with one part or with several. He treats them briefly here, having unfolded them more diligently in his books on arithmetic.

Quae vero sunt inaequalia, quinque inter se modis
(the-things)-which but are unequal in-five among themselves ways
inaequalitatis momenta custodiunt. Aut enim alterum ab altero
of-inequality the-measures keep either for the-one by the-other
multiplicate transcenditur, aut singulis partibus, aut pluribus
multiple-wise is-surpassed or by-single parts or by-several
aut multiplicitate et parte, aut multiplicitate et partibus. Et
or by-multiplicity and a-part or by-multiplicity and parts and
primum quidem inaequalitatis genus multiplex appellatur. Est
the-first indeed of-inequality kind multiple is-called it-is

vero multiplex ubi major numerus minorem numerum habet,
but multiple where the-greater number the-lesser number has
in se totum, vel bis, vel ter, vel quater, ac deinceps nihilque
in itself whole either twice or thrice or four-times and so-on and-nothing
deest, nihilque exuberat, appellaturque vel duplum, vel triplum,
is-lacking and-nothing overflows and-it-is-called either double or triple
vel quadruplum, atque ad hunc ordinem in infinita progreditur.
or quadruple and after this order into the-infinite it-advances
Secundum vero inaequalitatis genus est quod appellatur
the-second but of-inequality kind is (that)-which is-called
superparticulare, id est cum major numerus minorem
superparticular that is when the-greater number the-lesser
numerum habet in se totum, et unam ejus aliquam partem,
number has in itself whole and one of-it some part
eamque vel dimidiam, ut tres duorum, et vocatur sesquialtera
and-it either the-half as three of-two and it-is-called sesquialter
proportio, vel tertiam, ut quatuor ad tres, et vocatur
proportion or the-third-(part) as four to three and it-is-called
sesquitertia. Ad hunc etiam modum in posterioribus numeris
sesquitertian after this also manner in later numbers
pars aliqua a majoribus super minores numeros continetur.
part some by the-greater beyond the-lesser numbers is-contained
Tertium vero genus inaequalitatis est, quotiens major numerus
the-third but kind of-inequality is whenever the-greater number
totum intra se minorem continet, et ejus aliquantas insuper
whole within itself the-lesser contains and of-it several besides
partes, et si duas quidem supra continet, vocabitur
parts and if two indeed over-and-above it-contains it-will-be-called
proportio superbipartiens, ut sunt quinque ad tres. Sin vero
a-proportion superbipartient as are five to three but-if indeed
tres super se continet, vocabitur supertripartiens, ut sunt
three beyond itself it-contains it-will-be-called supertripartient as are

septem ad quatuor. Et in caeteris quidem eadem similitudo
seven to four and in the-rest indeed the-same likeness
esse potest. Quartum vero est inaequalitatis genus, quod ex
to-be can the-fourth but is of-inequality the-kind which from
multiplici et superparticulari conjungitur, cum scilicet major
the-multiple and the-superparticular is-joined-together when namely the-greater
numerus habet in se minorem numerum, vel bis vel ter vel
number has in itself the-lesser number either twice or thrice or
quotieslibet, atque ejus unam aliquam partem. Et si eum bis
as-often-as-you-please and of-it one some part and if it twice
habet, et ejus dimidiam partem, vocabitur duplex sesquialter,
it-has and of-it the-half part it-will-be-called double sesquialter
ut sunt quinque ad duo. Sin vero bis minor continebitur,
as are five to two but-if indeed twice the-lesser will-be-contained
et ejus tertia pars, vocabitur duplex sesquitertius, ut sunt
and of-it the-third part it-will-be-called double sesquitertian as are
septem ad tres. Sin vero tertio continebitur, et ejus dimidia
seven to three but-if indeed a-third-time it-will-be-contained and of-it the-half
pars, vocabitur triplex sesquialter, ut sunt septem ad duo.
part it-will-be-called triple sesquialter as are seven to two
Atque ad eumdem modum in caeteris et multiplicitatis et
and after the-same manner in the-rest and of-multiplicity and
superparticularitatis vocabula variantur. Quintum est genus
of-superparticularity the-names are-varied the-fifth is kind
inaequalitatis, quod appellatur multiplex superpartiens, quando
of-inequality which is-called multiple superpartient when
major numerus minorem numerum habet in se totum
the-greater number the-lesser number has in itself whole
plusquam semel, et ejus plusquam unam aliquam partem.
more-than once and of-it more-than one some part
Et si bis major numerus minorem numerum continebit,
and if twice the-greater number the-lesser number shall-contain

duasque ejus insuper partes, vocabitur duplex superbipartiens,
and-two of-it besides parts it-will-be-called double superbipartient
ut sunt octo ad tres, et rursus triplex superbipartiens, ut
as are eight to three and again triple superbipartient as
sunt undecim ad tres. Ac de his idcirco nunc strictim ac
are eleven to three and about these therefore now summarily and
breviter explicamus, quoniam in libris quos de Arithmetica
briefly we-unfold since in the-books which about Arithmetical
Institutione conscripsimus, diligentius enodavimus.
Instruction we-have-written more-diligently we-have-unknotted

Quae inaequalitatis species consonantiis aptentur

I.5 · WHICH INEQUALITIES FIT THE CONSONANCES

Boethius sets aside the last two kinds of inequality, since they are mixed from the earlier ones, and keeps the first three for study. The multiple seems to hold the greater power over the consonances, the superparticular the next; the superpartient is severed from harmony's concord — so it seems to some, though not to Ptolemy.

Ex his igitur inaequalitatis generibus postrema duo, quoniam
from these therefore of-inequality kinds the-last two since
ex superioribus mixta sunt, relinquamus. De tribus vero
from the-former mixed they-are let-us-leave-(them) about the-three but
prioribus speculatio facienda est. Obtinere igitur majorem ad
earlier speculation to-be-made is to-hold therefore a-greater toward
consonantias potestatem videtur multiplex, consequenter autem
consonances power seems the-multiple next moreover
superparticularis. Superpartiens vero ab harmoniae concinentia
the-superparticular the-superpartient but from of-harmony the-concord
separatur, ut quibusdam praeter Ptolemaeum videtur.
is-severed as to-certain-(men) except Ptolemy it-seems

Cur multiplicitas et superparticularitas consonantiis deputentur

I.6 · WHY MULTIPLES AND SUPERPARTICULARS FIT THE CONSONANCES

Boethius argues that what is simple by nature suits consonance. Number grows without end from a bounded unity, and multiplicity, whose growing has no end, preserves its nature; continuous quantity divides without end, and superparticularity, whose parts grow ever smaller, preserves its property. The superpartient departs from simplicity, so the Pythagoreans do not admit it to the consonances — though Ptolemy places it among them, as Boethius will show later.

Ea namque probantur comparationi consentanea, quae sunt
those for-indeed are-proved to-comparison agreeable which are
natura simplicia. Et quoniam gravitas et acumen in
by-nature simple and since lowness and highness in
quantitate consistunt, ea maxime videbuntur servare naturam
quantity consist those most will-seem to-preserve the-nature
concinentiae, quae discretae proprietatem quantitatis poterunt
of-concord which of-discrete the-property quantity will-be-able
custodire. Nam cum sit alia quidem discreta quantitas, alia
to-keep for since is one indeed discrete quantity another
vero continua, ea quae discreta est in minimo quidem
but continuous that which discrete is in the-least indeed
finita est, sed in infinitum per majora procedit.
bounded is but into the-infinite through greater-(numbers) it-proceeds
Namque in ea minima unitas eademque finita est. In
for-indeed in it the-least (is)-the-unit and-the-same bounded is into
infinitum vero modus pluralitatis augetur, ut numerus
the-infinite but the-measure of-plurality is-increased so-that number
qui cum finita incipiat unitate, crescendi non habet
which though from-a-bounded it-begin unit of-growing not has
finem. Rursus quae est continua, tota quidem finita
an-end again (that)-which is continuous whole indeed bounded

est, sed per infinita minuitur. Linea enim quae continua
is but through the-infinite is-lessened a-line for which continuous
est, in infinita semper partitione dividitur. Cum sit ejus
is in infinite always partition is-divided though be its
summa, vel bipedalis, vel quaecunque alia definita mensura.
total either two-foot or whatever other defined measure
Quocirca numerus semper in infinita concrescit, continua vero
wherefore number always into the-infinite grows continuous but
quantitas in infinita minuitur. Multiplicitas igitur quoniam
quantity into the-infinite is-lessened multiplicity therefore since
crescendi finem non habet, numeri maxime servat naturam.
of-growing an-end not has of-number most it-preserves the-nature
Superparticularitas autem, quoniam in infinitum minorem
superparticularity moreover since into the-infinite the-lesser
minuit proprietatem servat continuae quantitatis. Minuit
it-lessens the-property it-preserves of-continuous quantity it-lessens
autem minorem, cum semper eum continet, et ejus vel
moreover the-lesser since always it it-contains and of-it either
dimidiam partem, vel tertiam, vel quartam, vel quintam. Nam
the-half part or the-third or the-fourth or the-fifth for
semper pars a majore numero denominata ipsa decrescit.
always the-part by the-greater number named itself decreases
Nam cum tertia a tribus denominata sit, quarta vero a
for since the-third from three named be the-fourth but from
quatuor, cum quatuor tres superent, quarta potius quam
four since four three surpass the-fourth rather than
tertia minutior invenitur. Superpartiens vero jam quodammodo
the-third smaller is-found the-superpartient but now in-a-certain-way
a simplicitate discedit. Duas enim, vel tres, vel quatuor
from simplicity departs two for or three or four
habet insuper partes, et a simplicitate discedens exuberat
it-has besides parts and from simplicity departing it-overflows

ad quamdam partium pluralitatem. Rursus multiplicitas omnis
to a-certain of-parts plurality again multiplicity all
in integritate se continet. Nam duplum bis habet totum
in wholeness itself contains for the-double twice has whole
minorem; triplum item tertio continet totum minorem,
the-lesser the-triple likewise a-third-time contains whole the-lesser
atque ad eumdem modum et caetera. Superparticularitas vero
and after the-same manner also the-rest superparticularity but
nihil integrum servat, sed vel dimidio superat, vel tertia,
nothing whole preserves but either by-the-half surpasses or by-the-third
vel quarta, vel quinta. Sed tamen divisionem singulis ac
or by-the-fourth or by-the-fifth but nevertheless division by-single and
simplicibus partibus operatur. Superpartiens autem inaequalitas
simple parts it-works the-superpartient moreover inequality
nec servat integrum nec a singulis adimit partes. Atque
neither preserves the-whole nor by single-(parts) takes-away parts and
ideo secundum Pythagoricos, minime musicis consonantiis
therefore according-to the-Pythagoreans by-no-means to-musical consonances
adhibetur. Ptolemaeus tamen etiam hanc proportionem inter
it-is-applied Ptolemy nevertheless even this proportion among
consonantias ponit, ut posterius ostendam.
the-consonances places as later I-shall-show

Quae proportiones quibus consonantiis musicis aptentur

I.7 · WHICH PROPORTIONS FIT WHICH CONSONANCES

Every musical consonance lies in the double, triple, quadruple, sesquialter, or sesquitertian proportion. The sesquitertian in numbers is the diatessaron in sounds, the sesquialter the diapente, the double the diapason, the triple the diapason-plus-diapente, and the quadruple the double diapason. This is said generally for now; the full reckoning of the proportions will come clear later.

Illud tamen esse cognitum debet, quod omnes musicae
this nevertheless to-be known ought that all of-music
consonantiae aut in duplici, aut in triplici, aut in quadrupla,
the-consonances either in the-double or in the-triple or in the-quadruple
aut in sesquialtera aut in sesquitertia proportione consistunt.
or in the-sesquialter or in the-sesquitertian proportion consist
Et vocabitur quidem quae in numeris sesquitertia est,
and will-be-called indeed (that)-which in numbers sesquitertian is
diatessaron in sonis. Quae in numeris sesquialtera, diapente
diatessaron in sounds (that)-which in numbers sesquialter diapente
appellatur in vocibus. Quae vero in proportionibus dupla
is-called in voices (that)-which but in proportions double
est diapason in consonantiis. Tripla vero diapente ac
is diapason in consonances the-triple but (is)-diapente and
diapason. Quadrupla autem bis diapason. Et nunc quidem
diapason the-quadruple moreover (is)-twice diapason and now indeed
universaliter atque indiscrete dictum sit. Posterius vero
universally and without-distinction said let-it-be later but
omnis ratio proportionum lucebit.
all the-reckoning of-proportions will-shine-clear

Quid sit sonus, quid intervallum, quid concinentia

I.8 · SOUND, INTERVAL, AND CONCORD

Boethius defines his terms. A sound — the Greek phthongus — is a melodic falling of the voice upon one tension; an interval is the distance between a high sound and a low one. Consonance is a mixture of high and low that falls sweetly and uniformly on the ears; dissonance is a harsh, unpleasing striking, in which two sounds refuse to mingle and each obstructs the other.

Sonus igitur est vocis casus, emmeles, id est aptus melo,
sound therefore is of-the-voice a-falling melodic that is apt for-melody
in unam intensionem. Sonum vero non generalem nunc
upon one tension sound but not general now
volumus definire; sed eum qui graece dicitur phthongus,
we-wish to-define but that-(sound) which in-Greek is-called phthongus
dictus a similitudine loquendi φθέγγεσθαι. Intervallum
named from likeness of-speaking to-utter-sound an-interval
vero est soni acuti gravisque distantia. Consonantia
but is of-a-sound high and-low the-distance consonance
est acuti soni gravisque mixtura, suaviter uniformiterque
is of-a-high sound and-of-a-low a-mixture sweetly and-uniformly
auribus accidens. Dissonantia vero est duorum sonorum
on-the-ears falling dissonance but is of-two sounds
sibimet permixtorum ad aurem veniens aspera atque
with-themselves mixed-together to the-ear coming a-harsh and
injucunda percussio. Nam dum sibimet misceri nolunt,
unpleasing striking for while with-themselves to-be-mingled they-refuse
et quodammodo integer uterque nititur pervenire, cumque
and in-a-certain-way whole each strives to-arrive and-when
alter alteri officit, ad sensum insuaviter uterque transmittitur.
the-one the-other obstructs to the-sense unsweetly each is-carried-across

Non omne judicium dandum esse sensibus sed amplius rationi esse credendum in quo de sensuum fallacia

I.9 · THE SENSES NEED REASON

Boethius refuses to give all judgment to the senses, though hearing is the beginning of this art. Sense-perception is not the same in all men or constant in one man, and it is confounded by the greatest sounds and misses the least. The Pythagoreans therefore take a middle road: they measure the consonances themselves by ear, but commit their differences to rules and reason — sense the obedient servant, reason the judge.

Sed de his ita proponimus, ut non omne judicium sensibus
but about these thus we-set-forth that not all judgment to-the-senses
demus, quanquam a sensu aurium hujusce artis sumatur
we-give although from the-sense of-the-ears of-this art be-taken
omne principium. Nam si nullus esset auditus, nulla
all the-beginning for if no there-were hearing no
omnino disputatio de vocibus exstitisset. Sed principium
at-all discussion about voices would-have-arisen but the-beginning
quodammodo, et quasi admonitionis vicem tenet auditus.
in-a-certain-way and as-it-were of-admonition the-place holds hearing
Postrema ergo perfectio, agnitionisque vis in ratione
the-last therefore perfection and-of-recognition the-power in reason
consistit, quae certis regulis sese tenens nullo unquam
consists which by-sure rules itself holding by-no ever
errore prolabitur. Nam quid diutius dicendum est de errore
error slips-down for what longer to-be-said is about the-error
sensuum, quando nec omnibus eadem sentiendi vis, nec
of-the-senses when neither to-all the-same of-perceiving power nor
eidem homini semper aequalis? Frustra autem vario judicio
to-the-same man always (is)-equal in-vain moreover to-a-shifting judgment
quisquam committet, quod veraciter affectat inquirere. Idcirco
anyone will-entrust what truly he-aims to-search-out therefore

Pythagorici medio quodam feruntur itinere. Nam nec omne
the-Pythagoreans by-a-middle certain are-borne road for neither all
judicium dedunt auribus, et quaedam tamen ab eis non
judgment they-give-over to-the-ears and certain-(things) nevertheless by them not
nisi auribus explorantur. Ipsas etenim consonantias aure
except by-the-ears are-explored the-very for-indeed consonances by-the-ear
metiuntur. Quibus vero inter se distantiis consonantiae
they-measure by-what but among themselves distances the-consonances
differant, id jam non auribus, quarum sunt obtusa judicia,
differ this now not to-the-ears whose are blunted the-judgments
sed regulis rationique permittunt, ut quasi obediens quidem
but to-rules and-to-reason they-commit so-that as-it-were obeying indeed
famulusque sit sensus, judex vero atque imperans ratio.
and-a-servant may-be sense the-judge but and commanding reason
Nam licet omnium pene artium, atque ipsius vitae momenta,
for although of-all almost arts and of-the-very life the-measures
sensuum occasione producta sint, nullum tamen in his
of-the-senses by-the-occasion brought-forth be no nevertheless in these
judicium certum, nulla veri est comprehensio, si arbitrium
judgment (is)-certain no of-the-true there-is grasp if the-arbitration
rationis abscedat. Ipse enim sensus aeque maximis minimisque
of-reason withdraw the-very for sense equally by-greatest and-least-(things)
corrumpitur. Nam neque minima sentire propter ipsorum
is-corrupted for neither the-least to-perceive because-of of-the-very
sensibilium parvitatem potest, et majoribus saepe confunditur.
sensibles the-smallness it-can and by-the-greater often is-confounded
Ut in vocibus, quae si minimae sint, difficilius captat
as in voices which if least be with-more-difficulty catches
auditus: si sint maximae, ipsius sonitus intentione surdescit.
the-hearing if they-be greatest of-the-very sounding by-the-intensity it-grows-deaf

Quemadmodum Pythagoras proportiones consonantiarum investigaverit

I.10 · PYTHAGORAS AT THE FORGE

Pythagoras, trusting neither ears nor instruments, searches for a firm way to learn the measures of the consonances. Passing a smiths' workshop, he hears the struck hammers sound a concord and finds that the property follows the hammers' weights, not the men's arms; one hammer, inconsonant with all, is rejected. With weights of 12, 9, 8, and 6, the 12 and 6 sound the diapason, 12 to 9 and 8 to 6 the diatessaron, 9 to 6 and 12 to 8 the diapente, and 9 to 8 the tone.

Haec igitur maxima causa fuit cur, relicto aurium
this therefore the-greatest cause was why having-been-abandoned of-the-ears
judicio, Pythagoras ad regularum momenta migraverit, qui
the-judgment Pythagoras to of-rules the-measures passed-over who
nullis humanis auribus credens, quae partim natura, partim
no human ears trusting which partly by-nature partly
etiam extrinsecus accidentibus permutantur, partim ipsis
also from-without by-accidents are-changed partly by-the-very
variantur aetatibus, nullis etiam deditus instrumentis, penes
are-varied ages to-no also given-over instruments in-the-power-of
quae saepe multa varietas atque inconstantia nasceretur, dum
which often much variety and inconstancy would-be-born while
nunc quidem si nervos velis aspicere, vel aer humidior
now indeed if strings you-should-wish to-look-at either the-air more-humid
pulsus obtunderet, vel siccior exsiccaret, vel magnitudo
the-pulses would-blunt or drier would-dry-(them) or the-bigness
chordae graviorem redderet sonum, vel acumen subtilior
of-the-string a-lower would-render sound or the-highness a-finer-(string)
tenuaret, vel alio quodam modo statum prioris
would-draw-fine or in-another certain way the-state of-the-former
constantiae permutaret. Et cum idem esset in caeteris
constancy would-change and since the-same was in the-other

instrumentis, omnia haec inconsulta minimaeque aestimans
instruments all these unreliable and-of-least judging
fidei, diuque aestuans inquirebat quanam ratione firmiter
trust and-long seething he-searched by-what-possible method firmly
et constanter consonantiarum momenta perdisceret. Cum
and constantly of-consonances the-measures he-might-learn-through when
interea divino quodam motu praeteriens fabrorum officinas,
meanwhile by-a-divine certain impulse passing-by of-smiths the-workshops
pulsos malleos exaudivit, ex diversis sonis unam quodammodo
struck hammers he-heard-clearly from diverse sounds one in-a-certain-way
concinentiam personare. Ita igitur ad id quod diu inquirebat
concord to-sound-forth so therefore at that which long he-was-searching
attonitus, accessit ad opus, diuque considerans arbitratus est
astonished he-came-up to the-work and-long considering judged he-has
diversitatem sonorum ferientium vires efficere, atque ut
the-diversity of-the-sounds of-(the-men)-striking the-strengths to-produce and that
id apertius colliqueret, mutare inter se malleos imperavit.
it more-openly might-grow-clear to-change among themselves the-hammers he-commanded
Sed sonorum proprietas non in hominum lacertis haerebat,
but of-the-sounds the-property not in of-the-men the-arms was-sticking
sed mutatos malleos comitabatur. Ubi igitur id animadvertit,
but the-changed hammers it-followed when therefore this he-observed
malleorum pondus examinat. Et cum quinque essent forte
of-the-hammers the-weight he-weighs and since five they-were by-chance
mallei, dupli reperti sunt pondere qui sibi secundum
hammers double found were in-weight which to-themselves according-to
diapason consonantiam respondebant. Eumdem etiam qui
the-diapason consonance answered the-same-(hammer) also which
duplus esset alio, sesquitertium alterius comprehendit, ad
double was than-one sesquitertian of-another he-grasped to
quem scilicet diatessaron sonabat. Ad alium vero quemdam,
which namely the-diatessaron it-sounded to another but certain-(hammer)

qui eidem diapente consonantia jungebatur, eumdem
which to-the-same by-the-diapente consonance was-joined the-same
superioris duplum reperit esse sesquialterum. Duo vero hi,
of-the-former the-double he-found to-be sesquialter two but these
ad quos superior duplex sesquitertius et sesquialter esse
to which the-former double sesquitertian and sesquialter to-be
probatus est, ad se invicem sesquioctavam proportionem
proved was to themselves mutually the-sesquioctave proportion
perpensi sunt custodire. Quintus vero est rejectus, qui cunctis
weighed-out were to-keep the-fifth but was rejected which with-all
erat inconsonans. Cum igitur ante Pythagoram consonantiae
was inconsonant since therefore before Pythagoras the-consonances
musicae, partim diapason, partim diapente, partim diatessaron,
of-music partly diapason partly diapente partly diatessaron
quae est consonantia minima, vocarentur, primus Pythagoras
which is the-consonance least were-called first Pythagoras
hoc modo reperit, qua proportione sibimet haec sonorum
in-this way found by-what proportion to-itself this of-sounds
concordia jungeretur. Et ut sit clarius quod dictum est,
concord was-joined and that may-be clearer what said has-been
sint, verbi gratia, malleorum quatuor pondera, quae
let-there-be of-a-word for-the-sake of-hammers four weights which
subterscriptis numeris contineantur, 12, 9, 8, 6. Hi igitur
by-the-underwritten numbers be-contained 12 9 8 6 these therefore
mallei, qui 12 et 6 ponderibus vergebant, diapason in
hammers which with-12 and 6 weights weighed-down the-diapason in
duplo concinentiam personabant. Malleus vero 12 ponderum
the-double concord sounded-forth the-hammer but of-12 weights
ad malleum 9, et malleus 8 ponderum ad malleum
to the-hammer of-9 and the-hammer of-8 weights to the-hammer
6 ponderum, secundum epitritam proportionem diatessaron
of-6 weights according-to the-epitritic proportion by-the-diatessaron

consonantia jungebatur. Novem vero ponderum ad 6, et 12
consonance was-joined the-nine but of-weights to 6 and 12
ad 8 diapente consonantiam permiscebant. Novem vero ad
to 8 the-diapente consonance mingled-together the-nine but to
8, in sesquioctava proportione resonabant tonum.
8 in the-sesquioctave proportion resounded a-tone

Quibusdam modis varie a Pythagora proportiones consonantiarum perpensae sint

I.11 · WEIGHING THE PROPORTIONS

Pythagoras returns home and tests whether the whole reckoning of the symphonies rests in these proportions — fitting equal weights to strings, halving and doubling the lengths of reeds, dropping ladles of equal weight into vessels, striking vessels with a rod of bronze or iron, and weighing the length and thickness of strings. He is glad to find nothing come out differently. From this he finds the rule, so called because its inspection is fixed and firm and deceives no inquirer.

Hinc igitur domum reversus, varia examinatione perpendit
hence therefore home having-returned by-varied weighing he-pondered
an in his proportionibus ratio symphoniarum tota
whether in these proportions the-reckoning of-symphonies whole
consisteret. Nunc quidem aequa pondera nervis aptans,
might-consist now indeed equal weights to-strings fitting
eorumque consonantias aure dijudicans; nunc vero in
and-their consonances by-ear judging-between now but in
longitudine calamorum duplicitatem medietatemque restituens,
the-length of-reeds doubleness and-halfness restoring
caeterasque proportiones aptans, integerrimam fidem diversa
and-the-other proportions fitting most-whole assurance by-diverse
experientia capiebat. Saepe et pro mensurarum
experience he-was-taking often also according-to of-measures

modo cyathos aequorum ponderum acetabulis immittens;
the-manner ladles of-equal weights into-vessels dropping-in
saepe ipsa quoque acetabula diversis formata ponderibus
often the-very also vessels with-diverse formed weights
virga, vel aerea, ferreave percutiens, nihil esse diversum
with-a-rod either brazen or-iron striking nothing to-be different
invenisse laetatus est. Hinc etiam ductus, longitudinem
to-have-found glad he-was hence also led the-length
crassitudinemque chordarum ut examinaret aggressus est.
and-thickness of-strings that he-might-weigh set-about he-has
Itaque invenit regulam, de qua posterius loquemur, quae
and-so he-found the-rule about which later we-shall-speak which
ex re vocabulum sumpsit, non quod regula sit lignea,
from the-thing the-name took not because the-rule may-be wooden
per quam magnitudines chordarum sonumque metimur,
through which the-magnitudes of-strings and-the-sound we-measure
sed quod regula quaedam sit hujusmodi inspectio fixa
but because a-rule certain may-be of-this-kind an-inspection fixed
firmaque, ut nullum inquirentem dubio fallat judicio.
and-firm that no inquirer with-doubtful it-may-deceive judgment

De divisione vocum earumque explanatione

I.12 · THE DIVISION OF THE VOICE

Boethius divides the voice. The continuous voice is the one we speak and read prose with: it hastens through the words rather than dwelling on high and low sounds. The diastematic voice is the one we suspend in singing, slower, serving measures rather than speech. Albinus adds a third, middle kind for reading heroic poems, which runs neither like prose nor like song.

Sed de his hactenus; nunc vocum differentias colligamus.
but about these thus-far now of-voices the-differences let-us-gather
Omnis enim vox aut συνεχής est, quae continua, aut
every for voice either continuous is which (is)-continuous or
διαστηματική, quae dicitur cum intervallo suspensa. Et
diastematic which is-called with an-interval suspended and
continua quidem est, qua loquentes vel prosam
the-continuous indeed is (that)-by-which speaking or prose
orationem legentes verba percurrimus. Festinat enim tunc
discourse reading the-words we-run-through hastens for then
vox non inhaerere in acutis et gravibus sonis, sed
the-voice not to-cling on high and low sounds but
quam velocissime verba percurrere, expediendisque sensibus,
as most-swiftly the-words to-run-through and-for-unfolding the-meanings
exprimendisque sermonibus continuae vocis impetus operatur.
and-for-expressing the-speeches of-the-continuous voice the-onrush works
Diastematice autem est ea quam canendo suspendimus, in
the-diastematic moreover is that which in-singing we-suspend in
qua non potius sermonibus, sed modulis inservimus. Estque
which not rather to-speeches but to-measures we-serve and-is
vox ipsa tardior, et per modulandas varietates quoddam
the-voice itself slower and through to-be-modulated varieties a-certain

faciens intervallum, non taciturnitatis, sed suspensae ac tardae
making interval not of-silence but of-suspended and slow
potius cantilenae. His (ut Albinus autumat) additur tertia
rather song to-these as Albinus avers is-added a-third
differentia, quae medias voces possit includere, cum scilicet
difference which the-middle voices may-be-able to-include when namely
heroum poemata legimus, neque continuo cursu, ut prosam,
of-heroes the-poems we-read neither with-continuous course as prose
neque suspenso segniorique modo vocis, ut canticum.
nor with-suspended and-more-sluggish manner of-voice as song

Quod infinitatem vocum humana natura finierit

I.13 · NATURE LIMITS THE INFINITE VOICE

Both the continuous voice and the singing voice are in themselves unbounded: nothing in their nature limits the unrolling of speech or the raising and lowering of pitch. Human nature sets each its own end. Breath bounds the continuous voice, for each man speaks only as long as his natural breath allows; the compass of his voice bounds the diastematic, fixing how high and how low he can go.

Sed quae continua vox est, et ea rursus qua decurrimus
but (that)-which the-continuous voice is and that again by-which we-run-through
cantilenam, naturaliter quidem infinitae sunt. Consideratione
a-song naturally indeed unbounded they-are consideration
enim accepta, nullus modus vel evolvendis sermonibus fit,
for being-taken no limit either for-unrolling speeches comes-to-be
vel acuminibus attollendis gravitatibusque laxandis, sed
or for-heights for-raising and-for-depths for-letting-down but
utrisque natura humana fecit proprium finem. Continuae
for-both the-nature human has-made its-own end for-the-continuous
enim voci terminum humanus spiritus fecit, ultra quem
for voice a-bound the-human breath has-made beyond which

nulla ratione valet excedere. Tantum enim unusquisque
by-no means it-avails to-pass so-much for each-one
loquitur continue, quantum naturalis spiritus sinat. Rursus
speaks continuously as-much-as the-natural breath may-allow again
diastematice voci natura hominum facit terminum, quae
for-the-diastematic voice the-nature of-men makes a-bound which
acutam eorum vocem gravemque determinat. Tantum enim
the-high their voice and-low marks-off so-much for
unusquisque vel acumen valet extollere, vel deprimere
each-one either height avails to-raise or to-sink
gravitatem, quantum vocis ejus naturaliter patitur modus.
lowness as-much-as of-voice his naturally allows the-measure

Quis modus sit audiendi

I.14 · HOW HEARING WORKS

Boethius compares hearing to a stone cast into quiet water: the wave spreads in growing circles, each wavelet weaker than the last, until the wearied motion rests — and if anything blocks the waves, the motion turns back toward its center. So struck air makes a sound, drives the air next to it, and stirs a round billow that strikes the hearing of all who stand around. Whoever stands farther off hears a dimmer voice, because a weaker wave reaches him.

Nunc quis modus audiendi sit disseramus. Tale enim
now what the-manner of-hearing may-be let-us-discuss such for
quiddam fieri consuevit in vocibus, quale cum paludibus vel
a-something to-happen has-been-wont in voices as when in-marshes or
quietis aquis jactum eminus mergitur saxum. Prius enim in
quiet waters cast from-afar is-plunged a-stone first for into
parvissimum orbem undam colligit, deinde majoribus orbibus
a-very-small circle the-wave it-gathers then with-greater circles

undarum globos spargit, atque eo usque dum fatigatus motus
of-waves the-rounds it-scatters and so far until wearied the-motion
ab eliciendis fluctibus conquiescat. Semperque posterior et
from calling-forth billows may-rest and-always the-later and
major undula pulsu debiliore diffunditur. Quod si quid
greater wavelet with-impulse weaker is-spread-abroad but if something
sit, quod crescentes undas possit offendere, statim motus
there-be which the-growing waves may-be-able to-dash-against at-once motion
ille revertitur, et quasi ad centrum, unde profectus fuerat,
that turns-back and as-it-were to the-center whence set-out it-had
eisdem undulis rotundatur. Ita igitur cum aer pulsus
with-the-same wavelets it-is-rounded so therefore when air struck
fecerit sonum, pellit alium proximum, et quodammodo
shall-have-made a-sound it-drives another next and in-a-certain-way
rotundum fluctum aeris ciet. Itaque diffunditur et omnium
a-round billow of-air it-stirs and-so it-is-spread-abroad and of-all
circumstantium simul ferit auditum, atque illi est obscurior
standing-around at-once it-strikes the-hearing and to-him is dimmer
vox, qui longius steterit, quoniam ad eum debilior
the-voice who farther shall-have-stood since to him weaker
pulsi aeris unda pervenit.
of-the-struck air the-wave arrives

De ordine theorematum, id est speculationum

I.15 · THE ORDER OF STUDY

Boethius sets the order of what follows. Every song is woven in one of three genera — diatonic, chromatic, enharmonic — but these can be unfolded only after the tetrachords are discussed, and after telling how the number of strings grew to its present multitude. That in turn will come about only by first recalling by what proportions the musical symphonies are blended.

His igitur ita propositis dicendum videtur quot
these therefore thus set-forth to-be-said it-seems with-how-many
generibus omnis cantilena texatur, de quibus harmonicae
genera every song may-be-woven about which of-the-harmonic
inventionis disciplina considerat. Sunt autem haec diatonicum,
discovery the-discipline considers they-are moreover these the-diatonic
chromaticum, enharmonicum. De quibus ita demum
the-chromatic the-enharmonic about which so at-length
explicandum est, si prius de tetrachordis disseramus, et
to-be-unfolded it-is if first about tetrachords we-discuss and
quemadmodum auctus nervorum numerus, ad id quo
how increased of-strings the-number to that at-which
nunc pluralitas est, usque pervenerit. Id autem fiet,
now the-multitude is all-the-way it-has-arrived this moreover will-come-about
si prius commemoremus quibus proportionibus symphoniae
if first we-recall by-what proportions the-symphonies
musicae misceantur.
musical may-be-blended

De consonantiis proportionum, et tono, et semitonio

I.16 · CONSONANCES, TONE, AND SEMITONE

Boethius gives the consonances in numbers: diapason 1:2, diapente 2:3, diatessaron 3:4, and the tone 8:9. Diatessaron and diapente joined make one diapason, as 2, 3, 4. The tone cannot be halved: doubled to 16 and 18, it admits 17 between, and 17 against 16 adds a sixteenth part where 18 against 17 adds a seventeenth. Both parts are called semitones, not because they are equal halves, but because "semum" names what falls short of the whole. One is the greater semitone, the other the lesser.

Diapason consonantia est quae fit in duplo, ut haec
the-diapason consonance is which comes-to-be in the-double as this
est, 1, 2; diapente vero quae constat his numeris, 2, 3;
is 1 2 the-diapente but which consists of-these numbers 2 3
diatessaron vero est quae in hac proportione consistit 3, 4;
the-diatessaron but is which in this proportion stands 3 4
tonus vero sesquioctava proportione concluditur, sed in hoc
the-tone but by-the-sesquioctave proportion is-bounded but in this
nondum est consonantia, ut 8, 9; diapason vero et diapente
not-yet is consonance as 8 9 the-diapason but and diapente
tripla comparatione colligitur hoc modo, 2, 4, 6; bisdiapason
by-triple comparison is-gathered in-this manner 2 4 6 the-bisdiapason
quadrupla collatione perficitur, 2, 4, 8; diatessaron vero ac
by-quadruple collation is-completed 2 4 8 the-diatessaron but and
diapente unum perficiunt diapason hoc modo, 2, 3, 4.
the-diapente one complete diapason in-this manner 2 3 4
Nam si vox voci duplo sit acuta vel gravis, diapason
for if a-voice to-a-voice by-the-double be high or low the-diapason
consonantia fiet. Si vox voci sesquialtera proportione
consonance will-come-to-be if a-voice to-a-voice by-sesquialter proportion
sit, vel sesquitertia, vel sesquioctava, acutior graviorque diapente,
be or by-sesquitertian or by-sesquioctave higher and-lower the-diapente

vel diatessaron, vel tonum consonantiam reddet. Item si
or diatessaron or the-tone consonance it-will-render likewise if
diapason, ut 2 et 4, et diapente, ut 4 et 6, conjungantur,
the-diapason as 2 and 4 and the-diapente as 4 and 6 be-joined
triplam, et quae est diapason et diapente, efficiant
the-triple and which is diapason and diapente they-may-effect
symphoniam. Quod si bis diapason fiant, ut duo ad
symphony but if twice diapason they-be-made as two to
quatuor, et quatuor ad octo, quadrupla fiet consonantia,
four and four to eight quadruple will-come-to-be the-consonance
quae est bisdiapason. Quod si sesquialtera et sesquitertia, id
which is bisdiapason but if the-sesquialter and the-sesquitertian that
est diapente et diatessaron, ut duo ad tres, et tres ad quatuor
is diapente and diatessaron as two to three and three to four
conjungantur, dupla id est, diapason nimirum nascitur
be-joined the-double that is diapason assuredly is-born
concinentia. Quatuor enim ad tres sesquitertiam obtinent
concord four for to three the-sesquitertian hold
proportionem. Tres vero ad binarium sesquialtera collatione
proportion three but to the-two by-sesquialter collation
junguntur, et idem quaternarius ad binarium appositus
are-joined and the-same four to the-two set-beside
dupla ei comparatione copulatur. Sed sesquitertia diatessaron,
by-double to-it comparison is-coupled but the-sesquitertian the-diatessaron
sesquialtera proportio diapente consonantiam creant.
the-sesquialter proportion the-diapente consonance they-create
Dupla vero diapason efficit symphoniam. Diatessaron igitur
the-double but the-diapason effects symphony the-diatessaron therefore
ac diapente unam diapason consonantiam jungunt. Rursus
and the-diapente one diapason consonance join again
tonus in aequa dividi non potest, cur autem posterius
the-tone into equal-(parts) to-be-divided not is-able why moreover later

liquebit. Nunc hoc tantum nosse sufficiat, quod nunquam
will-be-clear now this only to-know let-it-suffice that never
tonus in gemina aequa dividitur. Atque ut id facillime
a-tone into twin equal-(parts) is-divided and that this most-easily
comprobetur, sit sesquioctava proportio 8 et 9. Horum
may-be-proved let-there-be a-sesquioctave proportion 8 and 9 of-these
nullus naturaliter medius numerus incidet. Hos igitur si
no naturally middle number will-fall these therefore if
binario multiplicemus, fiuntque bis 8 16, 9 18, inter 16
by-two we-multiply and-they-become twice 8 16 9 18 between 16
autem et 18 unus numerus naturaliter incidit qui est scilicet
moreover and 18 one number naturally falls which is namely
17. Qui disponantur in ordinem 16, 17, 18. Igitur 16 ac
17 these let-them-be-set in order 16 17 18 therefore 16 and
18 collati, sesquioctavam retinent proportionem, atque idcirco
18 compared the-sesquioctave retain proportion and therefore
tonum. Sed hanc proportionem 17 numerus medius non in
a-tone but this proportion 17 the-number middle not into
aequalia partitur. Comparatus enim ad 16, habet in se totum
equal-(parts) divides compared for to 16 it-has in itself the-whole
16, et ejus sextam decimam partem, scilicet unitatem. Si vero
16 and of-it the-sixth (and-)tenth part namely the-unit if but
ad eum, id est 17, tertius, id est 18, numerus comparetur,
to it that is 17 the-third that is 18 number be-compared
habet eum totum, et ejus decimam septimam partem. Non
it-has it whole and of-it the-tenth (and-)seventh part not
igitur iisdem partibus et minorem superat, et a majore
therefore by-the-same parts both the-lesser it-surpasses and by the-greater
superatur. Est enim minor pars septima decima, major sexta
it-is-surpassed is for the-lesser part the-seventh (and-)tenth the-greater the-sixth
decima. Sed utraque semitonia nuncupantur. Non quod omnino
(and-)tenth but both semitones are-named not because altogether

semitonia ex aequo sint media, sed quod semum dici
the-semitones from the-equal they-be halves but because diminished to-be-called
solet, quod ad integritatem usque non pervenit. Sed inter
is-wont (that)-which to wholeness all-the-way not arrives but between
haec unum majus semitonium nuncupatur, aliud minus.
these one the-greater semitone is-named the-other the-lesser

In quibus primis numeris semitonium constet

I.17 · THE SEMITONE'S FIRST NUMBERS

Boethius shows in what first numbers the semitone stands. He sets out 192, 216, 243, 256: the outer terms make the sesquitertian diatessaron, and 216 against 192 and 243 against 216 are each a sesquioctave tone. What remains, 256 against 243, differs by 13, and 13 taken eight times does not fill half of 243 — so the remainder is less than a semitone. The true lesser semitone is the comparison of 243 to 256.

Quod vero integrum non sit semitonium, aut in quibus
(the-fact)-that but whole not may-be a-semitone or in what
primis numeris constet, nunc evidentius explicabo. Id enim
first numbers it-may-consist now more-evidently I-will-unfold that for
quod de divisione toni dictum est, non ad hoc pertinet,
which about the-division of-the-tone said has-been not to this pertains
ut semitoniorum modos voluerimus ostendere. Sed ad id
that of-semitones the-measures we-should-have-wished to-show but to that
potius quod tonum in gemina aequa diceremus non posse
rather that the-tone into twin equal-(parts) we-were-saying not to-be-able
disjungi. Diatessaron, quae est consonantia, vocum quidem est
to-be-parted the-diatessaron which is a-consonance of-voices indeed is
quatuor, intervallorum trium. Constat autem ex duobus tonis,
four of-intervals of-three it-consists moreover of two tones
et non integro semitonio. Sit enim subjecta descriptio
and not a-whole semitone let-there-be for set-beneath a-diagram

192, 216, 243, 256, Si igitur 192 numerus, 256 comparetur,
192 216 243 256 if therefore 192 the-number to-256 be-compared
sesquitertia proportio fiet, ac diatessaron concinentiam
a-sesquitertian proportion will-come-to-be and the-diatessaron concord
resonabit. Sed si 216 ad 192 comparemus, sesquioctava
it-will-resound but if 216 to 192 we-compare a-sesquioctave
proportio est; est enim eorum differentia 24, quae est octava
proportion it-is is for of-them the-difference 24 which is the-eighth
pars 192. Est igitur tonus. Rursus si 243 ad 216 comparetur,
part of-192 it-is therefore a-tone again if 243 to 216 be-compared
erit altera sesquioctava proportio. Nam eorum differentia
there-will-be a-second sesquioctave proportion for of-them the-difference
27, pars 216, probatur octava. Restat comparatio 256 ad
27 part of-216 is-proved the-eighth there-remains the-comparison of-256 to
243, quorum differentia est 13. Qui octies facti, medietatem
243 of-which the-difference is 13 which eight-times made the-half
243 non videntur implere. Non est igitur semitonium, sed
of-243 not are-seen to-fill not it-is therefore a-semitone but
minus a semitonio. Tunc enim integrum esse semitonium
less than a-semitone then for whole to-be a-semitone
jure putaretur, si eorum differentia, quae est 13 facta
rightly it-would-be-thought if of-them the-difference which is 13 made
octies, medietatem 243 numerorum potuisset aequare.
eight-times the-half of-243 of-the-numbers had-been-able to-equal
Estque verum semitonium minus 243 ad 256 comparatio.
and-is the-true semitone lesser of-243 to 256 the-comparison

Diatessaron a diapente tono distare

I.18 · DIATESSARON AND DIAPENTE DIFFER BY A TONE

The diapente holds three tones and the lesser semitone. Boethius takes 192 and its sesquialter 288, setting 216, 243, and 256 between them: from 192 to 256 lie two tones and the semitone, and 256 to 288 is a sesquioctave, that is a tone. Since the diatessaron ran from 192 to 256 and the diapente runs from the same 192 to 288, the diapente overpasses the diatessaron by exactly a tone.

Rursus diapente consonantia, vocum quidem est quinque
again the-diapente consonance of-voices indeed is five
intervallorum quatuor, trium tonorum, et minore semitonio.
of-intervals four of-three tones and a-lesser semitone
Ponatur enim idem numerus 192, et ejus sesquialter sumatur,
let-be-set for the-same number 192 and of-it the-sesquialter let-be-taken
qui ad eum diapente faciat consonantiam. Sit igitur
which to it the-diapente may-make consonance let-it-be therefore
numerus 288. Igitur horum et superius deprehensorum 192,
the-number 288 therefore of-these and above discovered 192
ponantur in medio numeri hi 216, 243, 256, et sit hoc
let-be-set in the-middle numbers these 216 243 256 and let-be in-this
modo formata descriptio, 192, 216, 243, 256, 288. In superiore
manner formed the-diagram 192 216 243 256 288 in the-upper
igitur descriptione, 192 et 256 duos tonos et semitonium
therefore diagram (between)-192 and 256 two tones and a-semitone
contineri monstrati sunt. Restat igitur comparatio 256
to-be-contained shown they-have-been there-remains therefore the-comparison of-256
ad 288, quae est sesquioctava, id est tonus, eorumque differentia
to 288 which is sesquioctave that is a-tone and-of-them the-difference
est 32, quae est octava pars 256. Itaque monstrata est
is 32 which is the-eighth part of-256 and-so shown it-has-been

diapente consonantia ex tribus tonis semitonioque constare.
the-diapente consonance of three tones and-a-semitone to-consist
Sed dudum diatessaron consonantia a centum nonaginta
but just-now the-diatessaron consonance from a-hundred ninety
duobus numeris usque ad 256 venerat. Nunc vero diapente
two numbers all-the-way to 256 had-come now but the-diapente
ab eisdem 192 numeris usque ad 288 distenditur. Superatur
from the-same 192 numbers all-the-way to 288 is-stretched is-surpassed
igitur diatessaron consonantia a diapente, ea proportione
therefore the-diatessaron consonance by the-diapente by-that proportion
quae inter 256 et 288 numeros continetur, et est hic tonus.
which between 256 and 288 the-numbers is-contained and is this a-tone
Diatessaron igitur symphonia a diapente tono transcenditur.
the-diatessaron therefore symphony by the-diapente by-a-tone is-overpassed

Diapason quinque tonis et duobus semitoniis jungi

I.19 · THE DIAPASON'S FIVE TONES AND TWO SEMITONES

The diapason consists of five tones and two semitones that together fall short of a whole tone. Its parts, diapente and diatessaron, bring three tones with a semitone and two tones with a semitone; the two lesser semitones joined pass half a tone but never reach a whole one. Boethius asks moderate credence for now, promising each claim its own demonstration, and turns next to the cithara's strings, their names, and how they were added.

Diapason consonantia constat ex quinque tonis et duobus
the-diapason consonance consists of five tones and two
semitoniis, quae tamen unum non impleant tonum. Quoniam
semitones which however one not fill-up tone since
enim monstratum est diapason ex diapente et diatessaron
for shown it-has-been the-diapason of diapente and diatessaron

consistere, diatessaron vero probata est ex duobus tonis
to-consist the-diatessaron but proved has-been of two tones
et semitonio constare; diapente ex tribus tonis ac semitonio
and a-semitone to-consist the-diapente of three tones and a-semitone
simul juncta efficiunt quinque tonos. Sed quoniam illa duo
together joined they-effect five tones but since those two
semitonia non erant integrae medietatis, eorum conjunctio ad
semitones not were of-a-whole half of-them the-conjunction to
plenum usque non pervenit. Sed medietatem quidem superat,
the-full all-the-way not arrives but the-half indeed it-surpasses
ab integritate relinquitur. Estque diapason secundum hanc
from wholeness it-is-left-behind and-is the-diapason according-to this
rationem ex quinque tonis et duobus semitoniis. Quae sicut ad
reckoning of five tones and two semitones which just-as to
integrum tonum non aspirant, ita ultra integrum semitonium
a-whole tone not aspire so beyond a-whole semitone
prodeunt. Sed quae horum sit ratio, vel quemadmodum
they-go-forth but what of-these may-be the-reckoning or how
ipsae musicae consonantiae reperiantur, postea liquidius
the-very musical consonances may-be-found afterward more-clearly
explanabitur. Interea praesenti disputationi sub mediocri
will-be-explained meanwhile to-the-present discussion under moderate
intelligentia credulitas adhibenda est: tunc vero firma omnis
understanding credence to-be-applied is then but firm all
fides sumenda est, cum propria unum quodque demonstratione
faith to-be-taken is when by-its-own one each demonstration
claruerit. His igitur ita dispositis paulisper de cytharae
shall-have-grown-clear these therefore thus arranged for-a-little about of-the-cithara
nervis, ac de eorum nominibus, quoque modo sunt
the-strings and about their names and-in-what manner they-have-been
additi, disseramus, quaque eorum causa sit nominum. His
added let-us-discuss and-what their cause may-be of-names these

enim primum ad notitiam venientibus facile erit scientia,
for first to acquaintance coming easy it-will-be by-knowledge
quae sequuntur, amplecti.
(the-things)-which follow to-embrace

De additione chordarum earumque nominibus

I.20 · THE STRINGS AND THEIR NAMES

Boethius gives Nicomachus' history of the strings: music began simple, on four strings, and so lasted until Orpheus; Mercury is called the quadrichord's inventor. Choroebus added a fifth string, Hyagnis a sixth, Terpander a seventh after the likeness of the seven planets, and later hands raised the count to eleven and beyond. He explains each string's name, from hypate to nete, and ends with the five tetrachords and the added proslambanomenos, an octave below mese.

Simplicem principio fuisse musicam refert Nicomachus,
simple in-the-beginning to-have-been (that)-music relates Nicomachus
adeo ut quatuor nervis tota constaret. Idque usque ad
so-far that on-four strings whole it-consisted and-this up to
Orpheum duravit, ut primus quidem nervus et quartus
Orpheus it-lasted so-that the-first indeed string and the-fourth
diapason consonantiam resonarent. Medii vero ad
the-diapason consonance might-resound the-middle-(strings) but to
se invicem atque ad extremos diapente, ac diatessaron, ac
themselves mutually and to the-outermost diapente and diatessaron and
tonum. Nihil vero in eis esset inconsonum, ad imitationem
a-tone nothing but in them might-be unconsonant in imitation
scilicet musicae mundanae, quae ex quatuor constat elementis.
namely of-the-music of-the-world which of four consists elements
Cujus quadrichordi Mercurius dicitur inventor. Quintam vero
of-which quadrichord Mercury is-said the-inventor a-fifth but

chordam post Chorebus Athylis filius adjunxit, qui fuit
string afterward Choroebus of-Athys son joined-on who was
Lydorum rex. Hyagnis vero Phryx sextum his apposuit
of-the-Lydians king Hyagnis but the-Phrygian a-sixth to-these set-on
nervum. Sed septimus nervus a Terpandro Lesbio adjunctus
string but the-seventh string by Terpander the-Lesbian joined-on
est, secundum septem scilicet planetarum similitudinem.
was according-to seven namely of-planets the-likeness
Inque his quae gravissima quidem erat, vocata est
and-among these which lowest indeed was called was
hypate, quasi major atque honorabilior. Unde Jovem etiam
hypate as-it-were greater and more-honorable whence Jupiter also
Hypaton vocant. Consulem eodem quoque nuncupant nomine
Hypatos they-call Consul by-the-same also they-name name
propter excellentiam dignitatis, eaque Saturno est attributa
on-account-of the-excellence of-dignity and-it to-Saturn has-been assigned
propter tarditatem motus et gravitatem soni. Parhypate
on-account-of the-slowness of-motion and the-lowness of-sound parhypate
vero secunda, quasi juxta hypaten posita et collocata.
but the-second as-it-were beside hypate placed and stationed
Lichanos tertia idcirco, quoniam lichanos digitus dicitur,
lichanos the-third for-this-reason since lichanos the-finger is-called
quem nos indicem vocamus. Graecus a lingendo lichanon
which we the-index call the-Greek from licking lichanos
appellat. Et quoniam in canendo ad eam chordam, quae
names and since in playing at that string which
erat tertia ab hypate, index digitus, qui est lichanos
was third from hypate the-index finger which is lichanos
inveniebatur, idcirco ipsa quoque lichanos appellata est.
was-found for-this-reason itself also lichanos named was
Quarta dicitur mese, quoniam inter septem semper est
the-fourth is-called mese since among the-seven always it-is

media. Quinta est paramese, quasi juxta mediam collocata.
middle the-fifth is paramese as-it-were beside the-middle stationed
Septima autem dicitur nete, quasi neate, id est inferior.
the-seventh moreover is-called nete as-it-were neate that is lower
Inter quam neten et paramesen est sexta, quae vocatur
between which nete and paramese is the-sixth which is-called
paranete, quasi juxta neten locata. Paramese vero quoniam
paranete as-it-were beside nete placed paramese but since
tertia est a nete, eodem quoque vocabulo trite, id est
third it-is from nete by-the-same also word trite that is
tertia nuncupatur, ut sit descriptio haec. His octavam
the-third it-is-named so-that may-be the-diagram this to-these an-eighth
Samius Lichaon adjunxit, atque inter paramesen, quae etiam
the-Samian Lichaon joined-on and between paramese which also
trite dicitur, et paraneten nervum medium coaptavit, ut
trite is-called and paranete a-string middle he-fitted so-that
ipse tertius esset a nete. Et paramese quidem vocata
itself third it-might-be from nete and paramese indeed called
est eo solum quod post mediam collocabatur.
was for-this only (reason)-that after the-middle it-was-stationed
Trites vero nomen perdidit, posteaquam inter eam atque
of-trite but the-name it-lost after-that between it and
paraneten tertius a nete locatus est nervus, qui
paranete a-third from nete placed was string which
digne trites nomen exciperet, ut sit octochordum
worthily of-trite the-name might-take-up so-that there-may-be an-octochord
secundum Lichaonis additionem, hoc modo: In superioribus
according-to of-Lichaon the-addition in-this manner in the-above
igitur duabus dispositionibus heptachordi et octochordi.
therefore two arrangements of-the-heptachord and of-the-octochord
Heptachordum quidem dicitur synemmenon, quod est
the-heptachord indeed is-called synemmenon which is

conjunctum. Octochordum vero diezeugmenon, quod est
conjoined the-octochord but diezeugmenon which is
disjunctum. In heptachordo enim est unum tetrachordum,
disjoined in the-heptachord for there-is one tetrachord
hypate, parhypate, lichanos, mese. Aliud vero mese, paramese,
hypate parhypate lichanos mese another but mese paramese
paranete, nete, dum mesen nervum secundo numeramus,
paranete nete while mese the-string a-second-time we-count
atque ideo duo tetrachorda per mesen conjunguntur.
and therefore the-two tetrachords through mese are-conjoined
In octochordo vero quoniam octo sunt chordae, superiores
in the-octochord but since eight there-are strings the-upper
quatuor, id est hypate, parhypate, lichanos, mese, unum
four that is hypate parhypate lichanos mese one
tetrachordum explent. Ab hoc vero disjunctum atque
tetrachord fill-out from this but the-disjoined and
integrum inchoat a paramese, progrediturque per triten
whole begins from paramese and-proceeds through trite
et paraneten, et est finitum ad neten, et est disjunctio,
and paranete and is bounded at nete and there-is a-disjunction
quae vocatur diezeuxis. Tonusque est distantia meses et
which is-called diezeuxis and-a-tone is the-distance of-mese and
parameses. Hic igitur mese tantum quidem nomen obtinuit.
of-paramese here therefore mese only indeed the-name has-kept
Non enim est media positione, quia in octochordo duae
not for it-is middle in-position because in an-octochord two
quidem semper mediae reperiuntur, sed una media non
indeed always middle-(strings) are-found but one middle not
potest inveniri. Theophrastus autem Pierites ad graviorem
is-able to-be-found Theophrastus moreover the-Pierian on the-lower
partem unam addidit chordam, ut faceret totum
side one added string so-that he-might-make a-whole

enneachordum. Quae quoniam super hypaten est addita,
enneachord which since above hypate has-been added
hyperhypate est nuncupata. Quae prius quidem dum novem
hyperhypate has-been named which formerly indeed while of-nine
chordarum tantum esset cythara, hyperhypate vocabatur.
strings only was the-cithara hyperhypate was-called
Nunc autem lichanos hypaton dicitur aliis superadditis. In
now moreover lichanos hypaton it-is-called others being-added-above in
quo ordine atque instructione, quoniam ad indicem digitum
which order and array since to the-index finger
venit, lichanos appellata est. Sed hoc posterius apparebit.
it-comes lichanos named it-has-been but this later will-appear
Nunc vero enneachordi ordo sic se habet.
now but of-the-enneachord the-order thus itself holds
Hestiaeus Colophonius decimam in graviorem partem coaptavit
Histiaeus the-Colophonian a-tenth on the-lower side fitted
chordam, Timotheus vero Milesius undecimam. Quae quoniam
string Timotheus but the-Milesian an-eleventh which since
super hypaten atque parhypaten sunt additae, hypate quidem
above hypate and parhypate have-been added hypate indeed
hypaton vocatae sunt quasi maximae magnarum et
hypaton called they-have-been as-it-were greatest of-the-great and
gravissimae gravium, aut excellentes excellentium. Sed vocata
lowest of-the-low or surpassing of-the-surpassing but called
est prima inter undecim hypate hypaton. Secunda vero
has-been the-first among the-eleven hypate hypaton the-second but
parhypate hypaton, quoniam juxta hypaten hypaton collocata
parhypate hypaton since beside hypate hypaton stationed
est. Tertia quae dudum in enneachordo hyperhypate
it-has-been the-third which lately in the-enneachord hyperhypate
vocabatur, lichanos hypaton est nuncupata. Quarta vero
was-called lichanos hypaton has-been named the-fourth but

hypate antiquum tenuit nomen. Quinta parhypate. Sexta
hypate its-ancient kept name the-fifth parhypate the-sixth
lichanos antiquum scilicet habens vocabulum. Septima mese.
lichanos its-ancient namely having word the-seventh mese
Octava paramese. Nona trite. Decima paranete. Undecima nete.
the-eighth paramese the-ninth trite the-tenth paranete the-eleventh nete
Est igitur unum tetrachordum hypate hypaton, parhypate
there-is therefore one tetrachord hypate hypaton parhypate
hypaton, lichanos hypaton, hypate. Aliud vero hypate, parhypate,
hypaton lichanos hypaton hypate another but hypate parhypate
lichanos, mese, et haec quidem conjuncta sunt. Tertium vero
lichanos mese and these indeed conjoined are the-third but
est paramese, trite, paranete, nete. Sed quoniam inter superius
is paramese trite paranete nete but since between the-upper
tetrachordum, quod est hypate hypaton, parhypate hypaton,
tetrachord which is hypate hypaton parhypate hypaton
lichanos hypaton, hypate meson, et inter infimum, quod
lichanos hypaton hypate meson and between the-lowest which
est paramese, trite, paranete, nete, fit positione medium
is paramese trite paranete nete comes-to-be in-position the-middle
tetrachordum, quod est hypate, parhypate, lichanos, mese.
tetrachord which is hypate parhypate lichanos mese
Totum hoc medium tetrachordum meson vocatum est,
whole this middle tetrachord meson called has-been
quasi mediarum, vocaturque cum additamento hoc hypate
as-it-were of-the-middle-(strings) and-is-called with the-addition this hypate
meson, parhypate meson, lichanos meson, mese. Quoniam
meson parhypate meson lichanos meson mese since
vero inter hoc meson tetrachordum, et inferius, quod est
but between this meson tetrachord and the-lower which is
netarum, disjunctio est, meses scilicet et parameses, inferius
of-the-netes a-disjunction there-is of-mese namely and of-paramese the-lower

omne tetrachordum disjunctarum, id est diezeugmenon,
whole tetrachord of-the-disjoined-(strings) that is diezeugmenon
vocatum est, cum additamento scilicet hoc: paramese
called has-been with the-addition namely this paramese
diezeugmenon, trite diezeugmenon, paranete diezeugmenon,
diezeugmenon trite diezeugmenon paranete diezeugmenon
nete diezeugmenon, ut sit descriptio hoc modo: Est
nete diezeugmenon so-that may-be the-diagram in-this manner there-is
igitur hic inter paramesen ac mesen disjunctio, atque ideo
therefore here between paramese and mese a-disjunction and therefore
diezeugmenon tetrachordum hoc vocatum est. Quod si
diezeugmenon tetrachord this called has-been but if
paramese auferatur, et sit mese trite, paranete, nete,
paramese be-removed and there-be mese trite paranete nete
tunc conjuncta, id est synemmena, erunt tria tetrachorda,
then conjoined that is synemmena will-be three tetrachords
vocabiturque ultimum tetrachordum synemmenon hoc modo:
and-will-be-called the-last tetrachord synemmenon in-this manner
Sed quoniam in hac vel in superiore hendecachordi dispositione
but since in this or in the-earlier of-the-hendecachord arrangement
mese, quae propter mediam collocationem ita vocata est,
mese which on-account-of its-middle placement so called has-been
nete proxima accedit, et longe ab hypatis ultimis distat, nec
to-nete nearest comes and far from the-hypates outermost stands-off nor
proprium distantiae retinet locum, aliud unum tetrachordum
its-proper of-distance retains place another one tetrachord
adjunctum est, super neten diezeugmenon, quae quoniam
joined-on has-been above nete diezeugmenon which since
supervadebat acumine netas superius collocatas, omne illud
it-overpassed in-height the-netes earlier placed all that
tetrachordum hyperboleon vocatum est hoc modo: Sed
tetrachord hyperboleon called has-been in-this manner but

quoniam rursus mese non erat loco media, sed magis
since again mese not was in-place middle but rather
hypatis accedebat, idcirco super hypatas hypaton addita
to-the-hypates drew-near therefore above the-hypates hypaton added
est una chorda, quae dicitur proslambanomenos;
has-been one string which is-called proslambanomenos
ab aliquibus autem prosmelodos dicitur, tono integro
by some moreover prosmelodos it-is-called by-a-tone whole
distans ab ea quae est hypate hypaton. Et ipsa quidem, id
distant from that which is hypate hypaton and itself indeed that
est proslambanomenos a mese octava est, resonans cum ea
is proslambanomenos from mese the-eighth is resounding with it
diapason symphoniam, eademque ad lichanon hypaton resonat
the-diapason symphony and-the-same to lichanos hypaton it-resounds
diatessaron ad quartam, scilicet quae lichanos hypaton ad
diatessaron to the-fourth namely which lichanos hypaton to
mesen resonat diapente symphoniam, et est ab ea quinta.
mese it-resounds the-diapente symphony and it-is from it the-fifth
Rursus mese a paramese distat tono, quae eadem mese
again mese from paramese stands-off by-a-tone which the-same mese
ad neten diezeugmenon quintam facit diapente consonantiam.
to nete diezeugmenon the-fifth makes the-diapente consonance
Quae nete diezeugmenon ad neten hyperboleon quartam, facit
which nete diezeugmenon to nete hyperboleon the-fourth makes
diatessaron consonantiam. Et proslambanomenos ad neten
the-diatessaron consonance and proslambanomenos to nete
hyperboleon reddit bisdiapason consonantiam hoc modo:
hyperboleon renders the-bisdiapason consonance in-this manner

De generibus cantilenarum

I.21 · THE GENERA OF SONG

Boethius names the three genera of song. The diatonic is somewhat harder and more natural, and its voice climbs each tetrachord by semitone, tone, and tone. The chroma — the word means color, as a surface passes into another color — softens that natural tension and moves by semitone, semitone, and three semitones. The enharmonic, the most closely fitted, moves by diesis, diesis, and ditone, a diesis being half a semitone.

His igitur expeditis, dicendum est de generibus melorum.
these therefore dispatched to-be-said it-is about the-genera of-songs
Sunt autem tria diatonum, chroma, enharmonium, et
they-are moreover three the-diatonic the-chroma the-enharmonic and
diatonicum quidem, aliquanto durius et naturalius. Chroma
the-diatonic indeed somewhat harder and more-natural the-chroma
vero est jam quasi ab illa naturali intentione descendens,
but is now as-it-were from that natural tension descending
et in mollius decidens. Enharmonium vero optime atque apte
and into the-softer falling the-enharmonic but best and aptly
conjunctum. Cum sint igitur quinque tetrachorda, hypaton,
joined since there-be therefore five tetrachords hypaton
meson, synemmenon, diezeugmenon, hyperboleon; in his
meson synemmenon diezeugmenon hyperboleon in these
omnibus secundum diatonum cantilenae procedit vox per
all according-to the-diatonic of-song proceeds the-voice through
semitonium, tonum ac tonum, in uno tetrachordo. Rursus
a-semitone a-tone and a-tone in one tetrachord again
in alio tetrachordo per semitonium tonum et tonum
in another tetrachord through a-semitone a-tone and a-tone
ac deinceps. Ideoque vocatur diatonicum quasi quod per
and so-on and-therefore it-is-called diatonic as-it-were because through

tonum ac per tonum progrediatur. Chroma autem quod
a-tone and through a-tone it-advances chroma moreover which
dicitur color, quasi jam ab hujusmodi intentione prima
is-called color as-it-were now from of-this-kind tension a-first
mutatio cantatur per semitonium et semitonium et tria
change is-sung through a-semitone and a-semitone and three
semitonia. Tota enim diatessaron consonantia est duorum
semitones the-whole for diatessaron consonance is of-two
tonorum ac semitonii, sed non pleni. Tractum est autem
tones and a-semitone but not full drawn has-been moreover
hoc vocabulum ut diceretur chroma a superficiebus, quae
this word that it-should-be-called chroma from surfaces which
cum permutantur, in alium transeunt colorem. Enharmonium
when they-are-changed into another pass-over color the-enharmonic
vero quod est magis coaptatum, est quod cantatur in
but which is more fitted-together is (that)-which is-sung in
omnibus tetrachordis per diesin et diesin et ditonum. Diesis
all tetrachords through a-diesis and a-diesis and a-ditone a-diesis
autem est semitonii dimidium, ut sit trium generum
moreover is of-a-semitone the-half so-that there-may-be of-the-three genera
descriptio, per omnia tetrachorda discurrens hoc modo:
a-diagram through all tetrachords running in-this manner

De ordine chordarum nominibusque in tribus generibus

I.22 · THE ORDER OF STRINGS IN THE THREE GENERA

Boethius sets out the full order of the strings, from proslambanomenos up to nete hyperboleon. The fixed strings keep one name everywhere, while each tetrachord's lichanos or paranete takes a further title from the genus it serves — diatonic, chromatic, or enharmonic. Counting the strings alike in all genera together with the unlike ones brings the whole to twenty-eight, which a subjoined diagram displays.

Nunc igitur ordo chordarum disponendus est omnium quae
now therefore the-order of-the-strings to-be-set-out is of-all which
per tria genera variantur, vel inconstanti ordine disponuntur.
through three genera are-varied or in-an-unfixed order are-disposed
Prima est igitur proslambanomenos, quae eadem dicitur
the-first is therefore proslambanomenos which the-same is-called
prosmelodos; secunda, hypate hypaton; tertia parhypate
prosmelodos the-second hypate hypaton the-third parhypate
hypaton; quarta vero universaliter quidem lichanos appellatur.
hypaton the-fourth but universally indeed lichanos is-named
Sed si in diatono genere aptetur, dicitur lichanos hypaton
but if in the-diatonic genus it-be-fitted it-is-called lichanos hypaton
diatonos. Si vero in chromate, dicitur diatonos chromaticae,
diatonos if but in the-chroma it-is-called diatonos chromaticae
vel lichanos hypaton chromaticae. Si autem in enharmonio,
or lichanos hypaton chromaticae if moreover in the-enharmonic
dicitur lichanos hypaton enharmonios, vel diatonos hypaton
it-is-called lichanos hypaton enharmonios or diatonos hypaton
enharmonios. Post hanc vocatur hypate meson; dehinc parhypate
enharmonios after this is-called hypate meson then parhypate
meson; atque hic lichanos meson simpliciter. In diatonico
meson and here lichanos meson simply in the-diatonic

quidem genere, diatonos meson. In chromate, lichanos meson
indeed genus diatonos meson in the-chroma lichanos meson
chromaticae, vel diatonos meson chromaticae. In enharmonio
chromaticae or diatonos meson chromaticae in the-enharmonic
diatonos meson enharmonios, vel lichanos meson enharmonios.
diatonos meson enharmonios or lichanos meson enharmonios
Has sequitur mese. Post hanc sunt duo tetrachorda partim
these follows mese after this are two tetrachords partly
synemmenon, partim diezeugmenon. Et synemmenon est
synemmenon partly diezeugmenon and synemmenon is
quod post mesen ponitur, id est trite synemmenon.
(that)-which after mese is-placed that is trite synemmenon
Dehinc lichanos synemmenon, eadem in diatonico diatonos
then lichanos synemmenon the-same in the-diatonic diatonos
synemmenon. In chromate vero vel diatonos synemmenon
synemmenon in the-chroma but either diatonos synemmenon
chromaticae, vel lichanos synemmenon chromaticae. In
chromaticae or lichanos synemmenon chromaticae in
enharmonio vero vel diatonos synemmenon enharmonios,
the-enharmonic but either diatonos synemmenon enharmonios
vel lichanos synemmenon enharmonios. Post has nete
or lichanos synemmenon enharmonios after these nete
synemmenon. Si vero mese nervo non sit synemmenon
synemmenon if but to-mese to-the-string not there-be the-synemmenon
tetrachordum adjunctum, sed sit diezeugmenon, est post
tetrachord joined but there-be diezeugmenon there-is after
mesen paramese. Dehinc trite diezeugmenon, inde lichanos
mese paramese then trite diezeugmenon thence lichanos
diezeugmenon, quae, in diatono, diatonos diezeugmenon. In
diezeugmenon which in the-diatonic (is)-diatonos diezeugmenon in
chromate tum diatonos diezeugmenon chromaticae, tum lichanos
the-chroma now diatonos diezeugmenon chromaticae now lichanos

diezeugmenon chromaticae. In enharmonio vero tum diatonos
diezeugmenon chromaticae in the-enharmonic but now diatonos
diezeugmenon enharmonios, tum lichanos diezeugmenon
diezeugmenon enharmonios now lichanos diezeugmenon
enharmonios. Eadem vero dicitur et paranete, cum additione
enharmonios the-same but is-called also paranete with the-addition
vel diatoni, vel chromatis, vel enharmonii. Super has nete
either of-the-diatonic or of-the-chroma or of-the-enharmonic above these nete
diezeugmenon, trite hyperboleon, paranete hyperboleon, nete
diezeugmenon trite hyperboleon paranete hyperboleon nete
hyperboleon, et quae est paranete hyperboleon, eadem in
hyperboleon and (that)-which is paranete hyperboleon the-same in
diatono diatonos hyperboleon. In chromate vero chromaticae
the-diatonic (is)-diatonos hyperboleon in the-chroma but chromaticae
hyperboleon; in enharmonio vero enharmonios hyperboleon.
hyperboleon in the-enharmonic but enharmonios hyperboleon
Harum ultima ea est quae est nete hyperboleon, et
of-these the-last that(-one) is which is nete hyperboleon and
sit descriptio ejusmodi ut trium generum contineat
let-there-be a-diagram of-such-kind that of-the-three genera it-may-contain
dispositionem in quibus et similitudinem nominum et
the-disposition in which both the-likeness of-the-names and
differentiam pernotabis, ut si nervi similes in omnibus,
the-difference you-will-mark so-that if the-strings alike in all
cum eis qui sunt dissimiles colligantur, fiant simul omnes
with those which are unlike be-gathered they-become together all
octo et viginti. Hoc autem monstrat subjecta descriptio.
eight and twenty this moreover shows the-subjoined diagram

Quae sint inter voces in singulis generibus proportiones

I.23 · THE PROPORTIONS BETWEEN THE NOTES

Boethius states how each genus divides its tetrachords. The diatonic divides every tetrachord by two tones and a semitone, and its tone is called uncompounded because it stands whole, with no other interval joined to it. The chroma divides by semitone, semitone, and an uncompounded trihemitone; the diatonic's semitone plus tone could be called a trihemitone too, but not uncompounded, since two intervals make it. The enharmonic divides by diesis, diesis, and an uncompounded ditone.

Hoc igitur modo per singula tetrachorda in generum
in-this therefore manner through the-several tetrachords into of-the-genera
proprietates partitio facta est, ut omnia quidem diatonici
the-properties the-division made has-been so-that all indeed of-the-diatonic
generis quinque tetrachorda, duobus tonis ac semitonio
genus five tetrachords by-two tones and a-semitone
partiremur. Diciturque in hoc genere tonus incompositus,
we-might-divide and-(it)-is-called in this genus the-tone uncompounded
idcirco quoniam integer ponitur, nec aliquod ei aliud
for-this-reason because whole it-is-placed nor any to-it other
intervallum adjungitur. Sed in singulis intervallis integri
interval is-joined but in the-several intervals whole
insunt toni. In chromate vero semitonio, ac semitonio
are-within tones in the-chroma but on-a-semitone and on-a-semitone
incompositoque trihemitonio posita divisio est. Idcirco
and-on-an-uncompounded trihemitone placed the-division is for-this-reason
autem incompositum hoc trihemitonium appellavimus,
moreover uncompounded this trihemitone we-have-called
quoniam in uno collocatum est intervallo; potest autem
because in one set it-is interval it-can moreover
appellari trihemitonium in diatono genere semitonium ac
to-be-called a-trihemitone in the-diatonic genus a-semitone and

tonus; sed non est incompositum, duobus enim id perficitur
a-tone but not it-is uncompounded by-two for it is-completed
intervallis, et in enharmonio genere idem est. Constat
intervals and in the-enharmonic genus the-same is it-consists
autem ex diesi, et diesi, et ditono incomposito, quod
moreover of a-diesis and a-diesis and a-ditone uncompounded which
scilicet propter eamdem causam incompositum nuncupamus,
evidently for the-same cause uncompounded we-name
quoniam in uno collocatum est intervallo.
because in one set it-is interval

Quid sit synaphe

I.24 · WHAT SYNAPHE IS

Synaphe, which Latin can call conjunction, occurs whenever one shared boundary note makes two tetrachords continuous. Hypate meson is counted in both the hypaton and the meson tetrachords, standing as the highest note of the former and the lowest of the latter. The synaphe is thus the middle note of two tetrachords — one and the same string joining them.

Sed in his ita dispositis constitutisque tetrachordis synaphe
but in these thus disposed and-established tetrachords synaphe
est. Quam conjunctionem dicere Latina significatione
is which a-conjunction to-call with-Latin signification
possumus, quotiens duo tetrachorda unius termini medietas
we-can whenever two tetrachords of-one boundary the-mean
continuat atque conjungit, ut in hoc tetrachordo. Hoc igitur
makes-continuous and joins as in this tetrachord this therefore
est unum tetrachordum hypate hypaton, parhypate hypaton,
is one tetrachord hypate hypaton parhypate hypaton
lichanos hypaton, hypate meson. Aliud vero hypate meson,
lichanos hypaton hypate meson the-other but hypate meson

parhypate meson, lichanos meson, mese. In utrisque igitur
parhypate meson lichanos meson mese in both therefore
tetrachordis hypate meson annumerata est. Superiorisque
tetrachords hypate meson counted-in has-been and-of-the-former
tetrachordi ea est acutissima, posterioris vero gravissima.
tetrachord it is the-highest of-the-later but the-lowest
Estque ista conjunctio una eademque chorda hypate meson,
and-is that conjunction one and-the-same string hypate meson
duo tetrachorda conjungens, ut eadem hypaton ac meson
two tetrachords joining as the-same the-hypaton and meson
tetrachorda in superiore descriptione adjunxit. Est igitur
tetrachords in the-above diagram it-joined there-is therefore
synaphe, quae conjunctio dicitur, duorum tetrachordorum vox
synaphe which conjunction is-called of-two tetrachords the-note
media, superioris quidem acutissima, posterioris vero gravissima.
middle of-the-former indeed the-highest of-the-later but the-lowest

Quid sit diezeuxis

I.25 · WHAT DIEZEUXIS IS

Diezeuxis, which can be called disjunction, occurs whenever two tetrachords are separated by the mean of a tone: mese and paramese differ by a full tone, and the eight strings plainly form two tetrachords. A later treatise will unfold this more plainly. Looking carefully, one finds five tetrachords and no more — hypaton, meson, synemmenon, diezeugmenon, hyperboleon.

Diezeuxis vero appellatur, quae disjunctio dici potest,
diezeuxis but is-named which a-disjunction to-be-called can
quotiens duo tetrachorda toni medietate separantur, ut
whenever two tetrachords of-a-tone by-the-mean are-separated as
in his duobus tetrachordis. Duo igitur esse tetrachorda
in these two tetrachords (that)-two therefore to-be tetrachords

evidenter apparet, quoniam quidem octo sunt chordae.
evidently it-appears since indeed eight are the-strings
Sed diezeuxis est, id est disjunctio inter mesen et
but diezeuxis is that is a-disjunction between mese and
paramesen, quae inter se pleno differunt tono, de
paramese which between themselves by-a-full differ tone about
quibus evidentius explicabitur, cum unumquodque studiosius
which more-plainly it-will-be-unfolded when each-thing more-studiously
explanandum posterior tractatus assumpserit. Sed diligentius
to-be-explained the-later treatise shall-have-taken-up but more-carefully
intuenti quinque, non amplius tetrachorda reperiuntur,
to-(one)-looking five not more tetrachords are-found
hypaton, meson, synemmenon, diezeugmenon, hyperboleon.
hypaton meson synemmenon diezeugmenon hyperboleon

Quibus nominibus nervos appellaverit Albinus

I.26 · ALBINUS' NAMES FOR THE STRINGS

Albinus rendered the strings' names into Latin, calling the hypatae principal, the mesae middle, the synemmenae conjoined, the diezeugmenae disjoined, and the hyperboleae excelling. Boethius declines to linger in another man's work.

Albinus autem earum nomina Latina oratione ita interpretatus
Albinus moreover their names in-Latin speech thus rendered
est, ut hypatas principales vocaret, mesas medias,
has so-that the-hypatae principal he-might-call the-mesae middle
synemmenas conjunctas, diezeugmenas disjunctas, hyperboleas
the-synemmenae conjoined the-diezeugmenae disjoined the-hyperboleae
excellentes. Sed nobis in alieno opere non erit immorandum.
excelling but for-us in another's work not it-will-be to-be-lingered

Qui nervi quibus sideribus comparentur

I.27 · STRINGS AND HEAVENLY BODIES

Boethius likens the strings from hypate meson up to nete to the order of the heavens. Hypate meson is given to Saturn, mese to the Sun, and nete to the Moon's circle. Marcus Tullius reverses that order in the sixth book of On the Republic*: the swift star-bearing course of heaven sounds the most acute, the Moon the most grave, and the unmoving Earth stands as silence. Which strings are fixed and which movable is left for the division of the monochord.*

Illud tamen interim de superioribus tetrachordis addendum
that however meanwhile about the-higher tetrachords to-be-added
videtur, quod ab hypate meson usque ad neten quasi
seems (namely)-that from hypate meson up to nete as-if
quoddam ordinis disjunctionisque caelestis exemplar est.
a-certain of-order and-of-disjunction heavenly a-pattern there-is
Namque hypate meson Saturno est attributa, parhypate vero
for-indeed hypate meson to-Saturn has-been assigned parhypate but
Joviali circulo consimilis est, lichanon meson Marti tradidere,
to-Jove's circle like is lichanos meson to-Mars they-have-consigned
Sol mesen obtinuit, trite synemmenon, Venus habet, paranete
the-Sun mese obtained trite synemmenon Venus holds paranete
synemmenon Mercurius regit, nete autem Lunaris circuli tenet
synemmenon Mercury rules nete moreover of-the-lunar circle holds
exemplum. Sed Marcus Tullius contrarium ordinem facit. Nam
the-likeness but Marcus Tullius the-contrary order makes for
in vi lib. de Republica sic ait: Et natura fert ut
in the-sixth book on the-Republic thus he-says and nature brings(-it-about) that
extrema ex altera parte graviter, ex altera autem acute sonent.
the-extremes from the-one side low from the-other moreover high sound
Quam ob causam summus ille caeli stellifer cursus,
which on-account-of cause the-highest that of-heaven star-bearing course

cujus conversio est concitatior acuto, et excitato movetur
whose revolution is swifter with-a-high and roused is-moved
sono, gravissimo autem hic lunaris atque infimus. Nam terra
sound with-the-lowest but this lunar and lowest for the-earth
nona, immobilis manens, ima sede semper haeret; hic
ninth unmoving remaining in-the-lowest seat always clings here
igitur Tullius terram quasi silentium ponit, scilicet immobilem.
therefore Tullius the-earth as-if silence sets namely unmoving
Post hanc sui proximus a silentio est, dat Lunae
after this to-it the-nearest from silence is he-gives to-the-Moon
gravissimum sonum, ut sit Luna proslambanomenos.
the-lowest sound so-that may-be the-Moon proslambanomenos
Mercurius hypate hypaton, Venus parhypate hypaton, Sol
Mercury hypate hypaton Venus parhypate hypaton the-Sun
lichanos hypaton, Mars hypate meson, Jupiter parhypate
lichanos hypaton Mars hypate meson Jupiter parhypate
meson, Saturnus lichanos meson, Coelum ultimum mese. Quae
meson Saturn lichanos meson the-heaven outermost mese which
vero sint harum immobiles, quae vero in totum mobiles, quae
but may-be of-these unmoving which but in the-whole moving which
autem immobiles mobilesque consistant, cum de monochordi
moreover unmoving and-moving stand-fast when about of-the-monochord
regularis divisione tractavero, erit locus aptior explicandi.
regular the-division I-shall-have-treated will-be a-place fitter of-explaining

Quid sit natura consonantiarum

I.28 · THE NATURE OF CONSONANCE

The sense of the ears may judge consonance, but reason weighs it. When two strings, one the graver, are struck together and give back a mingled, sweet sound, the two voices coalescing as if into one, that is what is called consonance. When, struck together, each desires to go its own way and no single sweet compound reaches the ear, that is dissonance.

Consonantiam vero licet aurium quoque sensus dijudicet,
consonance but although of-the-ears too the-sense may-judge
tamen ratio perpendit. Quotiens enim duo nervi, uno
yet reason weighs(-it) whenever for two strings one(-of-them)
graviore, intenduntur, simulque pulsi reddunt permixtum
lower are-stretched and-together struck they-give-back a-mingled
quodammodo et suavem sonum, duaeque voces quasi conjunctae
in-some-wise and sweet sound and-the-two voices as-if conjoined
in unum coalescunt, tunc fit ea quae dicitur consonantia.
into one coalesce then comes-about that which is-called consonance
Cum vero simul pulsi, sibi quisque ire cupit, nec
when but together struck its-own(-way) each to-go desires nor
permiscent ad aurem suavem atque unum ex duobus
do-they-mingle to the-ear a-sweet and one out-of the-two
compositum sonum, tunc est quae dicitur dissonantia.
compounded sound then (there)-is (that)-which is-called dissonance

Ubi consonantiae reperiuntur

I.29 · WHERE CONSONANCE IS FOUND

Consonance is found in comparisons whose terms are commensurable — where a common measure counts both, as 2 measures both 2 and 4, and both 4 and 6. In the superpartient 5:3 the difference, 2, measures neither term, so that kind is severed from consonance by nature. The consonance-forming ratios also keep much likeness: the double is twice the simple, the sesquialter twice the half, the sesquitertian thrice the third part — which is hardly found in the other kinds.

In his autem comparationibus gravitatis atque acuminis
in these moreover comparisons of-lowness and of-highness
has consonantias necesse est inveniri, quae sibi
these consonances necessary it-is to-be-found which to-one-another
commensuratae sint, id est quae notam possunt communem
commensurate be that is which a-known can common
habere mensuram, ut in multiplicibus duplum, quod est ut
to-have measure as in the-multiples the-double that is that
illa pars metiatur, quae inter duos terminos est differentia, ut
that part may-measure which between the-two terms is the-difference as
inter duos et quatuor binarius utrosque metitur, inter duos
between two and four the-two both measures between two
atque sex, quae tripla est, binarius utrosque metiatur, inter
and six which triple is the-two both may-measure between
novem atque octo eadem unitas est, quae utrosque metiatur.
nine and eight the-same unit is which both may-measure
Rursus in superparticularibus si sesquialtera sit proportio,
again in the-superparticulars if sesquialter be the-proportion
ut quatuor ad sex, binarius est qui utrosque metiatur, qui
as four to six the-two is which both may-measure which
scilicet utrorumque est differentia. Quod si sesquitertia sit
namely of-both is the-difference but if sesquitertian be

proportio, ut si octo senario comparentur, idem binarius
the-proportion as if eight to-six be-compared the-same the-two
utrosque metitur; id vero non evenit in caeteris generibus
both measures this but not comes-about in the-other kinds
inaequalitatum, quae supra retulimus, ut in superpartiente.
of-inequalities which above we-have-recounted as in the-superpartient
Nam si quinarium ad ternarium comparemus, binarius qui
for if the-five to the-three we-compare the-two which
eorum differentia est, neutrum metitur. Nam si semel
their difference is neither measures for if once
ternario comparatus, minor est, duplicatus excedit.
to-the-three compared less it-is doubled it-exceeds
Item bis quinario comparatus, minor est, tertio vero
likewise twice to-the-five compared less it-is a-third-time but
supergreditur. Atque idcirco hoc primum inaequalitatis genus
it-oversteps and therefore this first of-inequality kind
a consonantia natura disjungitur amplius, quod in
from consonance by-nature is-severed further (the-fact)-that in
his quae consonantias formant, multa similia sunt, in
these which consonances form many like(-elements) there-are in
his vero minime, id probatur hoc modo: namque duplum
these but not-at-all this is-proved in-this way for-indeed the-double
nihil est aliud nisi bis simplum. Triplum nihil aliud nisi
nothing is other than twice the-simple the-triple nothing other than
tertio simplum. Quadruplum vero idem est quod quarto
a-third-time the-simple the-quadruple but the-same is as a-fourth-time
simplum. Sesquialterum, bis medietas. Sesquitertium vero
the-simple the-sesquialter twice the-half the-sesquitertian but
ter pars tertia, quod haud facile in caeteris inaequalitatum
thrice the-part third which not easily in the-other of-inequalities
generibus invenitur.
kinds is-found

Quemadmodum Plato dicat fieri consonantias

I.30 · PLATO ON HOW CONSONANCE ARISES

Plato says consonance arises in the ear. The swifter sound is necessarily the more acute; it outruns the grave one, enters the ear, and returns as if beaten back from the farthest part — but slower now than at its first impulse, and therefore graver. Returning graver, it meets the oncoming grave sound as its like, mingles with it, and, as Plato says, makes one consonance.

Plato vero hoc modo fieri in aure consonantiam
Plato but in-this way (that)-to-come-about in the-ear consonance
dicit. Necesse est, inquit, velociorem quidem esse acutiorem
he-says necessary it-is says-he (that)-the-swifter indeed to-be the-higher
sonum. Hic igitur cum gravem praecesserit, in aurem
sound this(-one) therefore when the-low it-has-outrun into the-ear
celer ingreditur, offensaque extrema ejusdem corporis parte
swift enters and-struck the-outmost of-the-same body part
quasi pulsus iterato motu revertitur. Sed jam segnior,
as-if beaten with-repeated motion it-returns but now slower
nec ita celer, ut primo impetu emissus cucurrit, quocirca
nor so swift as with-the-first impulse sent-forth it-ran wherefore
gravior quoque. Cum igitur jam gravior rediens, nunc
lower too when therefore now lower returning now
primum gravi sono venienti similis occurrit, miscetur
first to-the-low sound the-oncoming like it-meets it-is-mingled
ei, unamque, ut ait Plato, consonantiam miscet.
with-it and-one as says Plato consonance it-mingles

Quid contra Platonem Nicomachus sentiat

I.31 · NICOMACHUS AGAINST PLATO

Nicomachus does not think Plato spoke truly: consonance belongs not to similars but to dissimilars coming into one, and a graver sound mingled with a grave makes no consonance — unlikeness, not likeness, effects the concord. In his account a string struck once drives the air many times and makes many voices, so swiftly that no gap is felt and one voice comes to the ears. When the strikings of grave and acute sounds are commensurable, they mingle into one consonance.

Sed id Nicomachus non arbitratur dictum veraciter.
but this Nicomachus not thinks (to-have-been)-said truly
Neque enim similium esse consonantiam, sed dissimilium potius
neither for of-similars to-be consonance but of-dissimilars rather
in unum eamdemque consonantiam venientium; gravius vero
into one and-the-same consonance of-(those-)coming the-lower but
gravi si misceatur, nullam facere consonantiam, quoniam hanc
with-a-low if it-be-mingled no to-make consonance since this
canendi concordiam similitudo non efficit, sed dissimilitudo.
of-singing concord likeness not effects but unlikeness
Quae cum distet in singulis vocibus, copulatur in
which although it-stands-apart in the-single voices is-coupled in
mixtis. Sed hinc potius Nicomachus consonantiam fieri
the-mingled but from-this rather Nicomachus (that)-consonance to-come-about
putat. Non, inquit, unus tantum pulsus est, qui simplicem
thinks not says-he one only pulse there-is which a-simple
modum vocis emittat, sed semel percussus nervus saepius
measure of-voice may-send-forth but once struck the-string many-times
aerem pellens multas efficit voces. Sed quia ea velocitas est
the-air driving many produces voices but because such the-swiftness is
percussionis, ut sonus sonum quodammodo comprehendat,
of-the-striking that sound the-sound in-some-wise may-overtake

distantia non sentitur, et quasi una vox auribus venit. Si
the-gap not is-perceived and as-if one voice to-the-ears comes if
igitur percussiones gravium sonorum commensurabiles sint
therefore the-strikings of-low sounds commensurable be
percussionibus acutorum sonorum, ut in his proportionibus quas
with-the-strikings of-high sounds as in these proportions which
supra retulimus, non est dubium quin ipsa commensuratio
above we-have-recounted not is (there-)doubt but-that the-very commensuration
sibimet misceatur, unamque vocum efficiat consonantiam.
with-itself be-mingled and-one of-voices may-produce consonance

Quae consonantia quam merito praecedat

I.32 · WHICH CONSONANCE DESERVES FIRST PLACE

Boethius asks which consonance ear and reason alike should judge the better. The mind grasps quantities in an order of ease: given a number or a line, nothing is easier to behold than its double, then the half, then the triple, then the third part. Because the double is easiest, Nicomachus deems the diapason the best consonance, the diapente next, then the diapason-plus-diapente, which contains the triple, and judges the rest by the same form. Ptolemy does not, and his whole opinion comes later.

Sed inter omnes quas retulimus consonantias, habendum
but among all which we-have-recounted consonances to-be-held
judicium est ut in aure, ita quoque in ratione quam harum
a-judgment is as in the-ear so too in reason which of-these
meliorem oporteat arbitrari. Eodem namque modo auris
the-better it-behooves to-deem in-the-same for-indeed manner the-ear
afficitur sonis, vel oculus aspectu, quo animi judicium
is-affected by-sounds or the-eye by-sight as of-the-mind the-judgment
numeris vel continua quantitate. Proposito enim numero
by-numbers or by-continuous quantity set-forth for a-number

vel linea, nihil est facilius quam ejus duplum oculo vel
or a-line nothing is easier than its double with-the-eye or
animo contueri. Item post dupli judicium sequitur
with-the-mind to-behold likewise after of-the-double the-judgment follows
dimidii, post dimidium tripli, post triplum partis
(that)-of-the-half after the-half (that)-of-the-triple after the-triple of-the-part
tertiae. Quae ideo quoniam facilior est dupli
third which(-things) therefore since easier is of-the-double
descriptio, optimam Nicomachus putat diapason consonantiam,
the-delineation (as-)best Nicomachus deems the-diapason consonance
post hanc diapente, quae medium tenet, hinc diapente ac
after this the-diapente which the-middle holds thence the-diapente and
diapason, quae triplum continet, caeteraque secundum eumdem
diapason which the-triple contains and-the-rest according-to the-same
modum formamque dijudicat. Non vero hoc Ptolemaeus
manner and-form he-judges not but this Ptolemy
eodem modo, cujus omnem sententiam posterius explicabo.
in-the-same way whose whole opinion later I-shall-unfold

Quo sint modo accipienda quae dicta sunt

I.33 · HOW TO TAKE WHAT HAS BEEN SAID

Boethius says he has run through all this summarily, to accustom the reader's mind at the surface before a later treatment goes deeper. Pythagoras' pupils took their master's authority for reason until they could find the reason themselves, and the reader is asked to take on faith what has been set forth: the ratios of the consonances, the tone that cannot be halved because no superparticular can be, and the diapason that never reaches six tones. He promises to prove it all by number and by ear.

Omnia tamen quae dehinc diligentius expedienda sunt,
all however which hereafter more-carefully to-be-unfolded are
summatim nunc ac breviter attentemus, ut interim in
summarily now and briefly let-us-essay so-that meanwhile on
superficie quadam animum haec lectoris assuefaciant, qui
the-surface a-certain the-mind these(-things) of-the-reader may-accustom who
ad interiorem scientiam posteriore tractatione descendet. Nunc
to the-inner knowledge by-the-later treatment will-descend now
vero quod erat Pythagoricis in morem, ut cum quid
but (that)-which was to-the-Pythagoreans for a-custom that when something
a magistro Pythagora diceretur, hinc nullus rationes petere
by the-master Pythagoras was-said thereupon no-one reasons to-seek
audebat, sed erat ei ratio docentis auctoritas, idque
dared but was for-him the-reason the-teacher's authority and-this
fiebat quandiu discentis animus firmiore doctrina roboratus,
went-on so-long-as the-learner's mind by-firmer teaching strengthened
ipse eamdem rerum rationem nullo etiam docente reperiret;
itself the-same of-things reason with-no-one even teaching might-find
ita etiam nunc lectoris fidei quae proponimus,
so also now of-the-reader to-the-faith (the-things)-which we-set-forth
commendamus, ut arbitretur diapason in dupla, diapente
we-commend that he-may-deem (that)-the-diapason in the-double the-diapente

in sesquialtera, diatessaron in sesquitertia, diapente ac
in the-sesquialter the-diatessaron in the-sesquitertian the-diapente and
diapason in triplici, bis diapason in quadrupla proportione
diapason in the-triple twice diapason in the-quadruple proportion
consistere. Post vero et hoc ratio diligentius explicabit,
to-consist afterward but also this reason more-carefully will-unfold
et quibus modis aurium quoque judicio consonantiae
and by-what manners of-the-ears too by-the-judgment the-consonances
musicae colligantur, caeteraque omnia quae superius dicta
of-music are-gathered and-the-rest all which above said
sunt amplior tractatus edisseret, et tonum sesquioctavam
have-been a-fuller treatise will-set-forth and (that)-the-tone a-sesquioctave
facere proportionem, eumque in duo aequa dividi non
to-make proportion and-that-it into two equal(-parts) to-be-divided not
posse, sicut nullam ejusdem generis proportiones, id est
to-be-able as no of-the-same kind proportion that is
superparticularis; diatessaron etiam consonantiam duobus
superparticular (that)-the-diatessaron also consonance of-two
tonis semitonioque consistere, semitonia vero esse duo, majus
tones and-a-semitone to-consist (that)-semitones but to-be two a-greater
ac minus, diapente autem tribus tonis ac semitonio
and a-lesser (that)-the-diapente moreover by-three tones and a-semitone
minore contineri. Diapason vero quinque tonis ac duobus
lesser to-be-contained (that)-the-diapason but by-five tones and two
minoribus semitoniis expleri, neque ad sex tonos ullo modo
lesser semitones to-be-filled-up nor to six tones in-any way
pervenire. Haec omnia posterius et numerorum ratione
to-reach these all later both of-numbers by-the-reckoning
et aurium judicio comprobabo, atque haec hactenus.
and of-the-ears by-the-judgment I-shall-prove and these(-things) thus-far

Quid sit musicus

I.34 · WHAT A MUSICIAN IS

Boethius closes the book by asking what a musician is. Reason is naturally more honorable than handicraft: knowing what one does is greater than doing it, and the hand's work is vain unless reason commands. Of the three kinds busied about music, players serve their instruments, devoid of speculation; poets are borne to song by natural instinct rather than reason; the musician proper is the third kind, who by reason and speculation judges rhythms, melodies, and the poets' songs.

Nunc illud est intuendum quod omnis ars, omnisque etiam
now that is to-be-considered that every art and-every also
disciplina honorabiliorem naturaliter habeat rationem, quam
discipline a-more-honorable naturally holds reason than
artificium, quod manu atque opere artificis exercetur.
handicraft which by-hand and work of-the-craftsman is-plied
Multo enim est majus atque altius scire quod quisque
by-much for is greater and loftier to-know what each-man
faciat, quam ipsum illud efficere quod sciat; etenim
may-do than itself that to-effect what he-may-know for-indeed
artificium corporale, quasi serviens famulatur. Ratio vero quasi
handicraft bodily as-if serving does-service reason but as-if
domina imperat, et nisi manus secundum id quod ratio
a-mistress commands and unless the-hand according-to that which reason
sancit efficiat, frustra fit. Quanto igitur praeclarior
ordains effects in-vain it-comes-about by-how-much therefore more-splendid
est scientia musicae in cognitione rationis, quam in opere
is the-knowledge of-music in the-cognition of-reason than in the-work
efficiendi atque actu tantum, scilicet quantum corpus mente
of-effecting and in-act only namely by-as-much-as the-body by-the-mind
superatur! Quod scilicet rationis expers servitio degit, illa
is-surpassed (that)-which namely of-reason devoid in-servitude it-lives she

vero imperat, atque ad rectum deducit, quod nisi pareat ejus
but commands and to the-right leads(-it) and-this unless it-obey her
imperio, et expers rationis opus titubabit. Unde fit
command even devoid of-reason the-work will-totter whence it-comes-about
ut speculatio rationis operandi actu non egeat. Manuum
that the-speculation of-reason of-working the-act not may-need of-hands
vero opera nulla sint, nisi ratione ducantur. Jam vero quanta
but the-works naught may-be unless by-reason they-be-led now indeed how-great
sit gloria meritumque rationis hic intelligi potest,
may-be the-glory and-desert of-reason here to-be-understood can
quod caeteri, ut ita dicam, corporales artifices non ex
(from-the-fact)-that the-rest as so I-may-say bodily craftsmen not from
disciplina, sed ex ipsis potius instrumentis cepere vocabula.
the-discipline but from the-very rather instruments took their-names
Nam citharoedus ex cithara, vel tibicen ex tibia, caeterique
for the-citharode from the-cithara or the-piper from the-pipe and-the-rest
suorum instrumentorum vocabulis nuncupantur. Is vero est
of-their instruments by-the-names are-called he but is
musicus qui, ratione perpensa, canendi scientiam, non servitio
a-musician who by-reason weighed of-singing the-science not by-the-servitude
operis, sed imperio speculationis assumit. Quod scilicet
of-work but by-the-command of-speculation takes(-it)-up which namely
in aedificiorum bellorumque opera videmus, et in contraria
in of-buildings and-of-wars the-works we-see even in the-contrary
scilicet nuncupatione vocabuli. Eorum namque nominibus vel
namely naming of-the-name of-those for-indeed by-the-names either
aedificia inscribuntur, vel ducuntur triumphi, quorum imperio
buildings are-inscribed or are-led triumphs whose by-command
ac ratione instituta sunt, non quorum opere servitioque
and reason founded they-were not whose by-work and-servitude
perfecta. Tria sunt igitur genera quae circa artem musicam
completed three there-are therefore kinds which about the-art musical

versantur: unum genus est quod instrumentis agitur, aliud
are-busied one kind is (that)-which by-instruments is-plied another
fingit carmina, tertium quod instrumentorum opus carmenque
fashions songs a-third which of-instruments the-work and-song
dijudicat. Sed illud quidem quod in instrumentis positum
judges but that(-kind) indeed which in instruments placed
est, ibique totam operam consumit, ut sunt citharoedi,
is and-there its-whole effort spends as are the-citharodes
quique organo caeterisque musicae instrumentis artificium
and-(those)-who on-the-organ and-on-the-other of-music instruments their-craft
probant, a musicae scientiae intellectu sejuncti sunt,
display from the-musical science's understanding severed they-are
quoniam famulantur (ut dictum est), nec quidquam afferunt
since they-do-service as said has-been nor anything do-they-bring
rationis, sed sunt totius speculationis expertes. Secundum
of-reason but they-are of-the-whole speculation devoid the-second
vero musicam agentium est genus poetarum, quod non
but music of-(those-)practicing is kind of-poets which not
potius speculatione ac ratione quam naturali quodam
rather by-speculation and reason than by-a-natural certain
instinctu fertur ad carmen, atque idcirco hoc quoque genus
instinct is-borne to song and therefore this too kind
a musica segregandum est. Tertium est quod judicandi
from music to-be-set-apart is the-third is (that)-which of-judging
peritiam sumit, ut rhythmos cantilenasque eorumque
the-skill takes-up so-that rhythms and-melodies and-their
carmen possit perpendere. Quod scilicet quando totum in
song it-may-be-able to-weigh which namely since wholly in
ratione ac speculatione positum est, hoc proprie musico
reason and speculation placed is this properly to-the-musician
deputabitur. Isque musicus est cui adest facultas secundum
will-be-assigned and-he a-musician is to-whom is-present the-faculty according-to

speculationem rationemve propositam ac musicae convenientem,
speculation or-reason set-forth and to-music suited
de modis ac rhythmis, deque generibus cantilenarum, ac
about modes and rhythms and-about the-kinds of-melodies and
de permixtionibus, ac de omnibus de quibus posterius
about minglings and about all about which later
explicandum est, ac de poetarum carminibus judicandi.
to-be-unfolded is and about of-poets the-songs of-judging

BOOK II

Prooemium

II.1 · PROEM

Boethius opens the second book with a short proem. The first book set in order everything he now proposes to unfold more carefully. Before coming to what must be taught through its proper reasonings, he premises a few things so the hearer's mind arrives better prepared.

Superius volumen cuncta digessit, quae nunc diligentius
the-former volume all set-in-order which now more-carefully
explicanda esse proposui; itaque priusquam ad ea
to-be-unfolded to-be I-have-purposed and-so before to those(-things)
veniam quae propriis rationibus perdocenda sunt,
I-may-come which with-their-proper reasonings to-be-taught-through are
pauca praemittam, quibus elucubratior animus auditoris
a-few(-things) I-shall-premise whereby better-prepared the-mind of-the-hearer
ad ea quae dicenda sunt accipienda perveniat.
to those which to-be-said are to-be-received may-arrive

Quid Pythagoras esse philosophiam constituerit

II.2 · PYTHAGORAS' DEFINITION OF PHILOSOPHY

Pythagoras was the first to call the pursuit of wisdom philosophy. He set it down as knowledge of what properly and truly is — things that neither grow by stretching, shrink by lessening, nor change by any accident: forms, magnitudes, qualities, relations. Viewed in themselves these are unchangeable; joined to bodies they turn with the changeable.

Primus omnium Pythagoras sapientiae studium philosophiam
first of-all Pythagoras of-wisdom the-pursuit philosophy
nuncupavit, quam scilicet ejus rei notitiam ac disciplinam
named which namely of-that thing the-knowledge and discipline
ponebat, quae proprie vereque esse diceretur. Esse autem
he-set-down which properly and-truly to-be might-be-said to-be moreover
illa putabat quae nec intentione crescerent, nec
(that)-those he-thought which neither by-stretching would-grow nor
diminutione decrescerent, nec ullis accidentibus mutarentur.
by-lessening would-shrink nor by-any accidents would-be-changed
Haec autem esse formas, magnitudines, qualitates,
(that)-these moreover to-be forms magnitudes qualities
habitudines, caeteraque quae per se speculata
relations and-the-rest which through themselves viewed
immutabilia sunt, juncta vero corporibus permutantur, et
unchangeable are joined but to-bodies are-changed-about and
multimodis variationibus mutabilis rei cognatione vertuntur.
by-manifold variations of-the-changeable thing by-kinship are-turned

De differentiis quantitatis, et quae cui disciplinae sit deputata

II.3 · THE KINDS OF QUANTITY

According to Pythagoras, every quantity is continuous (magnitude) or discrete (multitude). Multitude begins from a finite quantity and grows without limit; magnitude is bounded above but can be halved without end. Each science takes its share: geometry studies unmoving magnitude, astronomy moving magnitude, arithmetic discrete quantity in itself, and music the discrete quantity that is related to something else.

Omnis vero quantitas secundum Pythagoram vel continua, vel
every but quantity according-to Pythagoras either continuous or
discreta est. Sed quae continua est magnitudo appellatur,
discrete is but (that)-which continuous is magnitude is-named
quae discreta est multitudo. Quorum haec est diversa et
(that)-which discrete is multitude of-these this is the-diverse and
contraria pene proprietas. Multitudo enim a finita inchoans
contrary well-nigh property multitude for from a-finite beginning
quantitate, crescens in infinita progreditur, ut nullus
quantity growing into the-infinite advances so-that no
crescendi finis occurrat. Estque ad minimum terminata,
of-growing limit may-meet(-it) and-it-is at the-least bounded
interminabilis ad majus, ejusque principium unitas est,
unboundable toward the-greater and-its beginning the-unit is
qua minus nihil est. Crescit vero per numeros atque in
than-which less nothing is it-grows but through numbers and into
infinita protenditur, nec ullus numerus, quo minus crescat,
the-infinite is-stretched-forth nor any number whereby the-less it-may-grow
terminum facit. Sed magnitudo finitam rursus suae
a-bound makes but magnitude a-finite again of-its-own
mensurae recipit quantitatem, sed in infinita decrescit. Nam
measure receives quantity but into the-infinite decreases for

si sit pedalis linea, vel cujuslibet alterius modi, potest
if there-be a-foot-long line or of-whatever other measure it-can
in duo aequa dividi, ejusque medietas in medietatem
into two equal(-parts) to-be-divided and-its half into a-half
secari, ejusque rursus medietatis in aliam medietatem, ut
to-be-cut and-of-its again half into another half so-that
nunquam ullus secandi magnitudinem terminus fiat.
never any of-cutting the-magnitude bound may-come-about
Ita magnitudo quantum ad majorem modum terminata est,
thus magnitude as-far-as to the-greater measure bounded is
fit vero cum decrescere coeperit infinita.
it-becomes but when to-decrease it-shall-have-begun unbounded
At contra numerus, quantum ad minorem modum finitus
but on-the-contrary number as-far-as to the-lesser measure finite
est, infinitus autem incipit esse cum crescit. Cum igitur haec
is infinite moreover it-begins to-be when it-grows although therefore these
ita sint infinita, tamen quasi de rebus finitis philosophia
thus be infinite however as-if about things finite philosophy
pertractat, inque rebus infinitis reperit aliquid terminatum,
treats and-in things infinite it-finds something bounded
de quo possit jure acumen propriae speculationis
about which it-may-be-able rightly the-keenness of-its-proper speculation
adhibere. Namque magnitudinis alia sunt immobilia, ut terra,
to-apply for-indeed of-magnitude some are unmoving as the-earth
ut quadratum, vel triangulum, vel circulus. Alia sunt mobilia,
as a-square or a-triangle or a-circle others are moving
ut sphaera mundi, et quidquid in eo rata celeritate
as the-sphere of-the-world and whatever in it with-fixed swiftness
convertitur. Discretae vero quantitatis alia sunt per se,
is-revolved of-discrete but quantity some are through themselves
ut tres vel quatuor, vel caeteri numeri; alia vero ad aliud, ut
as three or four or the-other numbers others but to another as

duplum, triplum, aliaque quae ex comparatione nascuntur.
the-double the-triple and-others which from comparison are-born
Sed immobilis magnitudinis geometria speculationem tenet.
but of-unmoving magnitude geometry the-speculation holds
Mobilis vero scientiam astronomia persequitur. Per se
of-the-moving but the-science astronomy pursues through itself
vero discretae quantitatis arithmetica auctor est. Ad aliquid
but of-discrete quantity arithmetic the-author is to something
vero relatae musica probatur obtinere peritiam.
but of-(the-)related music is-proved to-hold the-skill

De relatae quantitatis differentiis

II.4 · THE DIFFERENCES OF RELATED QUANTITY

Related quantity has three simple kinds — multiple, superparticular, superpartient — and two mixed from them. Comparing the natural numbers to unity weaves the order of multiples; comparing neighbors gives the superparticulars (3:2 sesquialter, 4:3 sesquitertian); skipping one, two, or three terms gives the superpartients. All of this, Boethius says, is treated more fully in the Arithmetic.

Ac de ea quidem quantitate discreta quae per se est in
and about that indeed quantity discrete which through itself is in
arithmetica sufficienter diximus. Relatae vero ad aliquid
arithmetic sufficiently we-have-spoken of-the-related but to something
quantitatis simplicia quidem genera tria sunt: unum quidem
of-quantity the-simple indeed kinds three are one indeed
multiplex, aliud vero superparticulare, tertium superpartiens.
the-multiple another but the-superparticular the-third the-superpartient
Cum vero multiplex superparticulari superpartientique
when but the-multiple with-the-superparticular and-with-the-superpartient
miscetur, fiunt duae aliae ex his, id est multiplex
is-mingled there-arise two others from these that is the-multiple

superparticularis, et multiplex superpartiens. Horum igitur
superparticular and the-multiple superpartient of-these therefore
omnium talis est regula, si unitatem cunctis in naturali
all such is the-rule if the-unit to-all in the-natural
numero volueris comparare, multiplicis ordo texetur.
number you-shall-have-wished to-compare of-the-multiple the-order will-be-woven
Duo enim ad unum duplus est, tres ad eumdem triplus,
two for to one double is three to the-same triple
quatuor quadruplus, et in caeteris eodem modo, ut subjecta
four quadruple and in the-rest in-the-same manner as the-subjoined
descriptio docet. Si vero superparticularem proportionem
diagram teaches if but a-superparticular proportion
quaeras, naturalem sibi compara numerum, detracta
you-seek the-natural to-itself compare number having-been-withdrawn
scilicet unitate, ut tres duobus sesquialter est, quatuor tribus,
namely the-unit as three to-two sesquialter is four to-three
qui sesquitertius est, quinarius quaternario sesquiquartus
which sesquitertian is the-five to-the-four sesquiquartan
est, et in caeteris eodem modo, quod monstrat subjecta
is and in the-rest in-the-same manner which shows the-subjoined
descriptio. Superpartientes autem tali modo reperies.
diagram the-superpartients moreover in-such manner you-will-find
Disponas naturalem numerum a ternario scilicet
you-should-set-out the-natural number from the-three namely
inchoantem. Si unum igitur intermiseris, superbipartientem
beginning if one therefore you-shall-have-skipped a-superbipartient
effici pernotabis; quod si duo, supertripartientem;
to-be-made you-will-mark-well but if two a-supertripartient
quod si tres, superquadripartientem, idemque in caeteris.
but if three a-superquadripartient and-the-same in the-rest
Ad hunc ordinem spectans et compositas ex multiplici
to this order looking also the-compounded from the-multiple

et superparticulari, et ex multiplici et superpartienti
and the-superparticular and from the-multiple and the-superpartient
proportiones, lector diligens speculabitur. Sed de his
proportions the-reader diligent will-survey but about these
tamen omnibus in arithmeticis expeditius dictum est.
nevertheless all in the-arithmetical-(books) more-fully said has-been

Cur multiplicitas antecellat

II.5 · WHY MULTIPLICITY EXCELS

The multiple kind of inequality is by far the most ancient of the three. It alone is built on comparison to unity, the first of numbers; superparticulars compare the numbers after unity, and superpartients compare skipped terms at unequal intervals. Multiplicity begins from one, superparticularity from two, the superpartient from three. Boethius closes by promising the axioms his proofs will need.

Sed in his illud est considerandum quod multiplex
but in these this is to-be-considered that the-multiple
inaequalitatis genus longe duobus reliquis videtur antiquius.
of-inequality kind by-far than-the-two remaining seems more-ancient
Naturalis enim numeri dispositio, in multiplicibus unitati
the-natural for of-number the-arrangement in the-multiples to-the-unit
quae prima est, comparatur. Superparticularis vero non unitatis
which first is is-compared the-superparticular but not of-the-unit
comparatione perficitur, sed ipsorum, qui post unitatem sunt
by-the-comparison is-perfected but of-the-very which after the-unit are
dispositi numerorum, ut ternarii ad binarium, quaternarii ad
arranged of-numbers as of-the-three to the-two of-the-four to
ternarium, et in caeteris ad hunc modum. Superpartientium
the-three and in the-rest after this manner of-the-superpartients
vero longe retro formatio est, quae nec continuis numeris
but far backward the-formation is which neither to-continuous numbers

comparatur, sed intermissis, nec semper aequali intermissione.
is-compared but to-skipped-(ones) nor always with-equal skipping
Sed nunc quidem una, nunc vero duabus, nunc tribus,
but now indeed by-one now but by-two now by-three
nunc quatuor, atque ita in infinitum succrescit. Amplius
now by-four and so into the-boundless it-grows-up further
multiplicitas ab unitate incipit, superparticularitas a binario,
multiplicity from the-unit begins superparticularity from the-two
superpartiens proportio a ternario initium capit. Sed
the-superpartient proportion from the-three (its-)beginning takes but
de his hactenus; nunc quaedam quae quasi axiomata
about these thus-far now certain-(things) which as-it-were axioms
Graeci vocant, praemittere oportebit, quae tum demum quo
the-Greeks call to-set-forth-first it-will-behoove which then at-last whither
spectare videantur intelligemus, cum de uniuscujusque rei
to-look they-may-seem we-shall-understand when about of-each-single thing
demonstratione tractabimus.
the-demonstration we-shall-treat

Qui sint quadrati numeri deque his speculatio

II.6 · SQUARE NUMBERS

A square number grows together from equal twin dimensions: twice two, thrice three, four times four. Subtract a square from the next square and the remainder equals the sum of the two sides, as 9 minus 4 leaves 5, which is 2 plus 3. Skip one square and the sides' sum is half the remainder; skip two and it is a third; the part is always named by one more than the numbers skipped.

Quadratus numerus est qui gemina dimensione in aequa
a-square number is which by-twin dimension in equal-(measure)
concreverit, ut bis duo, ter tres, quater quatuor,
shall-have-grown-together as twice two thrice three four-times four
quinquies quinque, sexies sex, quorum est ista descriptio:
five-times five six-times six of-which is this diagram
Superius igitur dispositus numerus naturalis, latus est
above therefore the-arranged number natural the-side is
quadratorum inferius descriptorum. Continui enim naturaliter
of-the-squares below of-(those-)drawn continuous for naturally
sunt quadrati, qui sese in subjecto ordine consequuntur, ut
are the-squares which one-another in the-subjoined order follow as
4, 9, 16, et caeteri. Si igitur continuum quadratum minorem
4 9 16 and the-rest if therefore a-continuous square lesser
a continuo quadrato majore sustulero, quod relinquitur
from the-continuous square greater I-shall-have-taken-away what is-left
tantum erit quantum est quod ab utrorumque quadratorum
so-much will-be as-much-as is what from of-both the-squares
lateribus jungitur; ut si quatuor auferam a novenario, quinque
the-sides is-joined as if four I-take-away from the-nine five
sunt reliqui, qui ex duobus et tribus, qui sunt utrorumque
are left which from two and three which are of-both

quadratorum latera, conjunguntur. Item si novenarium aufero
squares the-sides are-joined-together likewise if the-nine I-take-away
de eo qui sedecim numeris ascriptus est, 7 sunt
from that which at-sixteen among-the-numbers inscribed is 7 are
reliqui, qui scilicet ex ternario quaternarioque conjunctus est,
left which namely from the-three and-the-four joined is
qui praedictorum quadratorum latera sunt. Idemque est in
which of-the-aforesaid squares the-sides are and-the-same it-is in
caeteris. Quod si non sint continui quadrati, sed unus inter
the-rest but if not they-be continuous the-squares but one between
eos transmissus sit, fit ejus quod relinquitur medietas,
them passed-over be becomes of-that which is-left the-half
id quod ex utriusque lateribus efficitur, ut si quaternarium
that which from of-each the-sides is-made as if the-four
de 16 quadrato auferamus, 12 relinquuntur, quorum medietas
from 16 the-square we-take-away 12 are-left of-which the-half
est is numerus qui ex utrorumque lateribus confit.
is that number which from of-both the-sides is-made-up
Sunt autem utrorumque latera, duo et quatuor, quae senarium
are moreover of-both the-sides two and four which the-six
juncta perficiunt. Atque in caeteris idem modus est. Sin vero
joined complete and in the-rest the-same method is but-if indeed
duo intermittantur, tertia pars erit ejus quod relinquitur,
two be-skipped the-third part will-be of-that which is-left
id quod utrorumque latera conjungunt, ut si quatuor de 25
that which of-both the-sides join-together as if four from 25
auferam, intermissis duobus quadratis, reliqui 21 sunt. Eorum
I-take-away having-been-skipped two squares remaining 21 are of-them
vero latera sunt 2 et 5 qui efficiunt 7, qui sunt pars tertia
but the-sides are 2 and 5 which make 7 which are the-part third
numeri 21. Atque haec regula est, ut si tres intermissi sint,
of-the-number 21 and this the-rule is that if three skipped be

pars quarta sit, id quod ex utrorumque lateribus efficitur
the-part fourth may-be that which from of-both the-sides is-made
ejus quod subtracto minore a majore relinquitur.
of-that which having-been-subtracted the-lesser from the-greater is-left
Sin quatuor transmittantur, quinta, atque uno plus vocabulo
but-if four be-passed-over the-fifth and by-one more in-name
numeri partes venient, quam sit intermissio numerorum.
of-number the-parts will-come than may-be the-skipping of-the-numbers

Omnem inaequalitatem ex aequalitate procedere, ejusque demonstratio

II.7 · ALL INEQUALITY PROCEEDS FROM EQUALITY

As unity is the beginning of number, equality is the beginning of proportions. Three precepts — set the first equal to the first, the second to first plus second, the third to first, two seconds, and third — turn equal terms into multiples, as 1, 1, 1 becomes 1, 2, 4. Converted multiples beget superparticulars (4, 2, 1 gives 4, 6, 9), converted superparticulars superpartients (9, 6, 4 gives 9, 15, 25); left unconverted, they beget multiple superparticulars and multiple superpartients.

Est autem quemadmodum unitas pluralitatis numerique
is moreover just-as the-unit of-plurality and-of-number
principium, ita aequalitas proportionum. Tribus enim praeceptis
the-beginning so equality of-proportions by-three for precepts
(ut in arithmetica dictum est) multiplices proportiones
as in arithmetic said has-been the-multiple proportions
ex aequalitate producimus, ex conversis vero multiplicibus
from equality we-bring-forth from the-converted but multiples
superparticulares habitudines procreamus. Item ex conversis
the-superparticular relations we-beget likewise from the-converted
superparticularibus, superpartientes comparationes efficimus.
superparticulars the-superpartient comparisons we-effect

Ponantur enim tres unitates, vel tres binarii, vel tres ternarii,
let-there-be-set for three units or three twos or three threes
vel quotlibet aequi termini, et sit primus primo
or as-many-as-you-please equal terms and let-be the-first to-the-first
aequus in sequenti scilicet ordine constitutus. Secundus vero
equal in the-following namely row placed the-second but
primo ac secundo, tertius primo, duobus secundis ac tertio,
to-the-first and to-the-second the-third to-the-first to-two seconds and to-the-third
ita enim numero progresso, fit duplex multiplicitatis prima
so for the-number having-advanced arises the-double of-multiplicity the-first
proportio, ut haec subjecta descriptio monet. Nam unitas in
proportion as this subjoined diagram shows for the-unit in
secundo ordine constituta, aequa est primae unitati in superiore
the-second row placed equal is to-the-first unit in the-higher
loco dispositae. Item binarius aequus est unitati primae ac
place set-out likewise the-two equal is to-the-unit first and
secundae. Item quaternarius aequus est unitati primae, ac
second likewise the-four equal is to-the-unit first and
duabus unitatibus secundis atque unitati tertiae et est 1, 2, 4,
to-two units second and to-the-unit third and is 1 2 4
dupla proportio. Quod si de his idem feceris, tripla
the-double proportion but if from these the-same you-shall-have-done the-triple
comparatio procreabitur, ac de tripla quadrupla, de quadrupla
comparison will-be-begotten and from the-triple the-quadruple from the-quadruple
quincupla, ac deinceps talis currit habitudinum procreatio.
the-quintuple and thereafter such runs of-relations the-begetting
Rursus hisdem tribus praeceptis superparticulares fiunt, ut uno
again by-the-same three precepts the-superparticulars arise as by-one
probamus exemplo. Convertamus nunc, et priorem majorem
we-prove example let-us-convert now and first the-greater
numerum disponamus 4, 2, 1; ponatur igitur primus primo
number let-us-set-out 4 2 1 let-be-set therefore the-first to-the-first

aequus, id est 4; secundus primo scilicet et secundo, id est
equal that is 4 the-second to-the-first namely and to-the-second that is
6; tertius primo, duobus secundis, et tertio, id est 9; quibus
6 the-third to-the-first to-two seconds and to-the-third that is 9 which
dispositis sesquialtera notatur esse proportio. Atque id si
having-been-set-out the-sesquialter is-noted to-be proportion and this if
de triplis fiat, sesquitertia; si de quadruplis sesquiquarta,
from the-triples be-done the-sesquitertian if from the-quadruples the-sesquiquartan
consimilibusque in alterutra parte vocabulis proportionalitas ex
and-with-like in either part names proportionality from
multiplicitate nascetur, ex superparticularitate vero conversa
multiplicity will-be-born from superparticularity but converted
ducitur superpartiens habitudo. Disponatur enim conversim
is-derived the-superpartient relation let-be-set-out for conversely
sesquialtera comparatio, 9, 6, 4. Ponatur enim primus primo
the-sesquialter comparison 9 6 4 let-be-set for the-first to-the-first
aequus, id est 9; secundus primo et secundo, id est 15;
equal that is 9 the-second to-the-first and to-the-second that is 15
tertius primo duobus secundis ac tertio, id est 25; ac
the-third to-the-first to-two seconds and to-the-third that is 25 and
disponantur in ordinem hoc modo: Superbipartiens igitur
let-them-be-set-out into order in-this manner the-superbipartient therefore
ex conversis sesquialteris habitudo producta est; quod si
from the-converted sesquialters the-relation brought-forth has-been but if
quis ad hanc speculationem diligens scrutator accedat, ex
anyone to this speculation a-diligent searcher should-approach from
sesquitertiis conversis supertripartientem producit, caeterisque
sesquitertians converted the-supertripartient he-brings-forth and-with-the-rest
similibus vocabulis adaequatis cunctas ex superparticularitate
like names made-equal all from superparticularity
superpartientis species procreari mirabitur. Ex non conversis
of-the-superpartient the-species to-be-begotten he-will-marvel from not converted

autem superparticularibus, sed ita ut ex multiplici procreati
however superparticulars but so as from the-multiple begotten
sunt, manentibus, necesse est multiplices superparticulares
they-were (them-)remaining necessary it-is multiple superparticulars
creari. Ex manentibus vero superpartientibus, ita ut ex
to-be-created from remaining but the-superpartients so as from
superparticularibus prodierunt, non alii nisi multiplices
the-superparticulars they-came-forth not others unless multiple
superpartientes procreabuntur. Ac de his quidem hactenus,
superpartients will-be-begotten and about these indeed thus-far
diligentius enim in Arithmeticae libris de hac comparatione
more-diligently for in of-Arithmetic the-books about this comparison
est disputatum.
has-been discussed

Regula quotlibet continuas proportiones superparticulares inveniendi

II.8 · A RULE FOR CONTINUOUS SUPERPARTICULARS

A rule finds any number of continuous superparticular proportions. Each multiple, counted from unity, precedes as many superparticulars as its own place: the second double yields two sesquialters, the third three, and likewise for triples and sesquitertians. A second way multiplies the roots: from 2 and 3 come 4, 6, 9 — two sesquialters — and then 8, 12, 18, 27 for three.

Saepe autem accidit ut tres, vel quatuor, vel quotlibet
often moreover it-happens that three or four or as-many-as-you-please
aequas superparticularium proportiones de musica disputator
equal of-the-superparticulars proportions about music a-disputant
inquirat. Sed ne in casu atque inscitia facientes, error
may-seek but lest in chance and ignorance (us-)working error

ullus difficultatis impediat, hac regula quotlibet
any of-difficulty should-hinder by-this rule as-many-as-you-please
aequas proportiones ex multiplicitate ducemus. Unusquisque
equal proportions from multiplicity we-shall-derive each
multiplex ab unitate scilicet computatus, tot superparticulares
multiple from the-unit namely counted so-many superparticular
habitudines praecedit, sua scilicet in contrariam partem
relations precedes its-own namely in the-contrary direction
denominationis, quotus ipse ab unitate discesserit. Hoc
of-denomination how-manyeth itself from the-unit shall-have-departed in-this
modo ut duplex sesquialteras antecedat, triplex sesquitertias,
manner so-that the-double the-sesquialters may-precede the-triple the-sesquitertians
quadruplex sesquiquartas, ac deinceps in hunc modum.
the-quadruple the-sesquiquartans and thereafter in this manner
Sit igitur duplorum subjecta descriptio. In superiore igitur
let-there-be therefore of-the-doubles the-subjoined diagram in the-above therefore
descriptione binarius primus multiplex, unum ad se ternarium
diagram the-two the-first multiple one to itself the-three
habet, qui possit facere sesquialteram proportionem. Ternarius
has which can to-make the-sesquialter proportion the-three
vero non habet talium qui ejus possit esse sesquialter,
but not has (any)-of-such which of-it can to-be the-sesquialter
quoniam medietate deficit. Rursus quaternarius secundus
since by-a-half it-falls-short again the-four the-second
est duplex, hic duos sesquialteros antecedit senarium et
is double this-one two sesquialters precedes the-six and
novenarium, qui medietate caret. Atque idcirco nullus ei
the-nine which a-half lacks and therefore none to-it
habitudine sesquialtera comparatur, et in caeteris idem est.
by-relation sesquialter is-compared and in the-rest the-same is

Tripli vero eodem modo sesquitertios creant; sit enim
the-triples but in-the-same manner sesquitertians create let-there-be for
similis in triplo descriptio. In superiore igitur descriptione
a-like in the-triple diagram in the-above therefore diagram
sesquitertias proportiones ita natas videmus, ut primus
the-sesquitertian proportions so born we-see that the-first
triplex unum sesquitertium antecedat, secundus duos, tertius
triple one sesquitertian may-precede the-second two the-third
tres, semperque pars tertia in ultimo numero naturali
three and-always the-part third in the-last number by-a-natural
quodam fine claudatur. Quod si quadruplum statueris
certain end is-closed but if the-quadruple you-shall-have-set-up
eodem modo, sesquiquartos invenies, si quincuplum
in-the-same manner sesquiquartans you-will-find if the-quintuple
sesquiquintos. Ac deinceps singuli denominationis multiplicis
sesquiquintans and thereafter the-several of-denomination multiple
tot superparticulares praecedunt, quoto loco ipsi
so-many superparticulars precede in-how-manyeth place themselves
ab unitate discesserint. Unam vero tantum quadrupli
from the-unit they-shall-have-departed one but only of-the-quadruple
dispositionem ponemus, ut in ea sicut in caeteris lector
arrangement we-shall-set that in it just-as in the-rest the-reader
diligens acumen mentis exerceat. Haec igitur speculatio ad
diligent the-keenness of-mind may-exercise this therefore speculation for
hanc utilitatem videtur inventa, ut quotienscunque 4 vel
this usefulness seems devised that how-often-soever 4 or
6, vel quotlibet sesquialteros, vel sesquitertios, vel
6 or as-many-as-you-please sesquialters or sesquitertians or
sesquioctavos, vel quotlibet alias proportiones quis
sesquioctaves or as-many-as-you-please other proportions anyone
investigare voluerit, nullo errore labatur, utque non ei
to-search-out shall-have-wished by-no error may-be-slip and-that not to-that

numero primo tales proportiones quaerat aptare, qui quanti
number first such proportions he-may-seek to-fit which as-many-as
sint propositi, tot praecedere, et post se habere non possit.
may-be proposed so-many to-precede and after itself to-have not it-can
Sed disponat potius multiplices, videatque quantos
but let-him-set-out rather the-multiples and-let-him-see how-many
superparticulares requirit, eumque multiplicem respiciat, qui
superparticulars he-requires and-that multiple let-him-look-to which
eo loco ab unitate recesserit, ut est in superioribus
in-that place from the-unit shall-have-receded as is in the-above
descriptionibus, si tres sesquialteros fortasse quaesierit, non
diagrams if three sesquialters perhaps he-shall-have-sought not
a quaternario ingrediatur investigationem. Hic enim, quoniam
from the-four let-him-begin the-search this-one for since
secundus est duplus, duos tantum praecedit, tertiumque ei
the-second is double two only precedes and-a-third to-it
aptare non poterit, sed ut ab octonario medietates tentet
to-fit not he-will-be-able but that from the-eight halves let-him-try
apponere. Hic enim quoniam tertius est, tres quas quaerit
to-add this-one for since the-third is three which he-seeks
sesquialteras proportiones efficiet, et in caeteris eodem
sesquialter proportions he-will-effect and in the-rest in-the-same
modo. Est etiam alia augendi proportiones via hoc modo:
manner there-is also another of-increasing proportions way in-this manner
Radices proportionum dicuntur in eisdem comparationibus
the-roots of-proportions are-called in the-same comparisons
minimae proportionis. Disponatur enim numerus naturalis,
of-the-least of-proportion let-be-set-out for the-number natural
unitate multiplicatus. Minimae igitur proportiones sunt, ut
by-the-unit multiplied the-least therefore proportions are as
in sesquialtera 3 ad 2, in sesquitertia 4 ad 3, in sesquiquarta
in the-sesquialter 3 to 2 in the-sesquitertian 4 to 3 in the-sesquiquartan

5 ad 4, et deinceps in infinitum, et quaecunque se
5 to 4 and thereafter into the-infinite and whichever one-another
proportiones unitate praecesserint. Propositum igitur sit duas
proportions by-a-unit shall-have-preceded proposed therefore let-it-be two
sesquialteras proportiones continua comparatione producere.
sesquialter proportions in-continuous comparison to-produce
Sumo radicem sesquialteram, eamque dispono, 2 et 3.
I-take the-root sesquialter and-it I-set-out 2 and 3
Multiplico igitur binarium per binarium, fiunt 4. Item
I-multiply therefore the-two by the-two they-become 4 likewise
ternarius per binarium crescat, erunt 6. Rursus ternarium
the-three by the-two let-it-increase they-will-be 6 again the-three
in semetipsum ducemus, fiunt 9, qui disponuntur
into itself we-shall-draw they-become 9 which are-set-out
hoc modo: Invenimus igitur duas propositas sesquialteras
in-this manner we-have-found therefore the-two proposed sesquialter
proportiones 6 ad 4, et 9 ad 6. Sit nunc propositum tres,
proportions 6 to 4 and 9 to 6 let-it-be now proposed three
invenire. Dispono eosdem numeros quos supra in exquirendis
to-find I-set-out the-same numbers which above in seeking-out
duabus sesquialteris habitudinibus proposueram, ipsasque
the-two sesquialter relations I-had-proposed and-the-very
sesquialteras proportiones. Multiplico binario quaternarium,
sesquialter proportions I-multiply by-the-two the-four
fiunt octo. Rursus senarium binario, fiunt 12. Rursus
they-become eight again the-six by-the-two they-become 12 again
novenarium binario, fiunt 18. Rursus novenarium ternario,
the-nine by-the-two they-become 18 again the-nine by-the-three
fiunt 27. Disponantur igitur hoc modo: Atque hic
they-become 27 let-them-be-set-out therefore in-this manner and this
modus erit in caeteris, ut si sesquitertias proportiones
method will-be in the-rest that if the-sesquitertian proportions

velis extendere, ponas sesquitertiorum radices, quae sunt
you-wish to-extend you-may-set of-sesquitertians the-roots which are
quaternarius atque ternarius ad se invicem comparati.
the-four and the-three to themselves mutually compared
Atque ad hunc modum multiplices, quod si sesquiquartas
and after this manner you-may-multiply but if sesquiquartans
sesquiquartorum dispones radices eademque multiplicatione
of-sesquiquartans you-will-set-out the-roots and-by-the-same multiplication
sesquiquartos quotlibet extendes. Quantum autem
sesquiquartans as-many-as-you-please you-will-extend how-much moreover
nobis hae considerationes prosint, sequens ordo monstrabit.
to-us these considerations may-profit the-following order will-show

De proportione numerorum qui ab aliis metiuntur

II.9 · NUMBERS MEASURED BY OTHERS

If two numbers' difference measures both exactly, they stand in the proportion of the measuring counts: 5 measures 50 and 55 ten and eleven times, so they stand as 11 to 10. If the measuring leaves a remainder, the stripped numbers hold a greater proportion; if it overshoots, the increased numbers hold a lesser. The reason: a half is greater than a third, a third than a fourth — so among superparticulars, lesser numbers always carry the greater proportion.

Si duos numeros eorum differentia integre fuerit permensa,
if two numbers their difference wholly shall-have measured-through
in eadem sunt proportione numeri, quos sua differentia
in the-same are proportion the-numbers which their-own difference
mensa est, in qua erunt proportione etiam hi numeri
measured has in which they-will-be proportion also these numbers
secundum quos eos sua mensa est differentia. Sint enim
according-to which them their-own measured has difference let-there-be for

numeri 50, 55; hi ergo ad se invicem sesquidecima
the-numbers 50 55 these therefore to themselves mutually by-a-sesquitenth
habitudine comparantur, et est eorum differentia quinarius, qui
relation are-compared and is their difference the-five which
scilicet est pars decima numeri 50. Hic igitur metietur
namely is the-part tenth of-the-number 50 this-one therefore will-measure
quidem 50 numerum decies, 55 vero undecies; secundum
indeed the-50 number ten-times the-55 but eleven-times according-to
decem igitur atque 11 numeros 55 et 50 propria differentia,
ten therefore and 11 the-numbers 55 and 50 their-own difference
id est quinarius, permetietur, et sunt 11 ad 10 sesquidecima
that is the-five will-measure-through and are 11 to 10 by-a-sesquitenth
comparatione compositi. In eadem igitur sunt proportione
comparison composed in the-same therefore are proportion
numeri quos propria differentia integra permensa est, in qua
the-numbers which their-own difference entire measured-through has in which
sunt hi secundum quos eos propria differentia est permensa;
are these according-to which them their-own difference has measured-through
quod si qua differentia numerorum ita eos numeros quorum
but if any difference of-numbers so those numbers of-which
est differentia metiatur, ut eamdem mensuram numerorum
it-is the-difference should-measure that the-same measure of-the-numbers
pluralitas excedat, idemque in utrisque sit excessus, et sit
the-plurality should-exceed and-the-same in both be the-excess and be
diminutior differentiae mensura, quam est pluralitas numerorum,
smaller of-the-difference the-measure than is the-plurality of-the-numbers
majorem obtinebunt proportionem ad se invicem numeri,
a-greater will-obtain proportion to themselves mutually the-numbers
si eis illud quod relinquitur post mensionem, retractum sit,
if from-them that which is-left after the-measuring drawn-back be
quam fuerunt integri, cum eos propria differentia metiebatur.
than they-were whole when them their-own difference was-measuring

Sint enim numeri duo 53, 58; hos igitur quinarius, qui
let-there-be for numbers two 53 58 these therefore the-five which
est eorum differentia, metiatur. Metitur igitur 53 quinarius
is their difference let-it-measure it-measures therefore the-53 the-five
decies usque ad 50, relinquit vero ternarium. Rursus 58
ten-times up to 50 it-leaves but a-three again the-58
numerum metitur idem undecies usque ad 55, atque in eo
number measures the-same eleven-times up to 55 and in it
iterum ternarium derelinquit. Auferatur igitur ex utrisque
again a-three it-leaves-behind let-be-taken-away therefore from both
ternarius, fiunt 50 et 55, qui disponatur hoc modo: In
the-three they-become 50 and 55 which let-be-set-out in-this manner in
hoc igitur manifestum est majoris esse proportionis inter
this therefore manifest it-is of-a-greater to-be proportion between
se 50 et 55 quam 53 et 58. In minoribus enim numeris
themselves 50 and 55 than 53 and 58 in lesser for numbers
major semper proportio reperitur, quod paulo posterius
a-greater always proportion is-found which a-little later
demonstrabimus. Sin vero illa differentiae permensio
we-shall-demonstrate but-if indeed that of-the-difference measuring-through
numerorum multitudinem supervadat, eademque utrosque
of-the-numbers the-multitude should-overpass and-by-the-same both
numeri pluralitate praetereat, minores erunt proportiones
of-number plurality should-pass-by lesser will-be the-proportions
numeri superius mensi cum additione ejus summae qua
the-numbers above measured with the-addition of-that sum by-which
utrosque metiens differentia supervadit, quam fuerunt ante,
both measuring the-difference overpasses than they-were before
cum eos propria differentia metiebatur. Sint enim numeri
when them their-own difference was-measuring let-there-be for the-numbers
48 et 53, horum quinarius differentia est.
48 and 53 of-these the-five the-difference is

Metiatur igitur 48 numerum, quinarius decies fiunt
let-it-measure therefore the-48 number the-five ten-times they-become
50. Supervadit igitur 50 numerus 48 numerum binario,
50 overpasses therefore the-50 number the-48 number by-two
idem quinarius 53 undecies metiatur fiunt 55, qui
the-same the-five the-53 eleven-times let-it-measure they-become 55 which
eisdem rursus duobus 53 numerum supervadit; addatur
by-the-same again two the-53 number it-overpasses let-be-added
utrisque binarius et disponantur hoc modo: Minores igitur
to-both the-two and let-them-be-set-out in-this manner lesser therefore
sunt proportiones 50 ad 55 comparati, cum additione scilicet
are the-proportions 50 to 55 compared with the-addition namely
binarii quo differentia eos metiens supervadit, quam 48,
of-the-two by-which the-difference them measuring overpasses than 48
et 53 numeri quos eadem, quae tamen in eis supercrevit,
and 53 the-numbers which the-same which yet in them grew-over
quinarii differentia permensa est. Majores vero et minores
of-the-five difference measured-through has greater but and lesser
proportiones hoc modo intelliguntur. Dimidia pars major est
proportions in-this manner are-understood the-half part greater is
quam tertia. Tertia pars est major quam quarta. Quarta pars
than the-third the-third part is greater than the-fourth the-fourth part
major est quam quinta, ac deinceps eodem modo. Unde
greater is than the-fifth and thereafter in-the-same manner whence
fit ut sesquialtera proportio major sit sesquitertia, et
it-comes that the-sesquialter proportion greater be than-the-sesquitertian and
sesquitertia sesquiquartam vincat. Atque idem in caeteris.
the-sesquitertian the-sesquiquartan may-conquer and the-same in the-rest
Hinc evenit ut in numeris majoribus minor, et minoribus
hence it-comes-out that in numbers greater a-lesser and in-lesser
major semper videatur proportio superparticularium numerorum.
a-greater always may-be-seen proportion of-superparticular numbers

Quod apparet in numero naturali: disponatur enim numerus
which appears in the-number natural let-be-set-out for the-number
naturalis 1, 2, 3, 4. Binarius igitur ad unitatem duplus
natural 1 2 3 4 the-two therefore to the-unit double
est. Ternarius ad binarium sesquialter est. Quaternarius vero
is the-three to the-two sesquialter is the-four but
ad ternarium sesquitertius. Majores vero sunt numeri tres
to the-three sesquitertian greater but are the-numbers three
et 4. Minores vero binarius et unitas: in majoribus igitur
and 4 lesser but the-two and the-unit in the-greater therefore
minor, et in minoribus major proportio continetur: hinc
a-lesser and in the-lesser a-greater proportion is-contained hence
apparet quod si aliquibus numeris proportionem continentibus
it-appears that if some to-numbers a-proportion containing
superparticularem, aequa pluralitas addatur, majorem esse
superparticular an-equal plurality be-added greater to-be
proportionem ante aequae pluralitatis augmentum, quam
the-proportion before of-the-equal plurality the-increase than
posteaquam eis aequa sit addita pluralitas.
after to-them the-equal has-been added plurality

Quae ex multiplicibus et superparticularibus multiplicitate fiunt

II.10 · WHAT MULTIPLES AND SUPERPARTICULARS GENERATE

Boethius adds two rules that he will demonstrate a little later. A multiple interval multiplied by two yields a multiple, so a product that is not multiple proves the interval doubled was not multiple either. A superparticular multiplied by two yields neither a multiple nor a superparticular, so a product that is neither shows the interval doubled was superparticular or of some other kind, and not multiple.

Illud etiam praetermittendum non videtur, quod paulo post
this also to-be-passed-over not seems which a-little after
demonstrabitur, si multiplex intervallum binario fuerit
will-be-demonstrated if a-multiple interval by-two shall-have-been
multiplicatum, id etiam quod ex illa multiplicatione nascetur
multiplied that also which from that multiplication will-be-born
multiplex esse. Quod si id quod ex tali multiplicatione
multiple to-be but if that which from such multiplication
procreatum sit non fuerit multiplex, tunc illud non esse
begotten be not shall-have-been multiple then that not to-be
multiplex, quod binario fuerit multiplicatum. Item si
multiple which by-two shall-have-been multiplied likewise if
superparticularis proportio binario multiplicetur, id quod fit
a-superparticular proportion by-two be-multiplied that which arises
neque superparticulare esse, neque multiplex. Quod si id
neither superparticular to-be nor multiple but if that
quod ex tali multiplicatione nascetur, neque multiplex, neque
which from such multiplication will-be-born neither multiple nor
superparticulare est, tunc illud quod binario multiplicatum
superparticular is then that which by-two multiplied

est, vel superparticularis vel alterius generis est, non vero
has-been either of-the-superparticular or of-another kind is not indeed
multiplicis.
of-the-multiple

Qui superparticulares quos multiplices efficiant

II.11 · WHICH SUPERPARTICULARS MAKE WHICH MULTIPLES

The first two superparticulars joined make the first multiple: on 2, 3, 4, the sesquialter and the sesquitertian together create the double. The first multiple added to the first superparticular creates the second, as 2, 4, 6 shows the triple; a triple plus a sesquitertian makes a quadruple, a quadruple plus a sesquiquartan a quintuple, and so multiples are begotten without end.

His illud est addendum duos primos superparticulares
to-these this is to-be-added (that)-the-two first superparticulars
primam efficere multiplicem proportionem, ut sesquialter et
the-first to-effect multiple proportion as the-sesquialter and
sesquitertius si conjungantur duplicem creant. Sint enim
the-sesquitertian if they-be-joined the-double they-create let-there-be for
numeri 2, 3, 4, nam 3 ad 2 sesquialter est, 4 ad 3
the-numbers 2 3 4 for 3 to 2 sesquialter is 4 to 3
sesquitertius, quatuor ad duo duplus. Rursus primus multiplex
sesquitertian four to two double again the-first multiple
primo additus superparticulari secundum multiplicem creat.
to-the-first added superparticular the-second multiple creates
Sint enim numeri 2, 4, 6; 4 namque ad 2 duplex est,
let-there-be for the-numbers 2 4 6 4 for-indeed to 2 double is
primus scilicet multiplex. At 6 ad 4 sesquialter, qui est
the-first namely multiple but 6 to 4 sesquialter which is
primus superparticularis; 6 ad duos triplus, qui secundus
the-first superparticular 6 to two triple which the-second

est multiplex: quod si triplum sesquitertio addas, quadruplus
is multiple but if the-triple to-a-sesquitertian you-add a-quadruple
efficitur; si quadruplum sesquiquarto, quincuplus. Atque in
is-effected if the-quadruple to-a-sesquiquartan a-quintuple and in
hunc modum junctis proportionibus multiplicium ac
this manner having-been-joined the-proportions of-multiples and
superparticularium, in infinitum multiplices procreantur.
of-superparticulars into the-infinite the-multiples are-begotten

De arithmetica, geometrica, harmonica medietate

II.12 · THE ARITHMETIC, GEOMETRIC, AND HARMONIC MEANS

A proportion compares two terms, and a proportionality is a collection of equal proportions, so it needs three terms at least. The middle term joins the extremes in three ways: equal differences with unequal proportions give the arithmetic mean (1, 2, 3), equal proportions give the geometric (1, 2, 4), and the harmonic (3, 4, 6) sets the greater difference to the lesser as the greatest term is to the least. Boethius calls the geometric the proportionality proper, but uses the name for all three.

Quoniam vero de proportionibus quae erant interim tractandae
since but about the-proportions which were meanwhile to-be-treated
praediximus, nunc de medietatibus est dicendum. Proportio
we-have-said-before now about the-means (it)-is to-be-said a-proportion
enim est duorum ad se terminorum quaedam comparatio,
for is of-two to each-other terms a-certain comparison
terminos autem voco numerorum summas. Proportionalitas
terms moreover I-call of-numbers the-sums proportionality
est aequarum proportionum collectio. Proportionalitas vero in
is of-equal proportions a-collection proportionality but in
tribus terminis minimis constat. Constat autem plerumque
three terms (at)-fewest consists it-consists moreover often

in pluribus, ut in quatuor vel in sex terminis. Cum enim
in more as in four or in six terms when for
primus ad secundum terminum eamdem retinet proportionem
the-first to the-second term the-same retains proportion
quam secundus ad tertium, dicitur haec proportionalitas.
as the-second to the-third is-called this a-proportionality
Estque inter tres terminos medius, qui secundus est.
and-there-is between three terms a-middle-(term) which the-second is
Has igitur proportiones medii termini conjungentis, trina
these therefore proportions of-the-middle term conjoining-(them) threefold
partitio est. Aut enim aequa est differentia minoris termini
the-division is either for equal is the-difference of-the-lesser term
ad medium, et medii ad maximum, sed non aequa
to the-middle and of-the-middle to the-greatest but not equal
proportio, ut in his numeris 1, 2, 3; inter unum quippe ac
the-proportion as in these numbers 1 2 3 between one indeed and
duo et inter duo et tres tantum unitas differentiam tenet.
two and between two and three only the-unit the-difference holds
Non est autem aequa proportio. Duo quippe ad unum
not is however equal the-proportion two indeed to one
dupli sunt, ternarius ad duo sesquialter.
double are the-three to two sesquialter
Aut est aequa proportio in utrisque, non vero aequalibus
or is equal the-proportion in both not however with-equal
differentiis constituta, ut in his numeris 1, 2, 4. Nam duo
differences established as in these numbers 1 2 4 for two
ad unum ita sunt dupli, quemadmodum quaternarius ad
to one so are double just-as the-four to
binarium. Sed inter quaternarium binariumque binarius, inter
the-two but between the-four and-the-two the-two between
binarium atque unitatem unitas differentiam facit. Est vero
the-two and the-unit the-unit the-difference makes there-is but

tertium medietatis genus, quod neque eisdem proportionibus
a-third of-a-mean kind which neither in-the-same proportions
neque eisdem differentiis constat. Sed quemadmodum se
nor in-the-same differences consists but just-as itself
habet maximus terminus ad minimum, ita sese habet majorum
has the-greatest term to the-least so itself has of-the-greater
terminorum differentia ad minorum differentiam terminorum,
terms the-difference to of-the-lesser the-difference of-terms
ut in his numeris 3, 4, 6. Nam sex ad 3 duplus est, inter
as in these numbers 3 4 6 for six to 3 double is between
sex vero ac 4 binarius interest. Inter quaternarium vero ac
six but and 4 the-two lies-between between the-four but and
ternarium unitas. Sed binarius comparatus ad unitatem rursus
the-three the-unit but the-two compared to the-unit again
duplus est; ergo ut est maximus terminus in numeris ad
double is therefore as is the-greatest term in the-numbers to
minimum, ita majorum differentia ad minorum differentiam
the-least so of-the-greater the-difference to of-the-lesser the-difference
terminorum. Vocatur igitur illa medietas, in qua aequae sunt
of-terms is-called therefore that mean in which equal are
differentiae, arithmetica. Illa vero in qua aequae proportiones,
the-differences arithmetic that but in which equal the-proportions
geometrica. Illa autem quam tertiam descripsimus, harmonica.
geometric that moreover which (as)-third we-have-described harmonic
Quarum haec subjiciamus exempla: Non vero ignoramus
of-which these let-us-subjoin examples not indeed we-are-unaware
alias quoque esse proportionum medietates, quas quidem
other also (that-there)-be of-proportions means which indeed
in arithmetica diximus; sed ad praesentem tractatum hae
in the-arithmetic we-have-told but for the-present treatise these
sunt interim necessariae. Sed inter has tres medietates
are meanwhile necessary but among these three means

proportionalitas quidem proprie, et maxime geometrica
proportionality indeed properly and especially the-geometric
nuncupatur, idcirco quoniam aequis proportionibus tota
is-named for-this-reason since with-equal proportions wholly
contexitur. Sed tamen eodem utemur promiscue vocabulo,
it-is-woven but nevertheless the-same we-shall-use indiscriminately word
proportionalitates etiam caeteras nuncupantes.
proportionalities also the-rest naming

De continuis medietatibus et disjunctis

II.13 · CONTINUOUS AND DISJUNCT MEANS

A proportionality is continuous when one middle number serves both comparisons, as set out before. When there are two middles it is called disjunct, as in 1, 2, 3, 6, where 2 is to 1 as 6 is to 3. Continuous proportionality lives in as few as three terms, disjunct in four; a continuous one can run longer, as 1, 2, 4, 8, 16, and the proportions are always one fewer than the terms.

Sed in his alia continua est proportionalitas, alia
but among these one continuous is proportionality another
disjuncta. Continua quidem, ut superius disposuimus. Unus
disjunct continuous indeed as above we-have-set-out one
enim idemque numerus medius, nunc quidem majori
for and-the-same number the-middle now indeed to-the-greater
supponitur, nunc quidem minori praeponitur. Quotiens vero
is-set-under now indeed to-the-lesser is-set-before whenever but
duo sunt medii, tunc disjuncta proportionalitas nuncupatur,
two are the-middles then a-disjunct proportionality is-named
ut in geometrica hoc modo, 1, 2, 3, 6. Nam ut est binarius
as in the-geometric this way 1 2 3 6 for as is the-two
ad unitatem, ita senarius ad ternarium, et vocatur haec
to the-unit so the-six to the-three and is-called this

disjuncta proportionalitas; unde intelligi potest continuam
a-disjunct proportionality whence to-be-understood it-can (that)-continuous
quidem proportionalitatem in tribus et minimis terminis
indeed proportionality in three and the-fewest terms
inveniri, disjunctam vero in 4. Potest autem in 4 et in
to-be-found the-disjunct but in 4 (it)-can moreover in 4 and in
pluribus continua esse proportionalitas, siquidem hoc modo
more continuous to-be a-proportionality if-indeed this way
sit, 1, 2, 4, 8, 16. Sed hic non erunt duae proportiones, sed
it-be 1 2 4 8 16 but here not will-be two proportions but
plures, semperque una minus quam sint termini constituti.
more and-always one fewer than are the-terms established

Cur ita appellatae sint digestae superius medietates

II.14 · WHY THE MEANS ARE SO NAMED

Boethius explains the three names. The arithmetic mean is so called because the difference between its terms is equal according to number, the geometric because the quality of the proportion is alike, and the harmonic because it is fitted so that an equality of proportions is observed in both the differences and the terms. He treated all this more carefully in the Arithmetic and runs through it here only as a reminder.

Idcirco autem una eorum medietas arithmetica nuncupatur,
for-this-reason moreover one of-them mean the-arithmetic is-named
quod inter terminos, secundum numerum aequa est
because between the-terms according-to number equal is
differentia. Geometrica vero secunda dicitur, quod similis
the-difference the-geometric but second is-called because alike
est qualitas proportionis. Harmonica autem vocatur, quoniam
is the-quality of-the-proportion the-harmonic moreover is-called since
ita est coaptata, ut in differentiis ac terminis aequalitas
so it-is fitted-together that in the-differences and the-terms an-equality

proportionum consideretur. Et de his quidem diligentius
of-proportions may-be-observed and about these indeed more-carefully
in arithmeticis disputatum est, nunc vero ut
in the-arithmetical-(books) discussed (it)-has-been now but that
commemoremus, tantum ista percurrimus.
we-may-recall only these-things we-run-through

Quemadmodum ab aequalitate supradictae processerint medietates

II.15 · DERIVING THE MEANS FROM EQUALITY

Each mean is derived from equality by rules on three equal terms, since equality is to proportions what unity is to number. The harmonic derivation turns 1, 1, 1 into 3, 4, 6 — and there the musical consonances appear: 4 to 3 sounds the diatessaron, 6 to 4 the diapente, 6 to 3 the diapason. Multiplying the terms into one another yields 36, 24, 18, 16, 12, 9, which display the same consonances along with the triple, the quadruple, and the tone at 18 to 16.

Sed paulisper quemadmodum istae proportionalitates ab
but for-a-little-while how these proportionalities from
aequalitate procreantur dicendum est. Praedictum est
equality are-begotten to-be-said (it)-is said-before (it)-has-been
enim quod in numero valet unitas, idem in proportionibus
for that in number avails the-unit the-same in proportions
aequalitatem valere, et sicut numeri caput est unitas, ita
(that)-equality to-avail and just-as of-number the-head is the-unit so
proportionum aequalitatem esse principium. Quocirca hoc
of-proportions (that)-equality to-be the-beginning wherefore in-this
modo arithmetica medietas ab aequalitate nascetur: positis
way the-arithmetic mean from equality will-be-born having-been-set
enim tribus aequis terminis, hi duo modi sunt quibus haec
for three equal terms these two ways are by-which this

proportionalitas producatur. Ponatur enim primus primo
proportionality may-be-produced let-be-set for the-first to-the-first
aequus. Secundus primo et secundo. Tertius primo secundo
equal the-second to-the-first and to-the-second the-third to-the-first to-the-second
ac tertio, quod hoc monstratur exemplo: Sint unitates
and to-the-third which by-this is-shown example let-there-be units
tres. Ponatur igitur primus primo aequus id est unus.
three let-be-set therefore the-first to-the-first equal that is one
Secundus primo ac secundo, id est duo. Tertius primo
the-second to-the-first and to-the-second that is two the-third to-the-first
secundo ac tertio, id est tres, eritque dispositio talis:
to-the-second and to-the-third that is three and-will-be the-disposition such
Rursus sint tres binarii in aequalitate constituti, 2, 2, 2:
again let-there-be three twos in (an)-equality established 2 2 2
ponatur primus primo aequus, id est 2. Secundus primo et
let-be-set the-first to-the-first equal that is 2 the-second to-the-first and
secundo, id est 4. Tertius primo secundo ac tertio, id
to-the-second that is 4 the-third to-the-first to-the-second and to-the-third that
est 6, et erit dispositio haec: Sed in his hoc speculandum
is 6 and will-be the-disposition this but in these this to-be-observed
est, quodsi unitas fuerit ad aequalitatis principium
(it)-is that-if the-unit shall-have-been as of-the-equality the-beginning
constituta, unitas etiam erit in differentiis numerorum,
established a-unit also will-be in the-differences of-the-numbers
ipsi vero numeri inter se nullum intermittunt.
themselves but the-numbers between themselves no-(number) leave-between
Si vero binarius teneat aequalitatem, binarius est differentia,
if but the-two hold the-equality the-two is the-difference
et unus inter terminos semper numerus intermittitur.
and one between the-terms always number is-left-between
Sin vero ternarius idem differentia est, inter terminos
but-if indeed the-three the-same the-difference is between the-terms

vero duo naturaliter constituti intermittentur, ac deinceps
but two naturally established will-be-left-between and thenceforth
in hunc modum. Est etiam alia proportionalitatem
in this manner there-is also another the-proportionality
arithmeticam procreandi via. Ponantur enim tres aequi
arithmetic of-begetting way let-be-set for three equal
termini, constituaturque primus primo ac secundo aequus,
terms and-let-be-made the-first to-the-first and to-the-second equal
secundus primo ac duobus secundis, tertius primo et
the-second to-the-first and to-two seconds the-third to-the-first and
duobus secundis ac tertio, ut si sint tres unitates. Sit
to-two seconds and to-the-third as if there-be three units let-be
primus primo ac secundo aequus, id est 2, Secundus vero
the-first to-the-first and to-the-second equal that is 2 the-second but
primo, ac duobus secundis, id est 3. Tertius autem primo,
to-the-first and to-two seconds that is 3 the-third moreover to-the-first
duobus secundis ac tertio, id est quatuor. Hic igitur
to-two seconds and to-the-third that is four here therefore
terminorum differentiam unitas tenet; inter binarium enim et
of-the-terms the-difference the-unit holds between the-two for and
unitatem, atque inter ternarium ac binarium, unitas interest.
the-unit and-also between the-three and the-two the-unit lies-between
Nullus vero naturalis numerus intermittitur. Post unitatem
no indeed natural number is-left-between after the-unit
enim mox binarius est, ac post binarium ternarius
for straightway the-two is and after the-two the-three
naturaliter constitutus, idem rursus in binario fiat.
naturally established the-same again in the-two let-be-done
Sintque tres binarii, et sit primus primo ac secundo
and-let-there-be three twos and let-be the-first to-the-first and to-the-second
aequus, id est quaternarius; secundus vero primo et
equal that is the-four the-second but to-the-first and

duobus secundis, id est senarius; tertius autem primo,
to-two seconds that is the-six the-third moreover to-the-first
duobus secundis ac tertio, id est octonarius. Hic quoque
to-two seconds and to-the-third that is the-eight here too
binarius tenet differentiam terminorum, uno inter eos
the-two holds the-difference of-the-terms one between them
naturaliter intermisso. Nam inter 4 et 6 quinarius naturaliter
naturally left-between for between 4 and 6 the-five naturally
intermittitur, inter 6 atque 8 septenarius collocatur. Quod
is-left-between between 6 and 8 the-seven is-placed as-to-which
si ternarius aequalitatis principium sit, fiet ternarius
if the-three of-the-equality the-beginning be will-become the-three
differentia; verbi gratia: sint termini tres ad regulam
the-difference of-a-word for-the-sake let-be terms three by the-rule
superiorum subtus. In his ergo ternarius est differentia, et
of-the-above below in these therefore the-three is the-difference and
duo numeri intermissi, id est uno minus quam sit differentia
two numbers left-between that is (by)-one fewer than is the-difference
semper numeris intermissis, atque idem et in quaternario,
always the-numbers being-left-between and-also the-same also in the-four
quinarioque perspicitur, et quae nos propter brevitatem
and-the-five is-seen and what-(things) we on-account-of brevity
tacemus, iisdem regulis ex semetipso diligens lector inveniet.
pass-in-silence by-the-same rules from himself the-diligent reader will-find
Geometrica vero proportionalitas tunc quemadmodum inveniri
the-geometric but proportionality then how to-be-found
ab aequalitate possit ostendimus, quando quemadmodum
from equality may-be-able we-showed when how
ab aequalitate omnis inaequalitas profluit monstrabamus, nisi
from equality all inequality flows-forth we-were-demonstrating unless
tamen fastidium est, nunc quoque breviter repetendum est.
however weariness it-is now too briefly to-be-repeated (it)-is

Constitutis enim tribus aequis terminis, ponatur primus
having-been-established for three equal terms let-be-set the-first
primo aequus. Secundus primo ac secundo. Tertius primo
to-the-first equal the-second to-the-first and to-the-second the-third to-the-first
duobus secundis ac tertio. Idemque fiat continue atque
to-two seconds and to-the-third and-the-same let-be-done continually and
ita ex aequalitate geometrica proportionalitas principium
so from equality the-geometric proportionality its-beginning
sumat. Sed de harum proportionum proprietatibus quam
may-take but about of-these proportions the-properties as
diligentissime in arithmeticis diximus; quod si ad
most-carefully in the-arithmetical-(books) we-have-told as-to-which if to
haec illis instructus lector accedat, nullo dubitationis
these with-those equipped the-reader approach by-no of-doubt
errore turbabitur. Harmonica vero medietas, de qua nunc
error he-will-be-troubled the-harmonic but mean about which now
paulo latius tractandum est, hac ratione procreatur.
a-little more-broadly to-be-treated (it)-is by-this reckoning it-is-begotten
Constituatur enim, siquidem duplices curamus effingere, tribus
let-be-established for if-indeed doubles we-care to-fashion three
aequis terminis positis, primus primo ac duobus secundis
equal terms having-been-set the-first to-the-first and to-two seconds
aequalis. Secundus duobus primis et duobus secundis. Tertius
equal the-second to-two firsts and to-two seconds the-third
semel primo, bis secundo, et ter tertio. Atque hoc
once to-the-first twice to-the-second and thrice to-the-third and in-this
modo sint unitates. Constituatur igitur primus primo
way let-there-be units let-be-established therefore the-first to-the-first
ac duobus secundis aequalis, id est ternarius. Secundus
and to-two seconds equal that is the-three the-second
vero duobus primis et duobus secundis, id est quaternarius.
but to-two firsts and to-two seconds that is the-four

Tertius vero primo ac duobus secundis et tribus tertiis, id
the-third but to-the-first and to-two seconds and to-three thirds that
est sex. Et si in binariis aequalitas construatur, vel in ternariis
is six and if in twos the-equality be-constructed or in threes
eadem ratio medietatis apparet, duplo a se terminis
the-same reckoning of-the-mean appears by-a-double from each-other the-terms
differentiisque distantibus, ut subjectae descriptiones monent.
and-the-differences standing-apart as the-subjoined diagrams show
Quod si facienda est in extremitatibus tripla proportio, tribus
as-to-which if to-be-made is in the-extremes a-triple proportion three
aequis terminis constitutis, primus quidem faciendus est
equal terms having-been-established the-first indeed to-be-made is
ex primo ac secundo, secundus vero ex primo ac duobus
from the-first and the-second the-second but from the-first and two
secundis, tertius autem ex primo, duobus secundis ac tribus
seconds the-third moreover from the-first two seconds and three
tertiis, ut est subjecta descriptio. Sed ingressi harmonicam
thirds as is the-subjoined diagram but having-entered the-harmonic
disputationem, quae de ea diligentius dici possunt tacite
discussion what-(things) about it more-carefully to-be-said can silently
praetereunda esse non arbitror. Collocetur igitur harmonica
to-be-passed-over to-be not I-judge let-be-placed therefore the-harmonic
proportionalitas, inque ea descriptione superiore ordine
proportionality and-in that diagram in-the-upper row
terminorum inter se differentiae disponantur. Videsne igitur
of-the-terms among themselves the-differences let-be-arranged do-you-see therefore
ut quatuor ad tres diatessaron consonantiam prodant, sex ad
how four to three the-diatessaron consonance put-forth six to
quatuor diapente concordent; sex vero ad tres diapason misceant
four (in)-diapente accord six but to three the-diapason mingle
symphoniam, ipsaeque earum differentiae rursus eamdem
symphony and-the-very of-them differences again the-same

statuant consonantiam. Binarius enim ad unitatem duplus est in
may-establish consonance the-two for to the-unit double is in
diapason consonantia constitutus; quod si se extremitates
the-diapason consonance established as-to-which if themselves the-extremes
multiplicent, itemque medius sui multiplicitate succrescat,
multiply and-likewise the-middle of-itself by-the-multiplication grow
comparati numeri toni habitudinem concordiamque servabunt.
the-compared numbers of-the-tone the-relation and-concord will-keep
Ter enim sex efficiunt 18, quater quatuor fiunt 16. Sed
thrice for six they-make 18 four-times four they-become 16 but
18 numerus, 16 numerum minoris parte octava transcendit.
18 the-number 16 the-number of-the-lesser by-a-part eighth surpasses
Rursus minimus terminus si se ipse multiplicet, efficiet
again the-least term if itself (it)-itself multiply it-will-make
9. Quod si major terminus sui multiplicatione concrescat,
9 as-to-which if the-greater term of-itself by-the-multiplication grow-together
efficiet 36, qui sibimet comparati, quadruplam, id est bis
it-will-make 36 which to-themselves compared a-quadruple that is twice
diapason, concinentiam servant. Quod si haec diligentius
diapason concord keep as-to-which if these-(things) more-carefully
inspiciamus, haec erit omnis rei differentiarum vel
we-inspect this will-be of-the-whole matter of-the-differences or
terminorum in se invicem multiplicatio. Minimus enim
of-the-terms into themselves in-turn the-multiplication the-least for
terminus, si medio multiplicetur, fient 12. Item
term if by-the-middle be-multiplied they-will-become 12 likewise
minimus terminus, si maximo multiplicetur, fient 18.
the-least term if by-the-greatest be-multiplied they-will-become 18
Medius vero terminus, si maximi numerositate augeatur,
the-middle but term if of-the-greatest by-the-numerosity be-increased
fiant 24. Rursus terminus minimus, si seipso concrescat,
they-become 24 again the-term least if by-itself grow-together

fient 9. Eodem modo si medius, fiant 16. Senarius
they-will-become 9 in-the-same way if the-middle they-become 16 the-six
vero, qui maximus est, si seipsum multiplicet, 36 reddet:
but which greatest is if itself it-multiply 36 it-will-render
haec igitur in ordinem disponantur, 36, 24, 18, 16, 12, 9.
these therefore in order let-be-arranged 36 24 18 16 12 9
Sunt igitur diatessaron consonantiam resonantes 24 ad 18,
(there)-are therefore the-diatessaron consonance sounding-forth 24 to 18
et 12 ad 9, diapente vero 18 ad 12, et 24 ad 16, et 36 ad
and 12 to 9 the-diapente but 18 to 12 and 24 to 16 and 36 to
24. Tripla autem quae est diapason et diapente 36 ad 12.
24 the-triple moreover which is diapason and diapente 36 to 12
Quadrupla vero quae est bis diapason 36 ad 9. Epogdous
the-quadruple but which is twice diapason 36 to 9 the-epogdoal
vero qui tonus est 18 ad 16 comparatione servatur.
but which a-tone is 18 to 16 by-the-comparison is-kept

Quemadmodum inter duos terminos supradictae medietates vicissim collocentur

II.16 · PLACING THE MEANS BETWEEN TWO TERMS

Given two terms, each mean can be set between them; Boethius works all three between 10 and 40. Arithmetic: halve the difference of 30 and add 15 to the lesser — 10, 25, 40. Geometric: multiply the extremes to 400 and take the square side — 10, 20, 40. Harmonic: multiply the difference by the lesser term, divide the 300 by the sum, 50, and add the 6 to the lesser — 10, 16, 40.

Solent autem duo termini dari proponique, ut inter
are-wont moreover two terms to-be-given and-to-be-proposed that between
eos nunc quidem arithmeticam, nunc vero geometricam,
them now indeed the-arithmetic now but the-geometric

nunc harmonicam medietatem ponamus; de quibus in
now the-harmonic mean we-may-set about which in
arithmeticis quoque diximus, id tamen ipsum nunc
the-arithmetical-(books) too we-have-told this nevertheless very-(thing) now
etiam breviter explicemus. Si arithmetica medietas quaeritur,
also briefly let-us-unfold if the-arithmetic mean is-sought
datorum terminorum videnda differentia est, eademque
of-the-given terms to-be-seen the-difference is and-the-same
dividenda ac minori termino adjicienda. Sint enim decem et
to-be-divided and to-the-lesser term to-be-added let-be for ten and
40 altrinsecus termini constituti, horumque medietas secundum
40 on-either-side the-terms established and-of-these the-mean according-to
arithmeticam proportionalitatem quaeratur. Differentiam prius
the-arithmetic proportionality let-be-sought the-difference first
utrorumque respicio, quae est 30; hanc divido, fiunt 15;
of-both I-look-at which is 30 this I-divide they-become 15
hanc minori termino, id est decem appono, fiunt 25.
this to-the-lesser term that is ten I-set-to they-become 25
Si igitur hic inter 10 et 40 medius collocetur, fit
if therefore this-(number) between 10 and 40 as-middle be-placed it-becomes
arithmetica proportionalitas hoc modo, 10, 25, 40. Item
an-arithmetic proportionality in-this way 10 25 40 likewise
inter eosdem terminos medietatem geometricam collocemus.
between the-same terms a-mean geometric let-us-place
Extremos propria numerositate multiplico, ut 10 in 40
the-extremes by-their-own numerosity I-multiply as 10 into 40
fiunt 400; horum tetragonale latus assumo, fiunt
they-become 400 of-these the-square side I-take-up they-become
viginti. Vicies enim viginti fiunt 400. Hos igitur 20
twenty twenty-times for twenty they-become 400 these therefore 20
medios inter 10 ac 40 si collocem, fit geometrica
as-middles between 10 and 40 if I-place it-becomes a-geometric

medietas subjecta descriptione formata, 10, 20, 40. Si vero
mean by-the-subjoined diagram formed 10 20 40 if but
harmonicam medietatem quaeramus, sibimetipsos copulamus
the-harmonic mean we-seek to-themselves we-couple
extremos, ut 10 et 40 fient 50. Eorum differentiam
the-extremes as 10 and 40 they-will-become 50 of-them the-difference
quae est 30 in minorem terminum multiplicemus, scilicet in
which is 30 into the-lesser term let-us-multiply namely into
10, ut fiant decies 30, qui sunt 300; hos si secundum 50
10 that they-become ten-times 30 which are 300 these if by 50
partimur, fiunt 6. Quos cum minori termino addiderimus,
we-divide they-become 6 which when to-the-lesser term we-shall-have-added
fient 16. Hunc igitur numerum si inter 10 ac 40 medium
they-will-become 16 this therefore number if between 10 and 40 as-middle
collocemus, harmonica proportionalitas expedietur 10, 16, 40.
we-place the-harmonic proportionality will-be-unfolded 10 16 40

De consonantiarum modo secundum Nicomachum

II.17 · NICOMACHUS ON THE ORDER OF CONSONANCES

Boethius turns to how the Pythagoreans prove the consonances live in the stated proportions — noting Ptolemy withholds assent, as Boethius will report later. The diapason comes first because the double is the relation best known to sense. On the numbers 1 through 4: 2 to 1 is diapason, 3 to 1 diapason-plus-diapente, 4 to 1 double diapason, 3 to 2 diapente, 4 to 3 diatessaron. Nicomachus therefore orders them: diapason, diapason-plus-diapente, double diapason, diapente, diatessaron.

Sed his hactenus. Nunc illud addendum videtur
but with-these thus-far now that to-be-added seems
quemadmodum Pythagorici probant consonantias musicas
how the-Pythagoreans prove (that)-the-consonances musical

in praedictis proportionibus inveniri, in qua re scilicet
in the-aforesaid proportions to-be-found in which matter to-wit
eis Ptolemaeus non videtur assensus, de quo paulo
to-them Ptolemy not seems to-have-assented about which a-little
posterius dicemus. Haec enim ponenda est maxime
later we-shall-speak this for to-be-laid-down is most-of-all
esse prima suavisque consonantia, cujus proprietatem sensus
to-be first and-sweet consonance whose property the-sense
apertior comprehendit. Quale enim est unumquodque per
more-open grasps of-what-sort for is each-thing through
semetipsum, tale et deprehenditur sensu. Si igitur cunctis
its-very-self such also it-is-detected by-sense if therefore to-all
notior est ea consonantia quae in duplicitate consistit, non
better-known is that consonance which in doubleness consists not
est dubium primam esse omnium diapason consonantiam,
is doubt (that)-first to-be of-all the-diapason consonance
meritoque excellere, quoniam cognitione praecedat. Reliquae
and-deservedly to-excel since in-cognition it-precedes the-rest
vero hunc necessario secundum Pythagoricos, ordinem tenent,
but this necessarily according-to the-Pythagoreans order hold
quem dederint multiplicitatis augmenta et superparticularis
which have-given of-multiplicity the-increases and of-the-superparticular
habitudinis detrimenta. Monstratum quippe est quod
relation the-decreases shown indeed it-has-been that
multiplex inaequalitas superparticulares proportiones meriti
the-multiple inequality the-superparticular proportions of-merit
antiquitate transcendat. Quocirca naturalis numerus ab
by-antiquity surpasses wherefore the-natural number from
unitate usque ad quaternarium disponatur, 1, 2, 3, 4. Igitur
the-unit up to the-four let-be-arranged 1 2 3 4 therefore
uni binarius comparatus proportionem duplicem facit, et
to-one the-two compared a-proportion double makes and

reddit diapason consonantiam eam, quae est maxima, et
renders the-diapason consonance that-(one) which is greatest and
simplicitate notissima.
in-simplicity best-known
Si vero unitati ternarius comparetur, diapason ac diapente
if but to-the-unit the-three be-compared the-diapason and diapente
concordiam personabit. Quaternarius vero unitati comparatus
concord it-will-sound-forth the-four but to-the-unit compared
quadruplam tenet, scilicet bis diapason efficiens symphoniam;
a-quadruple holds namely twice diapason effecting symphony
quod si ternarius binario comparetur, diapente. Si vero
as-to-which if the-three to-the-two be-compared diapente if but
quaternarius ternario, diatessaron consonantiam supplet, isque
the-four to-the-three the-diatessaron consonance it-supplies and-this
est horum ordo cunctis ad se invicem comparatis. Nam
is of-these the-order all to each-other in-turn compared for
comparatio quae restat, si quaternarium binario comparemus,
the-comparison which remains if the-four to-the-two we-compare
cadet in duplicem proportionem, quam tenebat ad unitatem
will-fall into the-double proportion which was-holding to the-unit
binarius comparatus; itaque maxime distant soni in bis
the-two compared and-so most stand-apart the-sounds in the-twice
diapason cum a se quadrupla intervalli dimensione
diapason when from each-other by-the-quadruple of-the-interval measure
discedunt. Minimum vero, cum acutior graviorem tertia
they-depart least but when the-higher the-lower by-a-third
gravioris parte transcendit, ac stat deinceps concinentiarum
of-the-lower part surpasses and stands thenceforth of-the-concords
modus, qui neque ultra quadruplam possit extendi, neque
the-measure which neither beyond the-quadruple can to-be-stretched nor
intra partem tertiam coarctari; et secundum Nicomachum
within the-part third to-be-narrowed and according-to Nicomachus

quidem hic consonantiarum est ordo, ut sit prima diapason,
indeed this of-the-consonances is the-order that be first the-diapason
secunda diapason et diapente, tertia bis diapason, quarta
second diapason and diapente third twice diapason fourth
diapente, quinta diatessaron.
diapente fifth diatessaron

De ordine consonantiarum sententia Eubulidis et Hippasi

II.18 · EUBULIDES AND HIPPASUS ON THE ORDER OF CONSONANCES

Eubulides and Hippasus set another order: increases of multiplicity answer decreases of superparticularity step for step — no double without a half, no triple without a third. From the double comes the diapason, from its contrary division the diapente; mixed, they beget the triple; its contrary third part yields the diatessaron; triple joined to sesquitertian makes the quadruple. Their order: diapason, diapente, diapason-plus-diapente, diatessaron, double diapason.

Sed Eubulides atque Hippasus alium consonantiarum ordinem
but Eubulides and Hippasus another of-the-consonances order
ponunt: aiunt enim multiplicitatis augmenta superparticularitatis
set-down they-say for (that)-of-multiplicity the-increases of-superparticularity
diminutioni rato ordine respondere. Itaque non posse esse
to-the-diminution in-fixed order to-answer and-so not to-be-able to-be
duplum nisi dimidium, nec triplum praeter tertiam partem.
a-double unless a-half nor a-triple without a-third part
Quoniam igitur sit duplum, ex eo diapason consonantiam
since therefore there-is the-double from it (that)-the-diapason consonance
reddi. Quoniam vero sit dimidium, ex eo quasi contraria
to-be-rendered since but there-is the-half from it as-if by-a-contrary
divisione sesquialteram, id est diapente, effici proportionem.
division (that)-a-sesquialter that is diapente to-be-effected proportion

Quibus mixtis, scilicet diapason ac diapente, triplicem
these being-mixed namely the-diapason and diapente (that)-a-triple
procreari, quae utramque contineat symphoniam, sed rursus
to-be-begotten which each may-contain symphony but again
triplicis partem tertiam contraria divisione partiri. Ex quo
of-the-triple the-part third by-a-contrary division to-be-parted from which
rursus diatessaron symphonia nascetur. Triplicem vero atque
again the-diatessaron symphony will-be-born (that)-the-triple but and
sesquitertium junctos quadruplam comparationem proportionis
the-sesquitertian joined the-quadruple comparison of-proportion
efficere; unde fit ut ex diapason et diapente quae
to-effect whence it-comes-to-be that from diapason and diapente which
est una consonantia, et diatessaron quaedam continentia
is one consonance and diatessaron a-certain holding-together
conjungatur, quae in quadruplo consistens bis diapason nomen
be-conjoined which in the-quadruple consisting twice diapason the-name
accepit. Secundum hos quoque hic ordo est: Diapason,
has-received according-to these-(men) too this the-order is diapason
diapente, diapason ac diapente, diatessaron, bis diapason.
diapente diapason and diapente diatessaron twice diapason

Sententia Nicomachi quae quibus consonantiis apponantur

II.19 · NICOMACHUS ON WHICH RATIOS OPPOSE WHICH

Nicomachus rejects the pairing just described. As unity begins both increase and diminution, the diapason begins the consonances: the sesquialter is opposed to the triple, the sesquitertian to the quadruple, and the double, having no opposite, comes first. Still, he says, multiples rightly precede superparticulars. Sound comes from a pulse and a pulse from motion: moderately unequal motions yield the simple proportions, while widely or obscurely unequal ones yield dissonance.

Sed Nicomachus non eamdem esse eis arbitratur contrariam
but Nicomachus not the-same to-be for-them thinks the-contrary
positionem. Sed potius ut unitas in arithmeticis crementi
position but rather as the-unit in the-arithmetical-(books) of-increase
erat diminutionisque principium, ita etiam diapason
was and-of-diminution the-beginning so also (that)-the-diapason
symphoniam reliquarum esse principium, illas vero sibi
symphony of-the-rest to-be the-beginning (that)-those but for-themselves
e contraria divisione posse constitui; id vero facilius erit
from the-contrary division to-be-able to-be-established this but easier will-be
cognitu si prius praevideatur in numeris. Constituatur igitur
to-know if first it-be-foreseen in numbers let-be-established therefore
unitas, duaeque partes ab ea fluant, una multiplicis, alia
the-unit and-two parts from it let-flow one of-the-multiple the-other
divisionis. Sitque haec formula: Et ad hunc modum ad infinita
of-division and-let-be this the-scheme and after this manner to infinity
progressio est. Binarius enim unitatis duplex est. Contraria vero
the-progression is the-two for of-the-unit the-double is the-contrary but
ejus pars eisdem dimidium unitatis ostendit. Tres triplus
of-it part to-the-same the-half of-the-unit shows three (is)-the-triple
et contraria pars tertia. Quatuor quadruplus, parsque et
and the-contrary part a-third four the-quadruple and-the-part also

contraria quarta. Atque ita crescendi et decrescendi in simplici
the-contrary a-fourth and so of-increasing and of-decreasing in the-simple
est unitate principium. Idem igitur nunc ad consonantias
is unit the-beginning the-same therefore now to the-consonances
convertamus. Est igitur diapason quae dupla est supremi
let-us-turn is therefore the-diapason which double is of-the-supreme
loco principii, quae vero reliquae sunt, in contraria
in-the-place of-the-principle (the-ones)-which but remaining are in the-contrary
divisione hoc modo. Sesquialter quidem triplo, sesquitertius
division in-this way the-sesquialter indeed to-the-triple the-sesquitertian
vero quadruplo quod tali argumentatione probabitur.
but to-the-quadruple which by-such argumentation will-be-proved
Idem enim primum est sesquialter, qui primus triplus
the-same for first is sesquialter which (is)-the-first triple
principalis, scilicet unitatis. Nam ternarius idem primus
principal namely of-the-unit for the-three the-same the-first
triplus est, scilicet unitati. Idem primus sesquialter,
triple is namely to-the-unit the-same (is)-the-first sesquialter
si binario comparetur. Rursus idem ternarius ejusdem
if to-the-two it-be-compared again the-same three of-the-same
differentiae quam ad binarium facit, cujus naturaliter positus
difference which to the-two it-makes of-which naturally placed
probatur esse sesquialter, triplus est. Cum igitur jure
it-is-proved to-be sesquialter the-triple is since therefore rightly
sesquialter triplici opponatur, diapente consonantia, diapente
the-sesquialter to-the-triple is-opposed the-diapente consonance to-the-diapente
ac diapason consonantiae rationabiliter putatur opponi.
and diapason consonance reasonably is-thought to-be-opposed
Rursus quadruplus sesquitertii contrariam divisionem tenet.
again the-quadruple of-the-sesquitertian the-contrary division holds
Nam qui est primus quadruplus, idem rursus primus
for (that)-which is the-first quadruple the-same again (as)-first

sesquitertius invenitur hoc modo. Quaternarius quippe
sesquitertian is-found in-this way the-four indeed
primus est quadruplus, si unitati primus sesquitertius, si
first is quadruple if to-the-unit (as)-first sesquitertian if
ternario comparetur. Rursus ejus differentiae, quam inter se
to-the-three it-be-compared again of-that difference which between itself
ac ternarium tenet, ipse fit quadruplus. Unde fit
and the-three it-holds itself becomes the-quadruple whence it-comes-about
ut sesquitertia proportio, quae est diatessaron, quadruplae
that the-sesquitertian proportion which is diatessaron to-the-quadruple
proportioni, quae est bis diapason, in contrarium dividatur.
proportion which is twice diapason into the-contrary is-divided
Dupla vero quoniam nullam habet oppositam proportionem,
the-double but since no it-has opposed proportion
nec ullius ipsa sesquialtera est, aut exstat numerus
nor of-any itself sesquialter is or there-stands-forth a-number
cui possit binarius, qui primus est duplus, superparticulari
to-which could the-two which first is double in-superparticular
proportione conjungi, talem formam contrariae proportionis
proportion to-be-joined such a-form of-contrary proportion
excedit. Atque idcirco secundum Nicomachum diapason
it-exceeds and therefore according-to Nicomachus the-diapason
consonantiarum principium teneat hoc modo.
of-consonances the-beginning let-hold in-this way
Sed quamvis ita sese habeat, inquit tamen omnes
but although so itself it-may-have he-says yet (that)-all
melius multiplices proportiones consonantiarum praecedere,
better the-multiple proportions of-consonances to-precede
superparticulares sequi, sicut paulo ante descripsimus. Cum
the-superparticulars to-follow as a-little before we-described since
igitur consonantia sit duarum vocum rata permixtio, sonus
therefore consonance is of-two voices a-fixed mingling sound

vero modulatae vocis casus, una intentione productus,
but of-a-modulated voice a-falling by-one tension produced
sitque idem minima particula modulationis, omnis vero
and-is the-same the-least particle of-modulation every but
sonus constet impulsu, pulsus vero omnis ex motu fiat,
sound consists of-impulse pulse but every from motion comes-to-be
cumque motuum aliqui sint aequales, alii vero inaequales,
and-since of-motions some are equal others but unequal
inaequalium vero alii sint multo inaequales, alii vero minus,
of-the-unequal but some are by-much unequal others but less
alii vero mediocriter inaequales. Ex aequalitate quidem
others but moderately unequal from equality indeed
nascitur sonorum aequalitas. Ex inaequalitate vero ea quae
is-born of-sounds equality from inequality but that which
secundum mediocritatem distantiae inaequales sunt, manifestae,
according-to a-mean of-distance unequal are manifest
primaeque ac simpliciores eveniunt proportiones, quae sunt
and-first and simpler come-forth the-proportions which are
scilicet multiplices aut superparticulares, dupli, tripli,
namely multiple or superparticular of-the-double of-the-triple
quadrupli, sesquialteri, atque sesquitertii consonantiae. Ex
of-the-quadruple of-the-sesquialter and of-the-sesquitertian the-consonances from
his vero quae in reliquis proportionibus, vel multimodis,
those but which in the-remaining proportions either manifold
vel non ita claris, vel longe omnino a se distantibus
or not so clear or far altogether from each-other distant
inaequalitates fiunt, dissonantiae existunt. Nulla autem
inequalities come-to-be dissonances arise no moreover
sonorum concordia procreatur.
of-sounds concord is-begotten

Quid oporteat praemitti ut diapason in multiplici genere demonstretur

II.20 · PRELIMINARIES TO THE DIAPASON PROOF

Before the diapason proof comes a brief lemma: take from any superparticular the superparticular continuous with it — the lesser one — and what remains is less than half of the proportion subtracted. Subtracting the sesquitertian from the sesquialter leaves the sesquioctave, and a doubled sesquioctave falls short of a whole sesquitertian by the distance found in the semitone; so a single sesquioctave is not the sesquitertian's full half, and the same holds in the rest.

Hoc igitur ita distincto, demonstrabitur diapason
this therefore thus distinguished it-will-be-demonstrated (that)-the-diapason
consonantiam quae cunctarum optima est in multiplici
consonance which of-all the-best is in the-multiple
inaequalitatis genere, et in duplicitatis habitudine reperiri.
of-inequality genus and in of-doubleness the-relation to-be-found
Ac primum quidem illud demonstrandum, quemadmodum in
and first indeed that (is)-to-be-demonstrated in-what-way in
multiplicitatis genere diapason consonantia possit agnosci.
of-multiplicity the-genus the-diapason consonance can to-be-recognized
Praecurrendum est igitur ad breve quiddam, quo prius cognito
to-be-run-ahead it-is therefore to a-brief something which first being-known
facilior demonstratio fiat: ab omni superparticulari, si
easier the-demonstration may-become from every superparticular if
continuam ei superparticularem quis auferat proportionem;
the-continuous to-it superparticular anyone take-away proportion
quae est scilicet minor, id quod relinquitur minus est
which is namely the-lesser that which remains less is
ejus medietate, quae detracta est proportionis, ut in
of-that than-the-half which subtracted was proportion as in
sesquialtera ac sesquitertia. Quoniam sesquialtera major
the-sesquialter and sesquitertian since the-sesquialter greater

est, sesquitertiam de sesquialtera detrahamus, relinquitur
is the-sesquitertian from the-sesquialter let-us-subtract there-remains
sesquioctava proportio quae duplicata non efficit integram
the-sesquioctave proportion which doubled not makes a-whole
sesquitertiam proportionem sed ea distantia minor est
sesquitertian proportion but by-that distance less is
quae in semitonio reperitur; quod si duplicata sesquioctava
which in the-semitone is-found but if the-doubled sesquioctave
comparatio non est integra sesquitertiae, simplex sesquioctava
comparison not is the-whole of-the-sesquitertian the-single sesquioctave
non est sesquitertiae proportionis plena medietas; quod si
not is of-the-sesquitertian proportion the-full half but if
sesquiquartum sesquitertio auferas, id quod relinquitur
the-sesquiquartan from-the-sesquitertian you-take-away that which remains
medietatem sesquiquarti non efficit, idemque in caeteris.
the-half of-the-sesquiquartan not it-makes and-the-same in the-rest

Demonstratio per impossibile diapason in multiplici genere esse

II.21 · THE DIAPASON IS IN THE MULTIPLE GENUS

The diapason is proved multiple through the impossible. If it is not in the multiple genus it falls into the superparticular; then, taking away the continuous consonance diapente, the remaining diatessaron would not fill half of diapente, and twice diatessaron would be less than one diapente. But twice diatessaron will be shown to exceed diapente by a tone and a semitone — which is impossible, so the diapason cannot be placed in the superparticular genus.

Age nunc ad diapason consonantiam redeamus. Quod si
come now to the-diapason consonance let-us-return but if
ea non est in multiplici genere inaequalitatis, cadet in
it not is in the-multiple genus of-inequality it-will-fall into
superparticulare inaequalitatis genus. Sit igitur superparticularis
the-superparticular of-inequality genus let-be therefore a-superparticular
proportio diapason consonantia. Auferatur ab ea continua
proportion the-diapason consonance let-be-taken-away from it the-continuous
consonantia, id est diapente, relinquitur diatessaron. Bis igitur
consonance that is diapente there-remains diatessaron twice therefore
diatessaron minus est uno diapente, et ipsum diatessaron non
diatessaron less is than-one diapente and the-very diatessaron not
implet diapente consonantiae medietatem, quod est impossibile.
fills of-the-diapente consonance the-half which is impossible
Monstrabitur enim bis diatessaron tono ac semitonio
it-will-be-shown for (that)-twice diatessaron by-a-tone and a-semitone
consonantiam diapente transcendere. Quocirca nec diapason
the-consonance diapente to-exceed wherefore neither the-diapason
quidem in superparticulari inaequalitatis genere poni potest.
indeed in the-superparticular of-inequality genus to-be-placed can

Demonstratio per impossibile diapente diatessaron et tonum in superparticulari esse

II.22 · DIAPENTE, DIATESSARON, AND TONE ARE SUPERPARTICULAR

Diapente, diatessaron, and the tone are shown superparticular, again through the impossible. Set diatessaron in the double and diapente in the triple: the tone, their remainder, becomes sesquialter — but two sesquialters exceed a double, so two tones would surpass diatessaron, which in fact exceeds two tones by a semitone. Every other assignment to multiples fails the same way, so these relations rightly take their place in the superparticular genus.

Restat igitur ut diapente ac diatessaron, et tonum, in
it-remains therefore that diapente and diatessaron and the-tone in
superparticularitate ponenda esse monstremus. Nam etsi
superparticularity to-be-placed to-be we-may-show for although
id in prima quoque probatione ea qua diapason in
this in the-first also proof that by-which (that)-the-diapason in
superparticulari genere non esse ponenda monstravimus, id
the-superparticular genus not to-be to-be-placed we-showed this
quoque quodam rationis modo perclaruit; sigillatim tamen
too in-a-certain of-reason manner became-thoroughly-clear one-by-one yet
de eo ac diligentius pertractemus. Nam si in superparticulari
about it and more-carefully let-us-treat for if in the-superparticular
quis has habitudines ponendas esse non dixerit, in
anyone these relations to-be-placed to-be not shall-have-said in
multiplici genere fatebitur collocandas. Nam in superpartienti
the-multiple genus he-will-confess (them)-to-be-set for in the-superpartient
vel caeteris mixtis cur poni non possint, superius (ut
or the-other mixed-(kinds) why to-be-placed not they-can above as
arbitror) explanatum est. Ponantur igitur, si fieri potest,
I-think explained it-was let-them-be-placed therefore if to-be-done it-can
in multiplici genere. Et quoniam diatessaron consonantia
in the-multiple genus and since the-diatessaron consonance

minor est, diapente major, diatessaron duplici, diapente vero
lesser is the-diapente greater diatessaron to-the-double diapente but
triplici proportioni multiplicitatis aptetur. Verisimile est enim
to-the-triple proportion of-multiplicity let-be-fitted likely it-is for
ut est consonantia diatessaron consonantiae diapente continua,
as is the-consonance diatessaron to-the-consonance diapente continuous
ita si diatessaron in duplici statuitur, diapente in continua
so if diatessaron in the-double is-set (that)-diapente in the-continuous
duplicis poni, id est triplici. Tonus autem quoniam in
of-the-double to-be-placed that is the-triple the-tone moreover since in
habitudinibus musicis post diatessaron locatur, nimirum in
relations musical after diatessaron is-located doubtless in
ea proportione ponatur, quae est minor duplici.
that proportion let-it-be-placed which is less than-the-double
Haec autem in multiplicitatis genere non potest inveniri.
this moreover in of-multiplicity the-genus not can to-be-found
Restat igitur ut in superparticularitatis habitudinem cadat.
it-remains therefore that into of-superparticularity the-relation it-fall
Sit igitur prima, id est sesquialtera, toni proportio. Nam
let-be therefore the-first that is the-sesquialter of-the-tone the-proportion for
si duplicem auferamus, triplici, quod relinquitur, sesquialter
if the-double we-take-away from-the-triple what remains sesquialter
est. Quod si diatessaron quidem duplex est, diapente vero
is but if diatessaron indeed double is diapente but
triplum, ac sublato diatessaron a diapente tonus
triple and (with)-taken-away diatessaron from the-diapente the-tone
reliquus fit, nullo modo dubitari potest quin tonus
the-remainder becomes in-no way to-be-doubted it-can but-that the-tone
in sesquialtera debeat proportione constitui. Sed duae
in the-sesquialter ought proportion to-be-established but two
sesquialterae proportiones duplicem vincunt, quemadmodum
sesquialter proportions the-double conquer as

ex arithmeticis instructus sibi potest quisque colligere.
from the-arithmetical-(books) instructed for-himself can each-one to-gather
Duo igitur toni diatessaron superabunt, quod est inconveniens:
two therefore tones the-diatessaron will-surpass which is unfitting
diatessaron enim duos tonos semitonii spatio transcendit.
the-diatessaron for two tones of-a-semitone by-the-space exceeds
Non igitur fieri potest ut non diapente ac diatessaron
not therefore to-come-about it-can that not diapente and diatessaron
in superparticulari inaequalitatis genere collocetur. Quod si
in the-superparticular of-inequality genus be-set but if
quis tonum quoque in multiplici genere esse perscribat,
anyone (that)-the-tone too in the-multiple genus to-be should-write-down
quoniam quidem tonus minor est quam diatessaron,
since indeed the-tone less is than diatessaron
diatessaron vero minus est quam diapente, diapente quidem
diatessaron but less is than diapente diapente indeed
ponatur in quadrupla, diatessaron in tripla, tonus in duplici.
let-be-placed in the-quadruple diatessaron in the-triple the-tone in the-double
Sed diapente constat ex diatessaron et tono. Quadruplum
but diapente consists of diatessaron and a-tone the-quadruple
igitur secundum hanc rationem constabit ex triplo ac duplo,
therefore according-to this reckoning will-consist of the-triple and the-double
quod fieri nequit. Rursus statuatur diatessaron quidem
which to-come-about cannot again let-be-set diatessaron indeed
in triplici, et diapente in quadruplo. Si igitur auferamus
in the-triple and diapente in the-quadruple if therefore we-take-away
triplum a quadruplo, sesquitertius relinquetur. Rursus
the-triple from the-quadruple the-sesquitertian will-remain again
si diatessaron diapente consonantiae subtrahas, fit
if diatessaron from-the-diapente consonance you-subtract becomes
reliquus tonus. Tonus igitur secundum hanc rationem in
the-remainder a-tone the-tone therefore according-to this reckoning in

sesquitertia proportione constabit. Sed tres sesquitertii uno
the-sesquitertian proportion will-consist but three sesquitertians than-one
triplici sunt minores. Tres igitur toni unum diatessaron nulla
triple are lesser three therefore tones one diatessaron by-no
ratione supplebunt, quod est falsissimum. Duo enim toni
reckoning will-fill-up which is most-false two for tones
ac semitonium minus diatessaron consonantiam supplent.
and a-semitone lesser the-diatessaron consonance fill-up
Ex his igitur demonstratur diatessaron consonantiam
from these therefore is-demonstrated (that)-the-diatessaron consonance
non esse multiplicem. Dico autem quoniam nec diapente
not to-be multiple I-say moreover that neither the-diapente
consonantia in multiplici genere poterit collocari. Nam si in
consonance in the-multiple genus will-be-able to-be-set for if in
eo statuatur, quoniam est ei minor continua, id
it it-be-set since there-is to-it the-lesser continuous-(consonance) that
est diatessaron, non locabitur diapente in minimo multiplici,
is diatessaron not will-be-located diapente in the-least multiple
id est duplici scilicet ut sit locus quo diatessaron
that is the-double namely that there-be a-place in-which the-diatessaron
consonantia possit aptari.
consonance can to-be-fitted
Sed diatessaron consonantia multiplicis generis non est.
but the-diatessaron consonance of-the-multiple genus not is
Quocirca nec diapente, in majore habitudine multiplicis quam
wherefore neither diapente in a-greater relation of-the-multiple than
est dupla quae minima est, aptari potest. Si igitur diapente
is the-double which the-least is to-be-fitted can if therefore diapente
in minima, scilicet dupla, diatessaron vero, quae minor est, in
in the-least namely the-double diatessaron but which less it-is in
multiplici quidem aptari non potest. Non est enim quidquam
the-multiple indeed to-be-fitted not can not there-is for anything

minus a se duplici. Sit igitur sesquialtera, tonus vero
less than the-very double let-it-be therefore sesquialter the-tone but
sesquitertia, in continua enim proportione locabitur. Sed
sesquitertian in the-continuous for proportion it-will-be-located but
duo sesquitertii ampliores sunt uno sesquialtero. Duo igitur
two sesquitertians ampler are than-one sesquialter two therefore
toni unam diatessaron consonantiam vincent, quod nulla
tones one diatessaron consonance will-conquer which by-no
ratione continget. Ex his igitur approbatur diapente ac
reckoning will-befall from these therefore is-approved (that)-diapente and
diatessaron in multiplici genere collocari non posse: quocirca
diatessaron in the-multiple genus to-be-set not to-be-able wherefore
in superparticulari inaequalitatis genere jure ponentur.
in the-superparticular of-inequality genus rightly they-will-be-placed

Demonstratio diapente et diatessaron in maximis superparticularibus collocari

II.23 · THE GREATEST SUPERPARTICULARS

If diapente and diatessaron hold superparticular proportions, they must be the greatest ones, sesquialter and sesquitertian. The diapason has been shown to stand in the double proportion, and the double is composed of sesquialter and sesquitertian just as the diapason is coupled from diapente and diatessaron. No other pair of superparticulars joined completes one double, so no other pair could join to complete one diapason.

Illud quoque addendum necessario est, quoniam si diapente
that too to-be-added of-necessity is that if diapente
ac diatessaron superparticulares proportiones tenent, in
and diatessaron superparticular proportions hold in
maximis superparticularibus proportionibus collocantur. Sunt
the-greatest superparticular proportions they-are-set there-are

autem maximae sesquialtera et sesquitertia. Hoc vero
moreover the-greatest the-sesquialter and sesquitertian this but
approbabitur hoc modo. Nam si in minoribus proportionibus
will-be-approved in-this way for if in lesser proportions
quam sesquialtera vel sesquitertia diapente ac diatessaron
than the-sesquialter or sesquitertian the-diapente and diatessaron
consonantiae collocentur, non est dubium quin sicut aliae
consonances be-set not is doubt but-that as other
quaelibet proportiones superparticulares praeter sesquialteram
whatsoever proportions superparticular besides the-sesquialter
et sesquitertiam junctae non efficiunt unum duplum,
and sesquitertian joined not make one double
ita diapente ac diatessaron unum diapason nulla ratione
so diapente and diatessaron one diapason by-no reckoning
concludunt. Quoniam enim diapason in duplici proportione
conclude since for (that)-the-diapason in the-double proportion
esse monstratum est, duplex vero proportio ex sesquialtero
to-be shown it-was the-double but proportion from the-sesquialter
sesquitertioque componitur; diapason vero ex diatessaron ac
and-sesquitertian is-composed the-diapason but from diatessaron and
diapente copulatur, non est dubium quin, si totum diapason
diapente is-coupled not is doubt but-that if the-whole diapason
in duplici statuatur, diapente et diatessaron in sesquialtera
in the-double be-set diapente and diatessaron in the-sesquialter
sesquitertiaque proportione sint locandae. Aliter enim non
and-sesquitertian proportion are to-be-located otherwise for not
poterunt diapason junctas perficere, quae consonantia in
they-will-be-able the-diapason (when)-joined to-complete which consonance in
duplici proportione consistit, nisi in his duabus proportionibus
the-double proportion consists unless in these two proportions

steterint sesquialtera scilicet ac sesquitertia. Aliae enim
they-shall-have-stood the-sesquialter namely and sesquitertian other for
proportiones superparticulares hanc nulla ratione conjungent.
proportions superparticular this by-no reckoning will-conjoin

Diapente in sesquialtera, diatessaron in sesquitertia esse, tonum in sesquioctava

II.24 · SESQUIALTER, SESQUITERTIAN, SESQUIOCTAVE

Boethius assigns each consonance its own proportion: the greater fits the greater. Since the sesquialter is greater than the sesquitertian, and diapente greater than diatessaron, diapente stands in the sesquialter and diatessaron in the sesquitertian. Subtract diatessaron from diapente and the space called the tone remains; subtract the sesquitertian from the sesquialter and the sesquioctave remains — so the tone must be established in the sesquioctave ratio.

Dico autem quoniam proprie diapente in sesquialtera, et
I-say moreover that properly diapente in the-sesquialter and
diatessaron in sesquitertia proportione consistit. Quoniam
diatessaron in the-sesquitertian proportion consists since
enim inter utrasque proportiones, sesquialteram scilicet
for between both proportions the-sesquialter namely
et sesquitertiam, sesquialtera est major et sesquitertia
and sesquitertian the-sesquialter is the-greater and the-sesquitertian
minor, quoniamque in consonantiis diapente major
the-lesser and-since among the-consonances diapente the-greater
diatessaron minor, apparet majorem proportionem majori,
diatessaron the-lesser appears (that)-the-greater proportion to-the-greater
et minorem minori esse consonantiae aptandum. Erit
and the-lesser to-the-lesser to-be to-the-consonance to-be-fitted will-be
igitur diapente quidem in sesquialtera, diatessaron vero in
therefore diapente indeed in the-sesquialter diatessaron but in

proportione sesquitertia collocanda. Quod si diatessaron a
proportion the-sesquitertian to-be-set but if diatessaron from
diapente consonantiam subtrahamus, relinquitur spatium, quod
the-diapente consonance we-subtract there-remains a-space which
dicitur tonus. Sesquitertium vero si proportioni sesquialterae
is-called the-tone the-sesquitertian but if from-the-proportion sesquialter
minuamus, relinquitur sesquioctava proportio. Quo fit
we-diminish there-remains the-sesquioctave proportion whereby it-comes-about
ut tonus in sesquioctava debeat ratione constitui.
that the-tone in the-sesquioctave ought ratio to-be-established

Diapason ac diapente in tripla proportione esse, bis diapason in quadrupla

II.25 · THE TRIPLE AND QUADRUPLE CONSONANCES

The diapason is double and the diapente sesquialter, and those two joined beget the triple proportion — so diapason-plus-diapente stands in the triple. Join a sesquitertian relation to a triple and a quadruple arises: adding the diatessaron symphony to diapason-plus-diapente makes the quadruple space of voices, which was shown above to be double diapason.

Sed quoniam demonstratum est diapason quidem
but since demonstrated it-was (that)-the-diapason indeed
duplam, diapente vero sesquialteram, junctas vero duplam
(is)-double diapente but sesquialter (that)-joined but the-double
ac sesquialteram triplicem proportionem procreare, ex his
and sesquialter the-triple proportion to-beget from these
etiam apparet diapente ac diapason in triplici proportione
also it-appears (that)-diapente and diapason in the-triple proportion
constitui. Sed si quis triplici proportioni sesquitertiam
to-be-established but if anyone to-the-triple proportion the-sesquitertian

habitudinem jungat, quadruplam facit. Igitur si diapente
relation should-join the-quadruple he-makes therefore if to-the-diapente
ac diapason consonantiis diatessaron symphonia jungatur,
and diapason consonances the-diatessaron symphony be-joined
fit quadruplum spatium vocum, quod bis diapason
there-becomes the-quadruple space of-voices which twice diapason
supra esse monstravimus.
above to-be we-showed

Diatessaron ac diapason non esse consonantiam secundum Pythagoricos

II.26 · DIATESSARON-PLUS-DIAPASON IS NO CONSONANCE

Consonances laid under consonances make new ones, but diatessaron added to diapason makes none, on Pythagorean principles. Set the terms 3, 6, 8: the compound is 8 to 3, a multiple superbipartient, which keeps neither the order of multiplicity nor the simplicity of the superparticular, since no multiple proportion falls between the double and the triple. Add a tone and 8 becomes 9, giving the triple 9 to 3, which is diapason-plus-diapente; Boethius leaves Ptolemy's view for later.

Sed in his illud diligens lector agnoscat, quod consonantiae
but in these this the-careful reader may-recognize that consonances
consonantiis suppositae alias quasdam consonantias efficiant.
to-consonances placed-under other certain consonances produce
Nam diapente ac diatessaron juncta, diapason (ut dictum
for diapente and diatessaron joined the-diapason as said
est) creant; huic vero, id est diapason, rursus si diapente
it-was create to-this but that is the-diapason again if the-diapente
symphonia jungatur, fit consonantia, quae ex utrisque
symphony be-joined there-becomes a-consonance which from both
vocabulis nuncupatur, diapason scilicet ac diapente. Cui si
names is-called diapason namely and diapente to-which if

diatessaron addatur, fit bis diapason, quae quadruplam
diatessaron be-added there-becomes twice diapason which the-quadruple
proportionem tenet. Quid igitur si diatessaron ac diapason
proportion holds what therefore if diatessaron and diapason
consonantias jungamus, ullamne secundum Pythagoricos
consonances we-join any-at-all according-to the-Pythagoreans
efficient consonantiam? Minime. Mox enim in
will-they-produce consonance not-at-all straightway for into
superpartiens inaequalitatis genus cadit, nec servat vel
the-superpartient of-inequality genus it-falls nor keeps either
multiplicitatis ordinem, vel superparticularitatis simplicitatem.
of-multiplicity the-order or of-superparticularity the-simplicity
Age enim statuantur numeri quibus id facilius approbemus:
come then let-be-set numbers by-which this more-easily we-may-approve
sit enim ternarius, cujus sit senarius, duplus scilicet, in
let-be for the-three of-which let-be the-six the-double namely in
diapason proportione consistens. Huic aptetur sesquitertia
the-diapason proportion standing to-this let-be-fitted the-sesquitertian
quam diatessaron esse praediximus, ut octonarius. Is enim
which diatessaron to-be we-said-before as the-eight it for
ad senarium diatessaron proportionem tenet, qui octonarius
to the-six the-diatessaron proportion holds which eight
ad ternarium comparatus habet eum bis.
to the-three compared has it twice
Sed ne sit multiplex, habet etiam ejus aliquas partes, neque
but lest it-be multiple it-has also of-it some parts nor
eas simplices. Duabus enim eum supervenit unitatibus, quae
those simple by-two for it it-overpasses units which
sunt duae tertiae partes ternarii, quem primum terminum
are two third parts of-the-three which (as)-first term
minimumque locamus. Sint igitur termini hi, 3, 6, 8. Illud
and-least we-place let-be therefore the-terms these 3 6 8 that

quoque quod inter duas sibi continuas consonantias
too which between two to-each-other continuous consonances
cadit, etenim neque duplum est integrum, ut diapason
falls for-indeed neither the-double is whole that the-diapason
consonantiam prodat, neque triplum, ut diapason ac
consonance it-may-bring-forth nor the-triple that the-diapason and
diapente efficiat symphoniam. Cui si tonus addatur,
diapente it-may-effect symphony to-which if a-tone be-added
mox triplum modum proportionis efficiet. Quoniam enim
straightway the-triple measure of-proportion it-will-effect since for
diapason ac diapente sibi junctae efficiunt triplum,
diapason and diapente to-each-other joined effect the-triple
diatessaron vero et tonus diapente consonantiam jungunt, si
diatessaron but and a-tone the-diapente consonance join if
diapason consonantia addatur diatessaron, inconsonum fit,
the-diapason consonance be-added to-diatessaron unconsonant it-becomes
quoniam inter duplicem ac triplicem nulla potest naturaliter
since between the-double and triple no can naturally
proportio multiplicitatis intelligi. Quod si ei adjicio tonum,
proportion of-multiplicity to-be-understood but if to-it I-add a-tone
fit diapason, diatessaron, et tonus, quod nihil distabit
it-becomes diapason diatessaron and a-tone which in-nothing will-differ
utrum diapason ac diapente sit. Diatessaron enim et tonus
whether diapason and diapente it-be diatessaron for and a-tone
diapente constituunt. Sit enim diapason quidem 3 et 6;
the-diapente constitute let-be then diapason indeed 3 and 6
diatessaron 6 et 8; tonus 8 et 9; diapente 9 et 6; diapason
diatessaron 6 and 8 the-tone 8 and 9 diapente 9 and 6 diapason
ac diapente 3 ac 9: erit igitur sic tripla proportio 3, 6, 8,
and diapente 3 and 9 will-be therefore thus the-triple proportion 3 6 8
9. Sed quanquam de his multa Nicomachus, nos tamen
9 but although about these many-(things) Nicomachus we yet

qua potuimus brevitate partim ea ipsa quae Pythagorici
with-what we-could brevity partly those very-(things) which the-Pythagoreans
affirmant promentes, partim ex eisdem quaedam consequentia
affirm bringing-forth partly from the-same certain-(things) consequent
argumentantes, probavimus, si diatessaron consonantia diapason
arguing we-have-proved if the-diatessaron consonance to-the-diapason
addatur, consonantiam ex his conjungi non posse. Quid
be-added (that)-a-consonance from these to-be-conjoined not to-be-able what
vero sentiat de his Ptolemaeus, posterius apponam. Sed
but may-think about these Ptolemy later I-shall-set-down but
de his hactenus; nunc de semitoniis considerandum est.
about these thus-far now about semitones to-be-considered it-is

De semitonio in quibus minimis numeris constet

II.27 · THE SEMITONE'S MINIMAL NUMBERS

Semitones are so named not because they are true halves of tones but because they are incomplete tones; the ancients called this interval the limma or diesis. The semitone is what remains when two sesquioctave tones are taken from the sesquitertian diatessaron. Two continuous tones stand at 64, 72, 81; tripling those gives 192, 216, 243, and the sesquitertian of 192 is 256 — so the least semitone stands in 243 to 256.

Videntur enim semitonia nuncupata, non quod vere
they-seem for semitones to-have-been-named not because truly
tonorum sint medietates, sed quod sint non integri toni.
of-tones they-are halves but because they-are not whole tones
Hujusque spatii quod nunc quidem semitonium nuncupamus,
and-of-this space which now indeed a-semitone we-name
apud antiquiores autem limma vel diesis vocabatur, hic
among the-more-ancient however the-limma or a-diesis it-was-called this

modus est. Cum enim ex sesquitertia proportione quae
the-manner is when for from the-sesquitertian proportion which
diatessaron est, duae sesquioctavae habitudines, quae toni sunt,
diatessaron is two sesquioctave relations which tones are
auferuntur, relinquitur spatium quod semitonium nuncupatur.
are-taken-away there-remains a-space which a-semitone is-named
Quaeramus igitur duos tonos continua dispositione descriptos.
let-us-seek therefore two tones in-continuous disposition described
Sed, quoniam hi (ut dictum est) in sesquioctava
but since these as said it-was in the-sesquioctave
proportione consistunt, duasque sesquioctavas proportiones
proportion consist and-two sesquioctave proportions
continuas adhibere non possumus nisi multiplex ille, a
continuous to-apply not we-can unless the-multiple that from
quo hae derivari possint, reperiatur, sit unitas prima,
which these to-be-derived can be-found let-be the-unit first
eique octonarius octuplus primus. Ad hoc igitur unum
and-for-it the-eight octuple the-first to this therefore one
sesquioctavum potero derivare. Sed quia duos quaerimus,
sesquioctave I-shall-be-able to-derive but because two we-seek
fient octies octo, atque ex eo 64 explicentur. Erit
they-will-become eight-times eight and from it 64 let-be-unfolded it-will-be
igitur secundus octuplus a quo possumus duas sesquioctavas
therefore the-second octuple from which we-can two sesquioctave
proportiones educere. Namque octo quae est octava pars
proportions to-draw-forth for-indeed eight which is the-eighth part
64 unitatum eisdem addita totam summam 72 perficiunt;
of-64 units to-the-same added the-whole sum 72 they-complete
his vero si sua octava similiter apponatur, quae est
to-these but if its-own eighth likewise be-set-beside which is
novenarius, 81 reddunt. Eruntque duo toni continui principali
the-nine 81 they-render and-there-will-be two tones continuous in-the-principal

dispositione conscripti 64, 72, 81. Nunc igitur 64 unitatum
disposition written-out 64 72 81 now therefore of-64 units
sesquitertium conquiramus. Sed quoniam 64 probantur partem
the-sesquitertian let-us-seek-out but since 64 are-proved part
tertiam non habere, si omnes hi numeri ternario multiplicentur,
a-third not to-have if all these numbers by-three be-multiplied
mox eis pars tertia contingit, et omnes in eadem
straightway to-them part a-third befalls and all in the-same
proportione durabunt, in qua fuerunt antequam his ternarius
proportion will-endure in which they-were before to-these the-three
multiplicator accederet; fiant igitur ter 64, id est 192,
(as)-multiplier came let-become therefore thrice 64 that is 192
horum tertia 64 eisdem addita 256 reddet. Erit igitur
of-these the-third-(part) 64 to-the-same added 256 will-render will-be therefore
haec sesquitertia proportio diatessaron consonantiam tenens.
this the-sesquitertian proportion the-diatessaron consonance holding
Nunc igitur duas sesquioctavas proportiones ad 192 duobus
now therefore two sesquioctave proportions to 192 by-two
numeris continentes rato ordine collocemus; fiant igitur
numbers containing in-fixed order let-us-place let-become therefore
ter 72, id est 216. Rursus ter 81, qui sunt 243, qui inter
thrice 72 that is 216 again thrice 81 which are 243 which between
duos supra scriptos terminos collocentur hoc modo: 192, 216,
the-two above written terms let-be-placed in-this way 192 216
243, 256. In hac dispositione proportionum, primus numerus
243 256 in this disposition of-proportions the-first number
ad postremum diatessaron constituit consonantiam. Idem
to the-last the-diatessaron constitutes consonance the-same
vero primus ad secundum, et secundus ad tertium geminos
but the-first to the-second and the-second to the-third twin

constituunt tonos. Constat igitur spatium quod relinquitur
constitute tones is-established therefore the-space which remains
ex 243 ad 256, in quibus minimi semitonii forma consistit.
from 243 to 256 in which of-the-least semitone the-form consists

Demonstrationem non esse 243 ad 256 toni medietatem

II.28 · 243:256 IS NOT HALF A TONE

The distance from 243 to 256 is not a full half tone. Their difference, 13, is less than an eighteenth part of the smaller number and more than a nineteenth, while any true half of a tone must fall between a sixteenth and a seventeenth part. Doubling the ratio on itself gives 59049, 62208, 65536, and the extremes make no sesquioctave — two such parts joined do not fill a tone, so the semitone is less than the tone's half.

Approbo igitur 243 ad 256 distantiam non esse integram
I-prove therefore of-243 to 256 (that)-the-distance not to-be the-whole
toni medii dimensionem. Etenim ducentorum quadraginta
of-a-tone half measure for-indeed of-two-hundred forty
trium et ducentorum 56, differentia 13 tantum unitatibus
(and-)three and of-two-hundred (and-)56 the-difference 13 only in-units
continetur. Qui tredecim minus quidem quam minoris octavam
is-contained which thirteen less indeed than of-the-smaller the-eighth
decimam, plus vero quam nonam decimam obtinent partem. Si
(and-)tenth more but than the-ninth (and-)tenth occupy part if
enim octies decies tredecim ducas, efficies 234, qui
for eight-times (and-)ten-times thirteen you-multiply you-will-make 234 which
243 nullo modo aequabit; si decies novies multiplices,
243 in-no way will-equal if ten-times (and-)nine-times you-multiply
supervadent, cum oporteat omne semitonium, si tamen integrum
they-will-overpass since it-behooves every semitone if however the-whole

toni dimidium tenet, inter sextam decimam partem ac
of-the-tone half it-holds between the-sixth (and-)tenth part and
septimam decimam collocari. Quod posterius demonstrabitur.
the-seventh (and-)tenth to-be-placed which later will-be-demonstrated
Nunc illud liquebit talem semitonii distantiam sibimet
now this will-be-clear (that)-such of-the-semitone a-distance to-itself
geminatam, unum toni spatium non posse complere.
doubled one of-the-tone space not to-be-able to-fill-up
Age enim ut sese 256 ad 243 habent, tales duas sibimet
come for as themselves 256 to 243 are-disposed such two to-themselves
continuas proportiones, secundum superius descriptam regulam
continuous proportions according-to the-above described rule
disponamus; 200 enim 50 et 6 in semetipsos multiplicemus,
let-us-dispose 200 for (and-)50 and 6 into themselves let-us-multiply
et sit maximus terminus 65536. Item 243 propria numerositate
and let-be the-greatest term 65536 likewise 243 by-its-own numerosity
concrescant, et sit minimus terminus 59049.
let-them-grow-together and let-be the-smallest term 59049
Rursus igitur 256 ad ducentorum 43 multitudinem
again therefore 256 by of-two-hundred (and-)43 the-multitude
concrescant. Erit igitur numerus 62208; hic igitur
let-them-grow-together there-will-be therefore the-number 62208 this therefore
medius collocetur hoc modo: In eadem igitur sunt
in-the-middle let-it-be-placed in-this manner in the-same therefore are
proportione ducenti 56 et 243. In qua 65536 ad 62208;
proportion two-hundred (and-)56 and 243 in which 65536 to 62208
et item 62208 ad 59049. Sed maximus eorum terminus,
and likewise 62208 to 59049 but the-greatest of-them term
qui est 65536, ad minimum qui est 59049, unum integrum
which is 65536 to the-smallest which is 59049 one whole
non efficit tonum. Quod si primi ad secundum proportio,
not makes tone but if of-the-first to the-second the-proportion

quae est aequa secundi ad tertium proportioni, integri esse
which is equal of-the-second to the-third to-the-proportion of-a-whole to-be
semitonii probaretur, duo dimidia juncta unum necessario
semitone were-proved two halves joined one necessarily
efficerent tonum. Nunc autem cum non sit extremorum
would-make tone now however since not is of-the-extreme
terminorum sesquioctava proportio, manifestum est haec
terms a-sesquioctave proportion manifest it-is (that)-these
duo spatia proprie tonorum dimidia non videri. Quidquid
two spaces properly of-tones halves not to-be-seen whatever
enim cujuscunque est dimidium, id si duplicetur illud
for of-whatsoever is the-half that if it-be-doubled that-thing
efficit, cujus dicitur esse dimidium. Si vero illud implere
it-makes of-which it-is-said to-be the-half if but that to-fill-up
non possit geminata particula, minus est parte dimidia.
not it-can the-doubled particle less it-is than-the-part half
Si vero superfluat ac supervadat, plus est parte dimidia.
if but it-overflow and overpass more it-is than-the-part half
Praeterea probabuntur 65536 non facere sesquioctavam
besides will-be-proved 65536 not to-make a-sesquioctave
proportionem. Si 59049 unitatibus comparentur, si octava pars
proportion if 59049 to-units they-be-compared if the-eighth part
59049, eisdem secundum eas quae in arithmeticis dictae
of-59049 to-the-same according-to those which in the-arithmetical-(books) stated
sunt regulas aggeratur. Quae quoniam integris numeris
have-been rules be-heaped-on which since in-whole numbers
non consistit, idcirco eamdem octavam partem relinquimus
not consists therefore that-same eighth part we-leave
lectorum diligentiae computandam. Liquet igitur eam
of-readers to-the-diligence to-be-computed it-is-clear therefore (that)-that
proportionem quae in 256 et 243 est constituta, non esse
proportion which in 256 and 243 is established not to-be

integrum dimidium toni; quocirca id quod vere semitonium
the-whole half of-a-tone wherefore that which truly a-semitone
nuncupatur, pars toni minor est quam dimidia.
is-named a-part of-the-tone smaller is than the-half

De majore parte toni in quibus minimis numeris constet

II.29 · THE GREATER PART OF THE TONE

The remaining, greater part of the tone the Greeks call apotome, which Boethius renders decisio: by as much as the semitone falls short of the tone's true half, the apotome exceeds it. Since 243 has no eighth part, he multiplies 243 and 256 by eight to get 1944 and 2048, then adds 1944's own eighth to reach 2187. There 2048 to 1944 holds the lesser semitone, 2187 to 2048 the apotome, and 2187 to 1944 the whole tone — the least numbers in which the apotome stands.

Reliqua igitur pars, quae major est, apotome nuncupatur
the-remaining therefore part which greater is the-apotome is-named
a Graecis, a nobis vero potest vocari decisio. Id enim
by the-Greeks by us but it-can to-be-called a-cutting-off this for
natura fert, ut quotiens aliquid secatur, ita ut non aequis
nature brings that as-often-as something is-cut so that not into-equal
partibus dividatur, quanto minor pars dimidio minor est,
parts it-be-divided by-how-much the-lesser part than-the-half smaller is
tanto major pars eadem, quae auctior est, dimidium
by-so-much the-greater part that-same which larger is the-half
vincat. Quantum igitur semitonium minus integro dimidio
surpasses by-as-much-as therefore the-semitone lesser than-the-whole half
toni minus est, tanto apotome toni integrum superat
of-the-tone less is by-so-much the-apotome of-the-tone the-whole exceeds
dimidium et vincit. Et quoniam docuimus semitonium in
half and surpasses and since we-have-taught the-semitone in

256 et 243 principaliter stare, nunc ea quae apotome dicitur,
256 and 243 primarily to-stand now that which the-apotome is-called
in quibus possit minimis constare numeris approbemus.
in what it-may-be-able smallest to-consist numbers let-us-prove
Si igitur 243 partem recipere octavam possent cum ad eum
if therefore 243 a-part to-receive eighth they-could when to it
sesquioctavus numerus comparetur, tunc 256 habitudo ad
a-sesquioctave number were-compared then of-256 the-relation to
sesquioctavam summam minimi numeri comparata, apotomen
the-sesquioctave sum of-the-smallest number compared the-apotome
necessaria ratione monstraret. Nunc vero quoniam ei pars
by-necessary reckoning would-show now but since to-it a-part
octava deesse monstratur, utrique numeri octies fiant.
eighth to-be-lacking is-shown both numbers eight-times let-them-become
Et ex 243 quidem octies multiplicatis fit numerus 1944.
and from 243 indeed eight-times multiplied becomes the-number 1944
Quibus si propria conferatur octava, qui sunt 243, fient
to-which if its-own be-added eighth which are 243 they-will-become
2187. Rursus 256 per octonarium crescant, fient
2187 again 256 through the-number-eight let-them-grow they-will-become
igitur 2048. Atque hic suprascriptorum terminorum in
therefore 2048 and this of-the-above-written terms in
medio collocetur. Tertius igitur terminus ad primum toni
the-middle let-it-be-placed the-third therefore term to the-first of-a-tone
retinet proportionem. Secundus ad primum semitonii minoris.
keeps the-proportion the-second to the-first of-the-semitone lesser
Apotomes vero tertius ad secundum. Atque in hisdem primis
of-the-apotome but the-third to the-second and in these-same first-(numbers)
apotomes videtur constare proportio, cum semitonii in 256 et
of-the-apotome seems to-stand the-proportion since of-the-semitone in 256 and
243, minimis numeris spatium contineatur. Idcirco autem 1944
243 smallest numbers the-space is-contained therefore moreover 1944

et 2048 in eadem proportione sunt qua 243 ad ducentos 56,
and 2048 in the-same proportion are as 243 to two-hundred (and-)56
quoniam 256 et 243 octonario multiplicati sunt. Si enim
since 256 and 243 by-the-number-eight multiplied have-been if for
unus numerus duos quoslibet numeros multiplicet, qui
one number two whatsoever numbers should-multiply (those)-which
ex ea multiplicatione nascuntur, in eadem erunt proportione
from that multiplication are-born in the-same they-will-be proportion
qua fuerunt hi numeri quos prior numerus multiplicavit.
as were those numbers which the-former number multiplied

Quibus proportionibus diapente, diapason constent, et quoniam diapason sex tonis non constet

II.30 · THE DIAPASON IS LESS THAN SIX TONES

The diapente consists of three tones and a lesser semitone: on the series 192, 216, 243, 256, an eighth added to 256 gives 288, sesquialter to 192. The diapason therefore holds five tones and two lesser semitones that do not fill a tone — it is not the six tones Aristoxenus judges it to be. Six sesquioctave tones from 262144 reach 531441, while a diapason above 262144 reaches only its double, 524288; the excess of six tones over the diapason is the comma, in least numbers 524288 and 531441.

Sed quoniam de diatessaron consonantia latius diximus,
but since about the-diatessaron consonance more-broadly we-have-spoken
brevius et pene puris numeris de diapason ac diapente
more-briefly and almost with-bare numbers about the-diapason and diapente
consonantiis disseramus. Diapente enim constat ex tribus tonis
consonances let-us-discourse the-diapente for consists of three tones
ac semitonio, id est ex diatessaron et tono. Disponantur enim
and a-semitone that is of the-diatessaron and a-tone let-be-disposed for
numeri quos superior descriptio comprehendit, 192, 216, 243,
the-numbers which the-earlier diagram comprised 192 216 243

256. In hac igitur dispositione primus terminus ad secundum,
256 in this therefore disposition the-first term to the-second
et secundus ad tertium tonorum retinet proportiones. Sed
and the-second to the-third of-tones keeps the-proportions but
tertius ad quartum semitonii minoris, ut supra monstratum
the-third to the-fourth of-the-semitone lesser as above shown
est. Si igitur 256 octava eisdem quorum octava
it-has-been if therefore of-256 the-eighth to-the-same of-which the-eighth
est apponatur, fient 288. Quibus 192 comparati,
it-is be-added they-will-become 288 to-which 192 compared
sesquialterum spatium proportionis efficiunt. Quocirca tres
a-sesquialter space of-proportion they-make wherefore three
quidem toni sunt, si primus ad secundum, secundus ad
indeed tones there-are if the-first to the-second the-second to
tertium, quintus conferatur ad quartum. Semitonium vero
the-third the-fifth be-compared to the-fourth the-semitone but
minus tertii ad quartum terminum comparatio tenet; quod
lesser of-the-third to the-fourth term the-comparison holds but
si diatessaron quidem duorum tonorum est ac semitonii
if the-diatessaron indeed of-two tones is and of-a-semitone
minoris, diapente vero trium tonorum ac semitonii minoris.
lesser the-diapente but of-three tones and of-a-semitone lesser
Junctae vero diatessaron ac diapente unum diapason videntur
joined indeed the-diatessaron and diapente one diapason are-seen
efficere, erunt quinque toni et duo spatia semitoniorum
to-make there-will-be five tones and two spaces of-semitones
minora, quae unum tonum non videantur implere. Non est
lesser which one tone not may-seem to-fill-up not is
igitur diapason consonantia constans sex tonis, ut Aristoxenus
therefore the-diapason a-consonance consisting of-six tones as Aristoxenus
arbitratur, quod in numeris quoque dispositum evidenter
judges which in numbers also set-out evidently

apparet. Sex enim toni in ordinem disponantur, scilicet in
appears six for tones in order let-be-disposed namely in
sesquioctavis proportionibus constituti, sex vero sesquioctavae
sesquioctave proportions established six but sesquioctave
proportiones a sexto octuplo procreantur. Disponantur
proportions from the-sixth octuple are-begotten let-be-disposed
igitur sex octupli hoc modo: Ab hoc igitur ultimo numero
therefore six octuples in-this manner from this therefore last number
sex toni in sesquioctava proportione constituti locentur, hoc
six tones in sesquioctave proportion established let-be-placed in-this
modo dispositis, primum octuplis terminis, ut octavae
manner having-been-disposed first the-octuple terms so-that the-eighth
terminorum partes ipsorum terminorum lateribus adjungantur;
of-the-terms parts of-the-very terms to-the-sides may-be-joined
sit autem descriptio talis: Hujus igitur dispositionis haec
let-be moreover the-diagram such of-this therefore disposition this
ratio est. Continuus enim versus, qui limes I,
the-reckoning is the-continuous for row which boundary-line I
dicitur, octuplos numeros tenet. A sexto vero octuplo
is-called the-octuple numbers holds from the-sixth but octuple
sesquioctavae proportiones ducuntur. Ubi vero octavas partes
the-sesquioctave proportions are-drawn where but the-eighth parts
scripsimus, octavae sunt eorum numerorum partes, quibus
we-have-written the-eighths they-are of-those numbers the-parts to-which
adjacent. Quae si eisdem quibus adjacent apponantur,
they-lie-beside which if to-the-same to-which they-lie-beside be-added
posteriores numeros creant, ut in primo, qui est 262144.
the-later numbers they-create as in the-first which is 262144
Hujus octava 32768; hi sibimet si conjungantur posteriorem
of-this the-eighth 32768 these to-themselves if be-joined the-later
efficiunt numerum qui est 294912. Idemque in caeteris
they-make number which is 294912 and-the-same in the-rest

invenitur. Si igitur ultimus numerus qui est 531441 duplus
is-found if therefore the-last number which is 531441 double
esset prioris numeri qui est 262144, recte diapason sex
were of-the-former number which is 262144 rightly the-diapason of-six
tonis constare videretur. Nunc autem si minimi numeri,
tones to-consist would-be-seen now however if of-the-smallest number
id est prioris, duplicem conquiramus, minor erit eo
that is of-the-former the-double we-seek-out smaller it-will-be than-that
numero qui est maximus ac supremus. Nam 262144
number which is greatest and highest for of-the-262144
numeri duplus est qui ad eum scilicet diapason
number the-double is (that)-which to it namely the-diapason
consonantiam tenet 524288; hic igitur minor est eo
consonance holds 524288 this therefore smaller is than-that
numero qui sextum retinet tonum, eo scilicet qui est
number which the-sixth keeps tone that-(one) namely which is
531441. Minor est igitur diapason consonantia sex tonis.
531441 smaller is therefore the-diapason consonance than-six tones
Atque id quo sex toni diapason consonantiam supervadunt,
and that by-which six tones the-diapason consonance overpass
comma vero est, quod constat in minimis numeris 524288
the-comma indeed is which stands in smallest numbers 524288
et 531441. Sed de his quid Aristoxenus sentiat, qui auribus
and 531441 but about these what Aristoxenus may-think who to-the-ears
dedit omne judicium, alias commemorabo. Nunc voluminis
gave all judgment elsewhere I-shall-recount now of-the-volume
seriem fastidii vitator astringam.
the-course of-tedium an-avoider I-shall-draw-tight

Adversus Aristoxenum demonstratio superparticularem proportionem dividi in aequa non posse, atque ideo nec tonum

III.1 · AGAINST ARISTOXENUS: THE TONE CANNOT BE HALVED

Aristoxenus, trusting the ear, holds that a semitone is a true half tone. Boethius answers that no superparticular ratio divides into equal halves by a whole number, and that the ear misses differences too small to hear until they add up. The tone's 8 and 9, doubled to 16 and 18, admit 17, so the half lies between 17:16 and 18:17; but 17:16 doubled overshoots the tone, and 18:17 doubled falls short.

Superiore volumine demonstratum est diatessaron
in-the-former volume demonstrated it-has-been (that)-the-diatessaron
consonantiam ex duobus tonis copulari ac semitonio, diapente
consonance of two tones to-be-joined and a-semitone (that)-diapente
vero ex tribus tonis ac semitonio constare. Sed ea
but of three tones and a-semitone to-consist but (that)-those
semitonia dimidium toni integrum non posse perficere, si
semitones the-half of-a-tone whole not to-be-able to-complete if
sigillatim considerata tractentur, atque ideo diapason ad
one-by-one considered they-be-treated and therefore (that)-the-diapason to
sex tonos nullo modo pervenire. Sed quoniam Aristoxenus
six tones in-no way to-arrive but since Aristoxenus
musicus, judicio aurium cuncta permittens, haec semitonia
the-musician to-the-judgment of-the-ears all-things entrusting these semitones
non arbitratur esse secundum Pythagoricos contractiora
not judges to-be according-to the-Pythagoreans narrower
dimidio, sed sicut semitonia dicuntur, ita esse dimidietates
than-the-half but as semitones they-are-called so to-be halves
tonorum, de eisdem rursus paulisper est disputandum,
of-tones about the-same again for-a-little-while it-is to-be-disputed

demonstrandumque prius nullam superparticularem habitudinem
and-to-be-demonstrated first (that)-no superparticular relation
noto numero posse dividi in integram medietatem.
by-a-known number to-be-able to-be-divided into a-whole half
Inter duos enim numeros superparticularem proportionem
between two for numbers a-superparticular proportion
continentes, sive illi sint principales, quorum est unitas
containing whether those be primary of-which is the-unit
differentia, sive posteriores, nullus ita poterit medius numerus
the-difference or later-(ones) no so will-be-able middle number
collocari, ut quam minimus proportionem tenet ad
to-be-placed that what-(proportion) the-smallest proportion holds to
medium, eam medius teneat ad extremum, scilicet
the-middle that-(same) the-middle may-hold to the-extreme namely
ut in geometrica proportione, sed aut differentias aequas
as in geometric proportion but either differences equal
facere potest, ut sit aequalitas secundum arithmeticam
to-make it-can so-that there-be equality according-to the-arithmetical
medietatem, aut harmonicam inter eosdem terminos medius
mean or the-harmonic between the-same terms the-middle
numerus collocatus faciet medietatem, aut quamlibet aliam,
number placed will-make a-mean or whatsoever other
quarum in arithmeticis fecimus mentionem.
of-which in the-arithmetical-(books) we-have-made mention
Quod si id demonstrabitur, nec illud quidem constare poterit,
but if this shall-be-demonstrated nor that indeed to-stand will-be-able
sesquioctavam proportionem, quae tonus est in dimidia posse
(that)-the-sesquioctave proportion which the-tone is into halves to-be-able
discerni, quandoquidem sesquioctava omnis in superparticulari
to-be-parted since sesquioctave every in the-superparticular
inaequalitatis genere consistit. Id vero melius inductione
of-inequality genus consists this but better by-induction

monstrabitur. Nam si per singulas proportiones consideratione
will-be-shown for if through the-several proportions the-consideration
deducta, scilicet superparticulares, nulla prorsus occurrit quae,
led-down namely the-superparticulars no-(one) at-all occurs which
interposito medio termino, aequis proportionibus dividatur,
with-interposed middle term into-equal proportions may-be-divided
non est dubium quod superparticularis comparatio non possit
not is doubt that a-superparticular comparison not can
in aequa partiri. Quod si videtur consonum auribus
into equal-(parts) to-be-divided but if seems consonant to-the-ears
aliquid canere, cum cuilibet voci duobus tonis ac semitonio
something to-sound when to-any note by-two tones and a-semitone
integro distans vocula comparetur, id non esse consonum
whole distant little-voice be-compared (that)-this not to-be consonant
natura monstratur. Sed quoniam sensus omnis quae
by-nature is-shown but since sense every (the-things)-which
minima sunt comprehendere nequeat, idcirco hanc
smallest are to-grasp is-unable therefore (that)-this
differentiam, quae ultra consonum procedit, sensum aurium
difference which beyond the-consonant proceeds the-sense of-the-ears
non posse distinguere, fore autem ut deprehendatur,
not to-be-able to-distinguish (that-it)-will-be however that it-be-detected
si frequentissime talis particula per eosdem crescat errores.
if very-often such a-particle through the-same grows errors
Nam quod in minimo haud sane cernitur, compositum
for what in the-smallest not clearly is-discerned compounded
conjunctumque cum jam magnum esse coeperit pervidetur.
and-conjoined when now great to-be it-has-begun is-seen-through
A qua igitur proportione est ordiendum an compendium
from what therefore proportion is to-be-begun or a-shortcut
dabimus quaestioni, si ab eo de quo quaeritur ordiamur. Id
shall-we-give to-the-question if from that about which it-is-inquired we-begin this

vero est tonus, in duo aequa possit partiri necne.
but is the-tone into two equal-(parts) (whether)-it-can to-be-divided or-not
Nunc igitur de tono est pertractandum, et quemadmodum
now therefore about the-tone it-is to-be-treated-thoroughly and how
non possit in duo aequa dividi demonstrandum est.
not it-can into two equal-(parts) to-be-divided to-be-demonstrated is
Quam demonstrationem si quis ad reliquas superparticulares
which demonstration if anyone to the-remaining superparticular
comparationes transferat, similiter demonstrabitur
comparisons should-transfer similarly it-will-be-demonstrated
superparticularem in aequa noto atque integro numero
(that)-a-superparticular in equal-(parts) by-a-known and whole number
separari non posse. Primi igitur tonum continentes
to-be-separated not to-be-possible the-first therefore a-tone containing
numeri sunt 8 atque 9. Sed quoniam se isti ita naturaliter
numbers are 8 and 9 but since each-other these so naturally
consequuntur, ut medius inter eos numerus non sit, eosdem
follow that a-middle between them number not is the-same
binario, quo scilicet minimo possum, multiplico. Fiunt
by-two by-which namely smallest I-can I-multiply they-become
igitur 16 ac 18, inter hos vero naturaliter numerus cadit qui est
therefore 16 and 18 between these but naturally a-number falls which is
17; igitur 18 ad 16 tonus est. Sed 10 et 8 ad 10 et 7, comparatus
17 therefore 18 to 16 a-tone is but 10 and 8 to 10 and 7 compared
habet eum totum, et ejus septimam decimam partem.
has it whole and of-it the-seventh (and-)tenth part
Septima decima vero pars minor est sexta decima
the-seventh (and-)tenth but part smaller is than-the-sixth (and-)tenth
naturaliter. Major est igitur proportio quae sub 16 ac
naturally greater is therefore the-proportion which under 16 and
17 numeris continetur, quam ea quae sub 17 ac 18. Qui
17 numbers is-contained than that which under 17 and 18 which

disponantur hoc modo, et sit A 16, C 17, B 18. Medietas
let-be-disposed in-this manner and let-be A 16 C 17 B 18 the-half
igitur integra toni inter C ac B nullo modo cadet.
therefore whole of-the-tone between C and B in-no way will-fall
Minor est enim C B proportio C A proportione.
smaller is for the-C B proportion than-the-C A proportion
Ad majorem igitur partem medietas rata ponenda est.
toward the-greater therefore part the-half fixed to-be-placed is
Sit vero medietas D. Quoniam igitur D B quidem
let-be but the-half D since therefore the-D B indeed
proportio, quod est integrum dimidium toni, major est
proportion which is the-whole half of-the-tone greater is
C B proportione, quae est minor pars toni, A
than-the-C B proportion which is the-lesser part of-the-tone the-A
C autem proportio quae est major pars toni A
C moreover proportion which is the-greater part of-the-tone than-the-A
D proportione major est, quod est dimidium toni, est
D proportion greater is which is the-half of-the-tone is
autem A C proportio sesquisextadecima, C B autem
moreover the-A C proportion the-sesquisixteenth the-C B moreover
sesquiseptimadecima, non est dubium quin integra medietas
the-sesquiseventeenth not is doubt but-that the-whole half
inter sesquisextamdecimam ac sesquiseptimamdecimam cadat.
between the-sesquisixteenth and the-sesquiseventeenth falls
Sed hoc in integro numero nullo modo poterit inveniri.
but this in a-whole number in-no way will-be-able to-be-found
Quoniam vero ad 16 numerum 17 numerus comparatus,
since but to the-16 number the-17 number compared
sesquisextamdecimam obtinet proportionem, si ejusdem 17
the-sesquisixteenth occupies proportion if of-the-same 17
numeri sextam decimam requiramus, erit unitas, atque
number the-sixth (and-)tenth we-seek there-will-be a-unit and

unitatis pars 16, hanc si eidem 17 numero conjungamus,
of-a-unit the-part 16th this if to-the-same 17 number we-join
fiunt 18 et pars sexta decima. Si igitur 18 et
they-become 18 and a-part the-sixth (and-)tenth if therefore 18 and
pars sexta decima a 16 numero comparetur, recte
a-part the-sixth (and-)tenth from the-16 number be-compared rightly
toni mensuram videtur excedere, cum ad eum solus
of-the-tone the-measure it-seems to-exceed since to it alone
18 numerus sesquioctavam custodiat proportionem. Unde
the-18 number the-sesquioctave keeps proportion whence
fit ut quoniam supersesquisextadecima proportio tonum
it-comes that since the-supersesquisixteenth proportion the-tone
bis aucta transcendit, non sit integrum toni dimidium.
twice increased overpasses not it-is the-whole of-the-tone half
Quidquid enim bis ductum transcendit aliquid, id ultra
whatever for twice multiplied overpasses something that beyond
dimidium illius esse videbitur quod transcendit. Quocirca
the-half of-that to-be will-be-seen which it-overpasses wherefore
supersesquisextadecima, non erit toni dimidium. Ac per
the-supersesquisixteenth not will-be of-the-tone the-half and through
hoc, nec alia ulla major sesquisextadecima proportione toni
this nor other any greater than-the-sesquisixteenth proportion of-the-tone
poterit esse dimidium, cum ipsa sesquisextadecima integro
will-be-able to-be the-half since itself the-sesquisixteenth than-the-whole
toni dimidio sit major. Sed quoniam, sesquisextamdecimam
of-the-tone half is greater but since the-sesquisixteenth
proportionem continua sequitur sesquiseptimadecima, videamus
proportion continuous follows the-sesquiseventeenth let-us-see
an ea tonum bis multiplicata non impleat, 17 igitur
whether it the-tone twice multiplied not may-fill of-the-17 therefore
numeri sesquiseptimamdecimam partem tenet terminus 18.
number the-sesquiseventeenth part holds the-term 18

In eadem igitur proportione si ad 18 numerum alium
in the-same therefore proportion if to the-18 number another
comparemus, erit 19 et septimadecima pars. Quod
we-compare there-will-be 19 and a-seventeenth part but
si ad 17 terminum in sesquioctava proportione positum
if to the-17 term in the-sesquioctave proportion placed
numerum comparemus, fient 19, et pars octava.
a-number we-compare they-will-become 19 and a-part eighth
Major vero est pars octava parte septima decima.
greater but is the-part eighth than-the-part the-seventh (and-)tenth
Major igitur est proportio numerorum 17 ac 19 et octava
greater therefore is the-proportion of-the-numbers 17 and 19 and an-eighth
quam ea quae in 17 ac 19 et parte septima decima
than that which in 17 and 19 and a-part the-seventh (and-)tenth
continetur, quae sunt scilicet bis sesquiseptimaedecimae
is-contained which are namely twice sesquiseventeenth
proportiones. Duae igitur sesquiseptimaedecimae unum tonum
proportions two therefore sesquiseventeenths one tone
non videntur implere. Non est igitur sesquiseptimadecima
not are-seen to-fill not is therefore the-sesquiseventeenth
toni dimidium, quoniam quae duplicata non implet
of-the-tone the-half since (that)-which doubled not fills
integrum non continet dimidium. Semper enim dimidium
the-whole not contains the-half always for the-half
duplicatum ei cujus dimidium est coaequatur.
doubled that of-which the-half it-is equals

Ex sesquitertia proportione sublatis duobus tonis, toni dimidium non relinqui

III.2 · WHAT TWO TONES LEAVE OF THE DIATESSARON

Boethius takes the diatessaron in the numbers 192 and 256 and removes two tones, 192 to 216 and 216 to 243. What is left is the ratio 256:243. An eighteenth part of 243 is 13 and a half, which added gives 256 and a half, so 256:243 is smaller even than 19:18 — and since the tone's half lies between 17:16 and 18:17, this remainder falls far short of a half tone.

Jam vero si eos numeros disponamus qui de sesquitertia
now indeed if those numbers we-dispose which from the-sesquitertian
proportione duobus tonis retractis relinquuntur, in his
proportion with-two tones withdrawn are-left in these
considerare possumus utrum ea proportio quae post duos
to-consider we-can whether that proportion which after two
tonos relinquitur, integri loco semitonii censeatur;
tones is-left of-a-whole in-the-place semitone may-be-reckoned
quod si ita repertum sit, illud quoque est comprobatum
but if so found it-be that too is proved
diatessaron consonantiam duobus tonis atque integro
(that)-the-diatessaron consonance by-two tones and a-whole
semitonio copulari. Erat igitur superius terminus primus
semitone to-be-joined was therefore above the-term first
192. Ad hunc sesquitertiam proportionem tenebant 256. Sed
192 to this the-sesquitertian proportion held 256 but
ad primum terminum 216 faciunt tonum. Ad 216 rursus
to the-first term 216 they-make a-tone to 216 again
243 toni obtinent locum. Est igitur quod relinquitur ex
243 of-a-tone hold the-place is therefore what is-left from
diatessaron tota proportione, ea scilicet habitudo quae in 243
of-the-diatessaron whole proportion that namely relation which in 243

et 256 unitatibus constat. Haec igitur si probatur integri toni
and 256 units stands this therefore if is-proved of-a-whole tone
esse dimidium, dubitari non potest diatessaron ex duobus
to-be the-half to-be-doubted not it-can (that)-the-diatessaron of two
tonis semitonioque constare. Quoniam igitur demonstratum
tones and-a-semitone to-consist since therefore demonstrated
est toni dimidium inter sesquisextamdecimam
it-has-been (that)-of-the-tone the-half between the-sesquisixteenth
et sesquiseptimamdecimam proportionem locari, ab hac
and the-sesquiseventeenth proportion to-be-placed by this
comparatione etiam haec proportio metienda est.
comparison also this proportion to-be-measured is
Ne enim longius progrediamur, sumo ex 243 octavam decimam
lest for further we-proceed I-take from 243 the-eighth (and-)tenth
partem, ea fit 13 et semis. Hanc si eisdem apposuero,
part it becomes 13 and a-half this if to-the-same I-shall-have-added
fiunt 256 et semis. Apparet igitur minorem proportionem
they-become 256 and a-half it-appears therefore (that)-smaller the-proportion
esse 256 ad 243 sesquioctavadecima habitudine. Quod si
to-be 256 to 243 than-the-sesquieighteenth relation but if
dimidius tonus major quidem est in sesquisextadecima, minor
the-half tone greater indeed is in the-sesquisixteenth lesser
vero in sesquiseptimadecima proportione, sesquioctavadecima
but in the-sesquiseventeenth proportion the-sesquieighteenth
vero minor est sesquiseptimadecima habitudine, ducentorum
but smaller is than-the-sesquiseventeenth relation of-two-hundred
vero 56 ad ducentos 43 comparatio, quae scilicet
but (and-)56 to two-hundred (and-)43 the-comparison which namely
relinquitur ex diatessaron duobus retractis tonis, minor
is-left from the-diatessaron with-two withdrawn tones smaller

est sesquioctavadecima: non est dubium quin haec duorum
is than-the-sesquieighteenth not is doubt but-that this of-the-two
numerorum proportio semitonio longissime diminutior sit.
numbers proportion than-the-semitone by-far more-diminished is

Adversum Aristoxenum demonstrationes diatessaron consonantiam ex duobus tonis et semitonio non constare, nec diapason sex tonis

III.3 · THE DIATESSARON IS NOT TWO TONES AND A HALF

If, as Aristoxenus says, the diatessaron is two tones and a semitone, then two diatessarons make five tones. Boethius takes the six-tone numbers, 262144 with the fifth tone at 472392, stretches a diatessaron up from 262144 to 349525 and a third, and lets one down from 472392 to 354294. The two results should coincide; they differ by 4768 and two thirds, so the diatessaron is not two tones and a semitone.

Quod si (ut ait Aristoxenus) diatessaron consonantia ex duobus
but if as says Aristoxenus the-diatessaron consonance of two
tonis semitonioque conjungitur, duae diatessaron consonantiae
tones and-a-semitone is-joined two diatessaron consonances
necessario quinque tonos efficient, et diapente ac diatessaron
necessarily five tones will-make and the-diapente and diatessaron
junctae, sicut unum diapason jungunt, ita sex tonis
joined as one diapason they-join so to-six tones
continua proportione coaequantur. Et quoniam paulo ante sex
in-continuous proportion are-made-equal and since a-little before six
disposuimus tonos, quorum minimus erat numerus 262144, ad
we-disposed tones of-which the-smallest was number 262144 to
hunc vero ultimus in sexto collocabatur tono numerus 531441,
this but the-last in the-sixth was-placed tone number 531441
quintum vero retinebant tonum 472392, disponantur hoc
the-fifth but they-kept tone 472392 let-them-be-disposed in-this

modo: Nunc igitur de minoribus numeris, id est quinque
manner now therefore about the-smaller numbers that is five
tonis, loquamur. Si ergo diatessaron duobus tonis ac semitonio,
tones let-us-speak if then the-diatessaron of-two tones and a-semitone
bis vero diatessaron quinque consisteret tonis, cum ex 262144
twice but diatessaron of-five consisted tones when from 262144
diatessaron intenderem, cumque de 472392 aliud diatessaron
a-diatessaron I-stretched-up and-when from 472392 another diatessaron
remitterem, idem inter utramque intentionem remissionemve
I-slackened the-same between each stretching-up or-slackening
numerus inveniretur. Id autem fit hoc modo: a numero
number would-be-found this moreover is-done in-this manner from the-number
qui est 262144 diatessaron intendo, id est sesquitertium,
which is 262144 a-diatessaron I-stretch-up that is a-sesquitertian
qui fit in 349525 et triente. Rursus de 472392 numeris
which comes-to in 349525 and a-third again from the-472392 numbers
remitto sesquitertiam proportionem, quae sit in 354294, has
I-slacken a-sesquitertian proportion which is in 354294 these
igitur proportiones disponamus hoc modo, et sit primus
therefore proportions let-us-dispose in-this manner and let-be the-first
quidem numerus A, secundus vero B, tertius C, quartus D.
indeed number A the-second but B the-third C the-fourth D
Quoniam igitur A terminus ab D termino quinque
since therefore the-A term from the-D term by-five
remotus est tonis, quoniamque diatessaron in duobus tonis
removed is tones and-since the-diatessaron in two tones
ac semitonio jungitur, ut Aristoxenus arbitratur, unumque
and a-semitone is-joined as Aristoxenus judges and-one
diatessaron, inter A atque B, aliud vero inter C atque
diatessaron between A and B another but between C and
D positum est, B et C terminos non oportet esse
D placed is (that)-the-B and C terms not it-behooves to-be

diversos, sed unos atque eosdem, ut integre quinque toni
diverse but one and the-same so-that wholly five tones
ex duabus diatessaron consonantiis constare videantur. Nunc
of two diatessaron consonances to-consist may-be-seen now
vero quoniam est differentia 4768 et besse, arguitur
but since there-is a-difference 4768 and two-thirds it-is-proved
diatessaron minime tonis duobus ac semitonio conjungi.
(that)-the-diatessaron by-no-means from-tones two and a-semitone to-be-joined

Diapason consonantiam a 6 tonis commate excedi, et qui sit minimus numerus commatis

III.4 · SIX TONES EXCEED THE DIAPASON BY A COMMA

Boethius turns the fractional difference into whole numbers by adding to it its own half, 2384 and a third, which brings it to 7153. He adds their own halves to the terms A B C D as well, so the ratios stand unchanged and the same 7153 units still separate five tones from two diatessarons. Six tones therefore exceed the diapason by a comma, first contained in 7153 units.

Sed hanc si quaerimus in integris numeris differentiam
but this if we-seek in whole numbers the-difference
collocare, quoniam in ea parte quae est besse pars tertia
to-place since in that part which is two-thirds the-part third
si addatur, plenam efficit unitatem, quae pars tertia ejusdem
if it-be-added a-full it-makes unit which part third of-the-same
besse dimidium est, si totius differentiae dimidium eidem
two-thirds the-half is if of-the-whole difference the-half to-the-same
adjecero, quod est 2384 et triens sit omnis summa 7153.
I-shall-have-added which is 2384 and a-third let-be the-whole sum 7153
Quae dudum commatis proportionem tenebat. Comma enim
which lately of-the-comma the-proportion was-holding the-comma for

est quo sex toni superant diapason consonantiam, quod
is (that)-by-which six tones exceed the-diapason consonance which
in primis 7153 unitatibus continetur. Ut igitur differentiae
in the-first 7153 units is-contained as therefore of-the-difference
dimidium proprium adjecimus, ut in 7153 excresceret,
the-half its-proper we-added (so)-that into 7153 it-might-grow
ita etiam cunctis A B C D terminis medietates proprias
so also to-all A B C D the-terms halves their-proper
adjungamus, eadem erit in omnibus quae supra proportio,
let-us-join the-same will-be in all which-(was) above proportion
fietque eadem inter quinque tonos ac bis diatessaron
and-will-become the-same between five tones and twice the-diatessaron
differentia quae est inter sex tonos ac diapason consonantiam
the-difference which is between six tones and the-diapason consonance
differentia, scilicet 7153 unitates. Unde colligitur quinque
the-difference namely 7153 units whence it-is-gathered (that)-five
tonos bis diatessaron, et sex tonos unum diapason tantum
tones twice the-diatessaron and six tones one diapason only
commate superare. Quod in primis 7153 unitatibus invenitur.
by-a-comma to-exceed which in the-first 7153 units is-found
Id autem patefaciet haec subjecta descriptio:
this moreover will-make-plain this subjoined diagram

Quemadmodum Philolaus tonum dividat

III.5 · PHILOLAUS' DIVISION OF THE TONE

Philolaus starts the tone from 27, the cube of the first odd number, which stands a tone above 24, since 3 is an eighth of 24. He cuts it into a part greater than half, the apotome, and one less than half, the diesis, later called the lesser semitone; their difference is the comma. The diesis is 13 units, the difference of 256 and 243 and the sum of 9, 3, and 1 (square, line, and point); the apotome is 14, the comma 1, and the tone 27, the difference of 216 and 243.

Philolaus vero Pythagoricus alio modo tonum dividere
Philolaus but the-Pythagorean in-another manner the-tone to-divide
tentavit; statuens scilicet primordium toni ab eo numero
attempted establishing namely the-beginning of-the-tone from that number
qui primus cubum a primo impari (quod maxime
which first a-cube from the-first odd-(number) (a-thing)-which most
apud Pythagoricos honorabile fuit) efficeret. Nam cum
among the-Pythagoreans honorable was should-make for since
ternarius numerus primus sit impar, ter tres fiunt 9, atque id
the-ternary number the-first is odd thrice three become 9 and this
ter si duxeris, 27 necessario exsurgent qui ad 24
thrice if you-shall-have-multiplied 27 necessarily will-arise which to the-24
numerum tono distat, eamdem ternarii differentiam servans.
number by-a-tone is-distant the-same of-the-ternary difference keeping
Ternarius enim 24 summae octava pars est. Quae eisdem
the-ternary for of-the-24 sum the-eighth part is which to-the-same
addita primum a ternario cubum 27 reddit. Ex hoc igitur
being-added the-first from the-ternary cube 27 renders from this therefore
Philolaus duas efficit partes: unam quae dimidio sit major,
Philolaus two makes parts one which than-the-half is greater
eamque apotomen vocat; reliquam quae dimidio sit minor,
and-it the-apotome he-calls the-other which than-the-half is less

eamque rursus diesim dicit, quam posteri semitonium
and-it again a-diesis he-calls which the-later-(writers) a-semitone
minus appellavere. Horum vero differentiam comma
lesser have-named of-these but the-difference (he-calls)-a-comma
ac primum diesim in 13 unitatibus constare arbitratur,
and first the-diesis in 13 units to-consist he-judges
eo quod hoc inter 256 et 243 pervisa sit differentia.
for-this (reason)-that this between 256 and 243 observed is the-difference
Quodque idem numerus, id est 13, ex novenario ternario
and-(the-fact)-that the-same number that is 13 of the-novenary the-ternary
atque unitate consistat, quae unitas puncti obtineat locum.
and the-unit consists which unit of-a-point holds the-place
Ternarius vero primae imparis lineae. Novenarius primi
the-ternary but (is-that)-of-the-first odd line the-novenary of-the-first
imparis quadrati. Ex his igitur causis cum 13 diesim
odd square from these therefore causes since (at)-13 the-diesis
ponat, quod semitonium nuncupatur, reliquam 27 numeri
he-sets which a-semitone is-named the-remaining of-the-27 number
partem quae ex 14 unitatibus continetur, apotome esse
part which of 14 units is-contained the-apotome to-be
constituit. Sed quoniam inter 13 et 14 unitas differentiam
he-establishes but since between 13 and 14 a-unit the-difference
facit, unitatem loco commatis censet esse ponendam, totum
makes the-unit in-place of-a-comma he-deems to-be to-be-set-down the-whole
vero tonum in 27 unitatibus locat, eo quod inter 216
but tone in 27 units he-places for-this (reason)-that between 216
ac 243 qui inter se distant tono 27 sit differentia.
and 243 which between themselves are-distant by-a-tone 27 is the-difference

Tonum ex duobus semitoniis ac commate constare

III.6 · TWO SEMITONES AND A COMMA MAKE THE TONE

From this it follows that the tone consists of two lesser semitones and a comma. The whole tone is an apotome plus a semitone, and the apotome differs from the semitone by a comma, so the apotome is nothing but a lesser semitone and a comma. Take two lesser semitones from a tone, and a comma is left.

Ex quibus facile apparet tonum duobus semitoniis
from these-(things) easily it-appears (that)-the-tone of-two semitones
minoribus et commate constare. Nam si totus tonus ex
lesser and a-comma to-consist for if the-whole tone of
apotome constat ac semitonio, semitonium vero ab apotome
the-apotome consists and a-semitone the-semitone but from the-apotome
differt commate; nihil est aliud apotome nisi semitonium
differs by-a-comma nothing is else the-apotome except a-semitone
minus et comma. Si igitur duo semitonia minora de tono
lesser and a-comma if therefore two semitones lesser from the-tone
quis auferat, comma fit reliquum.
anyone should-take-away a-comma becomes the-remainder

Demonstratio, tonum duobus semitoniis commate distare

III.7 · PROOF OF THE COMMA OVER TWO SEMITONES

Boethius proves the same another way. The diapason is five tones and two lesser semitones, and six tones exceed the diapason by one comma; remove five tones from each, and two lesser semitones remain against one tone. The tone therefore surpasses the two semitones by a comma — found in the first 7153 units — and with the comma restored they equal it.

Idem vero hoc quoque probabitur modo. Nam si diapason
the-same but in-this too will-be-proved manner for if the-diapason
quinque tonis ac duobus minoribus semitoniis continetur,
by-five tones and two lesser semitones is-contained
superantque sex toni diapason consonantiam uno commate,
and-exceed six tones the-diapason consonance by-one comma
non est dubium quin tonis quinis ab utroque spatio
not is doubt but-that tones five-apiece from either interval
sublatis fiant reliqua ex diapason quidem duo
being-taken-away there-become remainders from the-diapason indeed two
semitonia minora, de sex vero tonis tonus. Atque hic tonus
semitones lesser from the-six but tones a-tone and this tone
haec duo semitonia quae relinquuntur vincit commate. Quod si
these two semitones which are-left surpasses by-a-comma but if
duobus eisdem semitoniis comma reponatur, aequabunt tonum;
to-the-two same semitones the-comma be-added-back they-will-equal the-tone
constat igitur unum tonum duobus semitoniis minoribus et
it-is-agreed therefore (that)-one tone to-two semitones lesser and
commati, quod in 7153 primis unitatibus invenitur, aequari.
to-a-comma which in 7153 first units is-found to-be-equal

De minoribus semitonii intervallis

III.8 · THE LESSER INTERVALS OF THE SEMITONE

Philolaus defines the smaller intervals: the diesis is the amount by which the sesquitertian ratio exceeds two tones, the comma the amount by which the sesquioctave exceeds two dieses, the schisma half a comma, the diaschisma half a diesis. The tone thus divides into a lesser semitone and an apotome, or two semitones and a comma, or four diaschismata and a comma; its exact half is two diaschismata and a schisma. The apotome differs from the lesser semitone by two schismata, that is, a comma.

Philolaus igitur haec atque his minora spatia talibus
Philolaus therefore these and than-these lesser intervals in-such
diffinitionibus includit. Diesis, inquit, est spatium quo major
definitions encloses a-diesis says-he is an-interval by-which greater
est sesquitertia proportio duobus tonis. Comma vero est spatium
is the-sesquitertian proportion than-two tones a-comma but is an-interval
quo major est sesquioctava proportio duabus diesibus, id est
by-which greater is the-sesquioctave proportion than-two dieses that is
duobus semitoniis minoribus. Schisma est dimidium commatis.
two semitones lesser a-schisma is the-half of-a-comma
Diaschisma vero dimidium dieseos, id est semitonii minoris.
a-diaschisma but the-half of-a-diesis that is of-a-semitone lesser
Ex quibus illud colligitur, quoniam tonus quidem dividitur
from these-(things) this is-gathered that the-tone indeed is-divided
principaliter in semitonium minus atque apotomen. Dividitur
principally into a-semitone lesser and the-apotome it-is-divided
etiam in duo semitonia et comma, quo fit ut dividatur
also into two semitones and a-comma whereby it-comes-about that it-be-divided
in quatuor diaschismata et comma. Integrum vero dimidium
into four diaschismata and a-comma the-entire but half
toni quod est semitonium, constat ex duobus diaschismatibus,
of-the-tone which is the-semitone consists of two diaschismata

quod est unum semitonium minus, et schismate, quod est
which is one semitone lesser and a-schisma which is
dimidium commatis. Quoniam ergo cum totus tonus ex
the-half of-a-comma since then when the-whole tone of
duobus semitoniis minoribus et commate conjunctus est, si
two semitones lesser and a-comma joined is if
quis id integre dividere velit, faciet unum semitonium
anyone it evenly to-divide should-wish he-will-make one semitone
minus commatisque dimidium. Sed unum semitonium minus
lesser and-of-a-comma the-half but one semitone lesser
dividitur in duo diaschismata. Dimidium vero commatis
is-divided into two diaschismata the-half but of-a-comma
unum schisma est. Recte igitur dictum est integre dimidium
one schisma is rightly therefore said it-is (that)-evenly the-half
tonum in duo diaschismata atque unum schisma posse
tone into two diaschismata and one schisma to-be-able
partiri. Quo fit ut integrum semitonium a minore
to-be-parted whereby it-comes-about that the-entire semitone from the-lesser
semitonio uno schismate differre videatur, apotome autem
semitone by-one schisma to-differ is-seen the-apotome moreover
a minore semitonio duobus schismatibus differt, differt
from the-lesser semitone by-two schismata differs it-differs
enim commate; sed duo schismata unum perficiunt comma.
for by-a-comma but two schismata one make-up comma

De toni partibus per consonantias sumendis

III.9 · TAKING PARTS OF THE TONE THROUGH CONSONANCES

Boethius shows how to take intervals through consonances alone, drawn as lines. A tone is found by stretching up a diapente and slackening a diatessaron, or the reverse toward the grave. The lesser semitone is what a diatessaron keeps after two such tones are removed; the apotome is what three tones keep after a diatessaron is removed; and a comma is taken by stretching up an apotome and slackening a lesser semitone.

Sed de his quidem hactenus. Nunc vero illud
but about these-(things) indeed thus-far now however this
videtur esse dicendum, quemadmodum per consonantias
seems to-be to-be-said how through consonances
musicas imperata possimus spatia nunc extendere, nunc
musical the-prescribed we-may-be-able intervals now to-stretch-out now
vero remittere. Id autem ut linealiter fiat, lineaeque
again to-slacken this moreover (so)-that by-lines it-may-be-done and-the-lines
quas describimus vocis accipiuntur loco. Sed sese jam
which we-describe of-the-voice are-taken in-place but itself now
ratio ipsa demonstret. Sit propositum toni spatium
the-reckoning itself let-show let-it-be proposed of-a-tone the-interval
per consonantiam sumere, in acutum scilicet atque
through consonance to-take toward the-high namely and
gravem. Sit sonus B, ab hoc intendo alium sonum qui
the-low let-there-be a-sound B from this I-stretch-up another sound which
diapente spatio ab eo quod est B distet ad eum
by-the-diapente interval from that which is B may-be-distant to that-(sound)
qui est C. Ad hoc remitto diatessaron consonantiam ad
which is C at this I-slacken the-diatessaron consonance to
id quod est D, et quoniam inter diapente ac diatessaron
that which is D and since between the-diapente and the-diatessaron

tonus differentiam facit, D B spatium tonus repertus
a-tone the-difference makes the-D B interval a-tone found
est. Ad graviorem vero partem ita modulabimur tonum.
has-been toward the-lower but part thus we-shall-measure-out a-tone
Ab eo quod est B diatessaron intendo ad F, et ab F
from that which is B a-diatessaron I-stretch-up to F and from F
diapente remitto ad K. Erit K B tonus. Animadvertet igitur
a-diapente I-slacken to K will-be K B a-tone will-observe therefore
diligens lector ad D B quidem ad acutam partem
the-careful reader (that)-at D B indeed toward the-high part
effectum tonum, ad K B autem ad gravem.
(to-have-been)-effected the-tone at K B however toward the-low
Sit propositum minorem toni partem per consonantiam
let-it-be proposed the-lesser of-the-tone part through consonance
sumere in acutam partem atque gravem. Minor vero
to-take toward the-high part and the-low the-lesser but
toni pars est spatium quo duos tonos diatessaron
of-the-tone part is the-interval by-which two tones the-diatessaron
consonantia transcendit. Sit enim sonus A, intendo ab
consonance surpasses let-there-be for a-sound A I-stretch-up from
A diatessaron ad B. Rursus intendo ab B diatessaron ad C,
A a-diatessaron to B again I-stretch-up from B a-diatessaron to C
ab C remitto diapente ad D. Tonus est igitur B D. Rursus
from C I-slacken a-diapente to D a-tone is therefore B D again
ab D intendo diatessaron ad E. Remitto iterum ab E
from D I-stretch-up a-diatessaron to E I-slacken again from E
diapente ad F. Tonus est igitur D F. Duo igitur sunt toni
a-diapente to F a-tone is therefore D F two therefore there-are tones
B D, D F. Et erat B A integrum diatessaron. Erit igitur
B D D F and was B A an-entire diatessaron will-be therefore
F A minor toni pars, quod semitonium nuncupatur.
F A the-lesser of-the-tone part which a-semitone is-named

Ad graviorem vero partem hoc modo: sit sonus A,
toward the-lower but part in-this way let-there-be a-sound A
intendo duos tonos per consonantiam ad C, diatessaron vero
I-stretch-up two tones through consonance to C a-diatessaron but
a C remitto ad K; erit igitur K A minor toni pars,
from C I-slacken to K will-be therefore K A the-lesser of-the-tone part
quod oportebat efficere. Si ergo a tribus tonis diatessaron
which it-behooved to-effect if therefore from three tones a-diatessaron
auferamus, apotome fit reliqua. Sint enim tres toni
we-take-away the-apotome becomes the-remainder let-there-be for three tones
A B, B C, C D; ab his auferatur A E diatessaron. Erit
A B B C C D from these let-be-taken-away A E a-diatessaron will-be
igitur E C semitonium minus; apotome igitur est E D.
therefore E C a-semitone lesser the-apotome therefore is E D
Hanc igitur apotomen si sit commodum sic sumemus. Ac
this therefore apotome if it-be convenient thus we-shall-take and
primum quidem ad acutum intendo tres tonos, ab A eos
first indeed toward the-high I-stretch-up three tones from A those
qui sunt ad B, et ab eo quod est B ad C diatessaron
which are at B and from that which is B to C the-diatessaron
consonantiam remitto, fit C A apotome reliqua. Quod
consonance I-slacken there-becomes C A the-apotome remaining but
si idem spatium ad gravem sonum velimus efficere,
if the-same interval toward the-low sound we-should-wish to-effect
fit hoc modo: sit sonus A, intendo semitonium
it-is-done in-this way let-there-be a-sound A I-stretch-up a-semitone
minus, id quod est A D, remitto ad D tonum qui est D E.
lesser that which is A D I-slacken at D a-tone which is D E
Erit igitur E A ea quam requirimus apotome. Sit
will-be therefore E A that which we-require apotome let-it-be
propositum in acutam partem comma sumere, sit sonus
proposed toward the-high part a-comma to-take let-there-be a-sound

A; intendo apotomen A B, remitto semitonium minus B C, et
A I-stretch-up an-apotome A B I-slacken a-semitone lesser B C and
quoniam semitonium apotome minus est commate, comma
since the-semitone than-the-apotome less is by-a-comma the-comma
erit C A. Rursus ad gravem partem hoc modo intendo:
will-be C A again toward the-low part in-this way I-stretch-up
ab A sono semitonium minus, id quod est A D, ad D vero
from the-A sound a-semitone lesser that which is A D at D but
remitto apotomen, id quod est D E. Erit igitur comma E A.
I-slacken an-apotome that which is D E will-be therefore the-comma E A

Regula sumendi semitonii

III.10 · A RULE FOR TAKING THE SEMITONE

These consonances must be known to the ear by practice as well as to reason. Boethius works one harder example: finding the lesser semitone on either side of a diatessaron A B, where chains of diatessarons and diapentes yield two tones and then a semitone toward the sharp and another toward the grave. The whole span — two tones and three lesser semitones — falls short of a diapente by one comma, and he closes by proposing to show in what ratio the comma itself stands.

Oportet vero has omnes consonantias rite esse animo atque
it-behooves but these all consonances duly to-be to-the-mind and
auribus notas; frustra enim haec ratione et scientia
to-the-ears known in-vain for these-(things) by-reckoning and by-knowledge
colliguntur; nisi fuerint usu atque exercitatione
are-gathered unless they-shall-have-been by-use and by-practice
notissima. Ut vero id quod institutione musicae adorsi
best-known (so)-that but that which by-the-instruction of-music undertaken
sumus non mox auribus, quod jam provectorum in
we-have not straightway by-the-ears (a-thing)-which now of-the-advanced in

musica est, sed ratione interim censeatur, unum dabimus
music is but by-reckoning meanwhile may-be-judged one we-shall-give
exemplum inveniendi spatii, quod videtur esse paulo
example of-finding an-interval which seems to-be a-little
difficilius, scilicet semitonii minoris, ut in utramque
harder namely of-the-semitone lesser (so)-that toward either
partem, acutam scilicet atque gravem, rato possit ordine
part the-high namely and the-low by-fixed it-may order
reperiri. Sit diatessaron A B: oportet igitur circa A
to-be-found let-there-be a-diatessaron A B it-behooves therefore around the-A
B consonantiam minus semitonium ad graviorem partem
B consonance a-lesser semitone toward the-lower part
acutioremque deducere. Intendo igitur B C diatessaron. Remitto
and-the-higher to-draw-out I-stretch-up therefore B C a-diatessaron I-slacken
rursus diapente C D. Erit igitur tonus B D; diatessaron enim
again a-diapente C D will-be therefore a-tone B D the-diatessaron for
consonantia a diapente consonantia tono superatur, et C
consonance by the-diapente consonance by-a-tone is-exceeded and the-C
B spatium ab C D spatio B D spatio transcenditur.
B interval by the-C D interval B D by-the-interval is-surpassed
Rursus intendo diatessaron D E, remitto autem diapente E F.
again I-stretch-up a-diatessaron D E I-slacken moreover a-diapente E F
Tonus est igitur D F. Sed D B tonus erat. Semitonium
a-tone is therefore D F but D B a-tone was the-semitone
igitur minus est A F, quod, substractis duobus tonis F D,
therefore lesser is A F which being-subtracted the-two tones F D
B D ab A B diatessaron spatio relinquitur. Rursus remitto
B D from the-A B diatessaron interval is-left again I-slacken
diatessaron A G, intendo diapente G H. Erit igitur A H
a-diatessaron A G I-stretch-up a-diapente G H will-be therefore A H
tonus. Sed erat A F semitonium. Erit igitur F H apotome.
a-tone but was A F a-semitone will-be therefore F H the-apotome

Rursus remitto diatessaron H K, intendo diapente K L: tonus
again I-slacken a-diatessaron H K I-stretch-up a-diapente K L a-tone
igitur est H L. Erat autem tonus H A. Semitonium igitur
therefore is H L was moreover the-tone H A the-semitone therefore
minus est L B. Sed erat tonus B D. Erit igitur L D apotome.
lesser is L B but was a-tone B D will-be therefore L D the-apotome
Rursus intendo diatessaron F M. Semitonium igitur est B M.
again I-stretch-up a-diatessaron F M a-semitone therefore is B M
Remitto diatessaron L N; semitonium igitur est N A. Per
I-slacken a-diatessaron L N a-semitone therefore is N A through
consonantiam igitur sumpta sunt circa A B diatessaron, duo
consonance therefore taken have-been around the-A B diatessaron two
semitonia, B M quidem ad acutum; N A vero ad gravem
semitones B M indeed toward the-high N A but toward the-low
partem; totumque M N minus est quam diapente. Constat enim
part and-the-whole M N less is than the-diapente it-consists for
ex quinque semitoniis et apotome geminata; ex duobus igitur
of five semitones and the-apotome doubled of two therefore
tonis et tribus semitoniis minoribus; et quoniam duo semitonia
tones and three semitones lesser and since two semitones
unum tonum implere nequeunt, sed relinquitur comma, totum
one tone to-fill are-unable but there-is-left a-comma the-whole
M N spatium minus est spatio diapente consonantiae
M N interval less is than-the-interval of-the-diapente consonance
uno commate. Quod facillime diligens lector intelliget.
by-one comma which most-easily the-careful reader will-understand
Sed quoniam paululum de commatis ratione praediximus,
but since a-little about of-the-comma the-reckoning we-have-said-before
non est diffugiendum et in quali proportione idem ipsum
not is (it)-to-be-shunned also in what proportion the-same very
comma contineatur ostendere. Est enim comma quod
comma may-be-contained to-show is for the-comma (that)-which

ultimum comprehendere possit auditus, dicendumque est
last to-grasp can the-hearing and-to-be-said it-is
semitonium minus ac semitonium majus, quantis sigillatim
the-semitone lesser and the-semitone greater of-how-many severally
commatibus constare videantur. Ipse quoque tonus quantis
commas to-consist they-are-seen itself too the-tone of-how-many
rursus commatibus conjungitur. Ac primum hinc conveniens
again commas is-compounded and first hence a-fitting
sumatur initium.
let-be-taken beginning

Demonstratio, Archytae superparticularem in aequa dividi non posse, ejusque reprehensio

III.11 · ARCHYTAS' PROOF, AND ITS FAULT

Archytas argues that no superparticular ratio can be split equally by a proportional mean: the least numbers in such a ratio differ only by a unit, so no middle number falls between them. Boethius reports the proof and calls it too loose — the least numbers of multiple ratios also differ by a unit, yet many multiples beyond the roots do admit an equal-splitting mean. The conclusion is true of the superparticular alone and must not be stated universally.

Superparticularis proportio scindi in aequa medio
a-superparticular proportion to-be-split into equal-(parts) a-middle
proportionaliter interposito numero non potest. Id vero
proportionally being-interposed number not is-able this but
posterius firmiter demonstrabitur. Quam enim demonstrationem
later firmly will-be-demonstrated the-which for demonstration
ponit Archytas, nimium fluxa est. Haec vero est hujusmodi:
sets-forth Archytas too loose is this but is of-this-kind
Sit, inquit, superparticularis proportio A B. Sumo in eadem
let-there-be says-he a-superparticular proportion A B I-take in the-same

proportione minimos C DE. Quoniam igitur sunt minimi
proportion the-least-(numbers) C DE since therefore they-are the-least
in eadem proportione C DE, et sunt superparticulares, DE
in the-same proportion C DE and are superparticular the-DE
numerus C numerum parte una sua ejusque transcendit.
number the-C number by-part one its-own and-of-it surpasses
Sit autem haec D. Dico quoniam D non erit numerus,
let-be moreover this-(part) D I-say that D not will-be a-number
sed unitas. Si enim est numerus D et pars ejus est qui est
but a-unit if for is a-number D and a-part of-it is which is
E, metietur D numerus DE numerum. Quocirca et E
E will-measure the-D number the-DE number wherefore also the-E
numerum metietur. Quo fit ut C quoque metiatur,
number it-will-measure whereby it-comes-about that C too it-may-measure
utrumque igitur C et DE numeros metietur D numerus,
both therefore C and DE the-numbers will-measure the-D number
quod est impossibile. Qui enim sunt minimi in
(a-thing)-which is impossible (those)-which for are the-least in
eadem proportione quibuslibet aliis numeris, hi primi ad
the-same proportion as-whatever other numbers these prime to
se invicem sunt, et solam differentiam retinent unitatem;
themselves mutually are and a-sole difference they-retain (as)-the-unit
unitas igitur est D: igitur DE numerus C numerum
a-unit therefore is D therefore the-DE number the-C number
unitate transcendit. Quocirca nullus incidet medius numerus,
by-a-unit surpasses wherefore no will-fall-in middle number
qui eam proportionem aequaliter scindat. Quo fit
which that proportion equally may-split whereby it-comes-about
ut nec inter eos qui eamdem his proportionem tenent,
that not-even between those which the-same as-these proportion hold
medius possit numerus collocari, qui eamdem proportionem
a-middle can number to-be-placed which the-same proportion

aequaliter scindat. Et secundum Archytae quidem rationem
equally may-split and according-to of-Archytas indeed the-reasoning
idcirco in superparticulari nullus medius terminus cadit,
for-this-cause in a-superparticular no middle term falls
qui aequaliter dividat proportionem, quoniam minimi
which equally may-divide the-proportion since the-least-(numbers)
in eadem proportione sola differunt unitate. Quasi vero non
in the-same proportion by-a-sole differ unit as-if indeed not
etiam in multiplici proportione minimi eamdem unitatis
also in the-multiple proportion the-least-(numbers) the-same of-a-unit
differentiam sortiantur. Cum plures videamus esse multiplices
difference should-obtain since many we-see to-be multiples
praeter eos qui in radicibus collocati sunt, inter quos medius
besides those which in the-roots placed are between which a-middle
terminus scindens aequaliter eamdem proportionem possit
term splitting equally the-same proportion can
aptari. Sed haec qui arithmeticos nostros diligenter
to-be-fitted but these-(things) (he)-who the-arithmetic-(books) our diligently
inspexerit, facilius intelliget. Addendum vero est id
shall-have-examined more-easily will-understand to-be-added but is (that)-this
ita evenire, ut Archytas putat, in sola superparticulari
so to-come-about as Archytas thinks in the-sole superparticular
proportione. Non autem universaliter est dicendum. Nunc
proportion not however universally is (it)-to-be-said now
autem ad sequentia convertamur.
moreover to what-follows let-us-turn

In qua numerorum proportione sit comma, et quoniam in ea quae major sit quam 75 ad 74, minor quam 74 ad 73

III.12 · THE NUMBERS OF THE COMMA

Boethius bounds the comma: its numbers are 531441 and 524288, with difference 7153. Multiplied by 73 the difference gives 522169, less than 524288, and by 74 it gives 529322, greater — so the difference is less than a seventy-third part and more than a seventy-fourth part of the smaller term. The comma's ratio is therefore greater than 75:74 and less than 74:73, which a second argument from adding and subtracting differences confirms.

Primum igitur dico quoniam hi numeri qui comma continent
first therefore I-say that these numbers which the-comma contain
majorem inter se retinent proportionem quam 75 ad
a-greater between themselves hold proportion than 75 to
74, minorem quam 74 ad 73. Id vero ita demonstrabitur:
74 a-lesser than 74 to 73 this but thus will-be-demonstrated
ac primo quidem illud reminiscendum est, quod sex toni
and first indeed that to-be-recalled is that six tones
diapason commate transcendunt. Sit igitur A quidem
the-diapason by-a-comma exceed let-be therefore A indeed
262144, B autem diapason ad eum contineat consonantiam
262144 B moreover the-diapason to it let-contain consonance
in duplici scilicet constituta 524288, C vero sex tonis
in the-double namely established 524288 C but by-six tones
ab A numero discedat et sit 531441. Quae omnia ex
from the-A number let-depart and let-be 531441 which all-(things) from
secundi voluminis tonorum dispositione sunt colligenda. Inter
the-second volume's of-the-tones arrangement are to-be-gathered between
B igitur atque C commatis proportio continetur. Aufero
B therefore and C the-comma's proportion is-contained I-take-away
igitur B numerum de numero C, et relinquitur D in 7153
therefore B the-number from the-number C and is-left D in 7153

unitatibus collocatus. Qui numerus minor D est quam sit
units placed which number smaller D is than may-be
septuagesima tertia pars B numeri, major vero est quam
the-seventy third part of-the-B number greater but it-is than
septuagesima quarta sit. Nam si eumdem D numerum,
the-seventy fourth may-be for if the-same D number
qui est in 7153, septuagies ter multiplicem fiet
which is in 7153 seventy-times thrice I-multiply there-will-become
mihi E numerus in 522169 unitatibus constitutus.
for-me the-E number in 522169 units established
Si eum septuagies quater multiplicem, fit numerus F
if it seventy-times four-times I-multiply becomes the-number F
529322. Quorum quidem E, qui per septuaginta tres auctus
529322 of-which indeed E which by seventy three increased
est, minor est B numero; F autem qui per 74, major
has-been smaller is than-the-B number F moreover which by 74 greater
est B numero. Recte igitur dictum est D, ejus
is than-the-B number rightly therefore said it-has-been (that)-D of-that
quod est B, minorem quidem esse quam septuagesimam tertiam
which is B smaller indeed to-be than the-seventy third
partem, majorem vero quam septuagesimam quartam. Quodcirca
part greater but than the-seventy fourth wherefore
et C numerus B numerum minore quidem parte ejus
also the-C number the-B number by-a-smaller indeed part of-that
quod est B eumdem B superat, quam septuagesima tertia,
which is B the-same B surpasses than the-seventy third
majore vero quam septuagesima quarta. Ejus igitur quod
by-a-greater but than the-seventy fourth of-that therefore which
est C proportio ad id quod est B major quidem est quam
is C the-proportion to that which is B greater indeed is than
75 ad 74, minor vero quam 74 ad 73. Nam in priore unitas
75 to 74 lesser but than 74 to 73 for in the-former the-unit

septuagesima quarta est minoris, in posteriore vero eadem
the-seventy fourth-(part) is of-the-smaller in the-latter but the-same
unitas septuagesima tertia. Idem aliter explicandum, illo
the-unit the-seventy third the-same otherwise to-be-explained with-that
prius praesumpto, quod si cui proportioni propria numerorum
first presupposed that if to-any proportion its-own of-the-numbers
differentia aequaliter augeatur, minor inter eos qui post
difference equally be-increased a-lesser between those which after
additionem fiunt proportio continebitur quam inter priores
the-addition come-to-be proportion will-be-contained than between the-former
qui ante additionem illam quadam proportione distabant.
which before the-addition that by-a-certain proportion stood-apart
Ut sex et quatuor si utrisque binarius, quae est differentia
as six and four if to-both the-two which is difference
sua apponatur, fient 8 et 6, sed inter 6 et 4,
their-own be-set-beside they-will-become 8 and 6 but between 6 and 4
sesquialtera, inter 8 et 6 sesquitertia proportio continetur,
a-sesquialter between 8 and 6 a-sesquitertian proportion is-contained
minor vero est proportio sesquitertia sesquialtera proportione;
lesser but is the-proportion sesquitertian than-the-sesquialter proportion
hoc igitur ita praedicto, disponantur superiores numeri qui
this therefore thus stated-before let-be-arranged the-above numbers which
proportionem commatis continebant, id est 531441, et sit
the-proportion of-the-comma contained that is 531441 and let-be
A. Sit etiam B 524288; horum differentia sit C 7153;
A let-be also B 524288 of-these the-difference let-be C 7153
C igitur numerus majorem numerum eo qui est
the-C therefore number a-greater number than-that-(one) which is
A septuagies quinquies metiatur. Si igitur C numerum A
A seventy-times five-times let-measure if therefore the-C number A
septuagies quinquies multiplices, fiet mihi D qui est
seventy-times five-times you-multiply there-will-become for-me D which is

536475. Igitur D numerus eum qui est A, numero
536475 therefore the-D number that-(one) which is A by-the-number
eo qui est E antecedit, id est 5034. Rursus C numerus
that which is E outstrips that is 5034 again the-C number
eum qui est B metiatur septuagies quater multipliceturque,
that-(one) which is B let-measure seventy-times four-times and-let-be-multiplied
fiet igitur numerus F 529322. Qui F eo qui
there-will-become therefore the-number F 529322 which F than-that which
est B major est eodem E numero qui est 5034.
is B greater is by-the-same E number which is 5034
Ergo D numerus eum qui est A transcendit E
thus the-D number that-(one) which is A exceeds by-the-E
numero, B autem numerus ab eo qui est F vincitur
number the-B moreover number by that which is F is-overcome
eodem E numero. Si igitur A numero eumdem E
by-the-same E number if therefore to-the-A number the-same E
apponamus, fiet D. Si vero B numero eumdem
we-set-beside there-will-become D if but to-the-B number the-same
E apponamus, fiet F. Sed D numerus septuagies
E we-set-beside there-will-become F but the-D number seventy-times
quinquies auctus est, per C scilicet multiplicatum. F
five-times increased has-been through C namely multiplied F
autem septuagies quater multiplicato C crevit; obtinent igitur
moreover seventy-times four-times by-the-multiplied C has-grown they-hold therefore
inter se proportionem D atque F quam habent 75 ad
between themselves the-proportion D and F which have 75 to
74. Sed D atque F sunt A atque B uno eis addito E,
74 but D and F are A and B with-one to-them added E
majorem igitur necesse est proportionem contineri inter A
(that)-a-greater therefore necessary it-is proportion to-be-contained between A
atque B quam inter D atque F. Namque A atque B numerus,
and B than between D and F for-indeed A and B the-number

uno E addito, effecti sunt D atque F. Minor igitur
with-one E added made have-been D and F a-lesser therefore
proportio est inter D atque F quam inter A atque B. Sed
proportion is between D and F than between A and B but
inter D atque F eadem proportio est quae inter 75 et 74.
between D and F the-same proportion is as between 75 and 74
Inter A igitur atque B major proportio est quam inter
between A therefore and B a-greater proportion is than between
75 et 74. Ac A atque B comma continent. Major igitur
75 and 74 and A and B the-comma contain greater therefore
proportio est commatis quam 75 atque 74. Quoniam igitur
the-proportion is of-the-comma than 75 and 74 since therefore
ostendimus commatis proportionem majorem esse quam eam
we-have-shown the-comma's proportion greater to-be than that
quam 75 continent ad 74 comparati; nunc ostendendum
which 75 contain to 74 compared now to-be-shown
est quemadmodum minorem inter se proportionem
it-is how a-lesser between themselves proportion
contineant numeri spatium commatis continentes, quam
may-contain the-numbers the-interval of-the-comma containing than
74, ad 73 comparati. Id vero monstrabitur hoc modo:
74 to 73 compared this but will-be-shown in-this manner
reminiscendum prius est quod secundo volumine dixerimus,
to-be-recalled first it-is what in-the-second volume we-said
cum de mensura differentiae loquebamur. Si enim ex
when about the-measure of-the-difference we-were-speaking if for from
qualibet proportione differentiam eorum numerorum, qui eam
any proportion the-difference of-those numbers which it
continent, auferamus, hi qui relinquuntur majorem obtinebunt
contain we-take-away these which are-left a-greater will-hold
proportionem his numeris qui erant ante differentiae
proportion than-these numbers which were before the-difference's

diminutionem. Sint enim 8 et 6. Ab his propriam aufero
diminution let-be for 8 and 6 from these their-proper I-take-away
differentiam, id est 2, fiunt 6 et 4. Sed in superioribus
difference that is 2 they-become 6 and 4 but in the-higher-(pair)
sesquitertia, in hac sesquialtera proportio continetur.
a-sesquitertian in this a-sesquialter proportion is-contained
Major vero est sesquialtera proportio sesquitertia proportione.
greater but is the-sesquialter proportion than-the-sesquitertian proportion
Sint igitur iidem A atque B, qui sunt superius descripti,
let-be therefore the-same A and B which are above described
quorum est differentia C. Multiplico differentiam C numeri
of-which is the-difference C I-multiply the-difference C of-the-number
septuagies quater, sit mihi numerus, F, scilicet 529322.
seventy-times four-times let-be for-me the-number F namely 529322
Qui A numero comparatus vincitur numero G, scilicet
which to-the-A number compared is-overcome by-the-number G namely
2119. Rursus idem C multiplicetur septuagies ter, efficiet
2119 again the-same C let-be-multiplied seventy-times thrice it-will-make
numerum K, id est 522169. Qui comparatus B numero
the-number K that is 522169 which compared to-the-B number
vincitur eodem G, scilicet 2119. Sublato igitur G
is-overcome by-the-same G namely 2119 (there-being)-taken-away therefore G
de numeris A atque B, effecti sunt F atque K. Minorem
from the-numbers A and B made have-been F and K a-lesser
igitur proportionem retinebit A atque B quam F atque K. Sed
therefore proportion will-hold A and B than F and K but
F atque K eam retinent proportionem quam 74 ad 73, hi
F and K that hold proportion which 74 to 73 these
enim multiplicato C effecti sunt. Minor est igitur proportio
for by-the-multiplied C made have-been lesser is therefore the-proportion
A atque B numerorum comma continentium quam 74 ad
of-the-A and B numbers the-comma containing than 74 to

73. Sed paulo ante monstratum est eamdem commatis
73 but a-little before shown it-has-been (that)-the-same comma's
proportionem majorem esse quam 75 ad 74. Monstrati sunt
proportion greater to-be than 75 to 74 shown have-been
igitur numeri qui comma continent, majorem quidem inter
therefore the-numbers which the-comma contain a-greater indeed between
se habere proportionem quam 75 ad 74, minorem vero
themselves to-have proportion than 75 to 74 a-lesser but
quam 74 ad 73, quod oportebat ostendere.
than 74 to 73 which it-behooved to-show

Quod semitonium minus majus quidem sit quam 20 ad 19, minus quam 19 1/2 ad 18 1/2

III.13 · THE BOUNDS OF THE LESSER SEMITONE

The same method bounds the lesser semitone, which stands in 256:243 with difference 13. Taken 19 1/2 times the difference gives 253 1/2, and 18 1/2 times gives 240 1/2, which shows 256:243 less than 19 1/2 to 18 1/2; taken 20 times it gives 260, and 19 times 247, which shows it greater than 20:19. Boethius then turns to compare this semitone with the comma, the last interval the hearing can grasp.

Quod si ad semitonium minus talis speculatio convertatur, ejus
but if to the-semitone lesser such speculation be-turned its
quoque proportionem facillime reperiemus. Quae constat inter
too proportion most-easily we-shall-find which stands between
256 et 243. Sit igitur 256 A, 243 B, horum differentia 13 C.
256 and 243 let-be therefore 256 A 243 B of-these the-difference 13 C
Dico quoniam A ad B minorem retinet proportionem quam 19
I-say that A to B a-lesser holds proportion than 19
1/2 ad 18 1/2. Metiatur enim C 19 semis, id quod est A fiunt
1/2 to 18 1/2 let-measure for C 19 a-half that which is A they-become

253 1/2 quod sit D, qui scilicet comparatus ad A, eodem
253 1/2 which let-be D which namely compared to A by-the-same
A duobus semisque transcenditur. Sitque haec differentia F,
A by-two and-a-half is-exceeded and-let-be this difference F
scilicet 2 et 1/2 rursus eadem C differentia B numerum
namely 2 and 1/2 again the-same C difference the-B number
metiatur octies decies semis, fient 240 1/2 quod sit
let-measure eight-times ten-times a-half they-will-become 240 1/2 which let-be
E. Igitur E comparatus ad B eodem F transcenditur, id est
E therefore E compared to B by-the-same F is-exceeded that is
duobus et semis. D igitur ab eo quod est A, et rursus E ab
by-two and a-half D therefore from that which is A and again E from
eo quod est B, eadem F differentia sunt minores. Subtracto
that which is B by-the-same F difference are smaller (with)-subtracted
igitur F ab eo quod est A atque B, facti sunt D atque E.
therefore F from that which is A and B made have-been D and E
Majorem igitur proportionem tenent inter se D atque
a-greater therefore proportion hold between themselves D and
E, quam A atque B. Sed D atque E eamdem retinent
E than A and B but D and E the-same hold
proportionem inter se quam 19 1/2 ad 18 1/2. A igitur ad
proportion between themselves as 19 1/2 to 18 1/2 A therefore to
B minorem retinet proportionem quam 19 1/2 ad 18 1/2 quod
B a-lesser holds proportion than 19 1/2 to 18 1/2 which
oportebat ostendere. Videtur tamen eadem proportio 256 ad
it-behooved to-show it-seems however the-same proportion of-256 to
243 major esse ab ea quam continent 20 et 19. Sint enim A
243 greater to-be than that which contain 20 and 19 let-be for A
B C idem qui superius descripti sunt. Metiatur igitur C
B C the-same which above described have-been let-measure therefore the-C
differentia A terminum vigies, fient 260, qui sunt
difference the-A term twenty-times they-will-become 260 which are

D, qui comparati ad id quod est A eumdem quaternario
D which compared to that which is A the-same by-a-four
transcendunt, hic sit F. Rursus idem C metiatur B decies
exceed this let-be F again the-same C let-measure B ten-times
novies, fient 247, hic sit E, qui comparati ad B eodem
nine-times they-will-become 247 this let-be E which compared to B by-the-same
F transcendunt. D igitur numerus, A numerum, et E
F exceed the-D therefore number the-A number and the-E
numerus numerum B, eodem F transcendunt. Adjecto
number the-number B by-the-same F exceed (there-being)-added
igitur F his qui sunt A atque B, facti sunt D atque E.
therefore F to-these which are A and B made have-been D and E
Major igitur est proportio eorum qui sunt A atque B,
greater therefore is the-proportion of-those which are A and B
quam eorum qui sunt D atque E. Sed D atque E vigies
than of-those which are D and E but D and E twenty-times
atque decies novies multiplicatus, C numerum efficit.
and ten-times nine-times multiplied the-C number makes
Major igitur est proportio eorum qui sunt A atque B, qui
greater therefore is the-proportion of-those which are A and B which
scilicet semitonium continent, quam ea quae est 20 ad 19.
namely the-semitone contain than that which is of-20 to 19
Demonstratum igitur est semitonium minus majorem
demonstrated therefore it-has-been (that)-the-semitone lesser a-greater
quidem habere proportionem quam 20 ad 19, minorem vero
indeed to-have proportion than 20 to 19 a-lesser but
quam 19 1/2 ad 18 1/2. Nunc idem minus semitonium
than 19 1/2 to 18 1/2 now the-same lesser semitone
commati comparemus quod est ultimum, auditui subjacens
to-the-comma let-us-compare which is the-last to-the-hearing lying-under
ultimaque proportio.
and-the-last proportion

Semitonium minus majus quidem esse tribus commatibus, minus vero quatuor

III.14 · THE SEMITONE IN COMMAS

Boethius shows the lesser semitone greater than three commas and less than four. From A 262144 he lays out five tones to B 472392, the diapason at C 524288, and six tones at D 531441, with the comma of 7153 units between C and D; two tones down and diatessarons up place the semitone between F 497664 and C, with difference 26624. Three commas make 21459 and four make 28612, and 26624 lies between them.

Igitur demonstrandum proponimus semitonium minus, majus
therefore to-be-demonstrated we-propose (that)-the-semitone lesser greater
quidem esse commatibus tribus, minus vero quatuor, quod hinc
indeed to-be than-commas three lesser but than-four which hence
facillime possis agnoscere. Sint tres numeri ita dispositi,
most-easily you-may-be-able to-recognize let-be three numbers so arranged
ut inter se proportionem contineant diapason, et eam
that between themselves the-proportion they-may-contain of-the-diapason and that
quae dicitur sex tonorum. Sint enim A 262144. Intendantur
which is-called of-six tones let-be for A 262144 let-be-stretched
igitur ab A quidem quinque toni continui, et sit B 472392;
therefore from A indeed five tones continuous and let-be B 472392
ad C autem diapason consonantia referatur, et sit C 524288.
to C moreover the-diapason consonance let-be-referred and let-be C 524288
Ad D autem sex toni intendantur, sitque D 531441. His ita
to D moreover six tones let-be-stretched and-let-be D 531441 with-these thus
dispositis et constitutis, manifestum est inter C atque D
arranged and established manifest it-is (that)-between C and D
comma constitui, eorumque differentiam esse 7153; id autem
the-comma to-be-established and-their difference to-be 7153 that moreover
sit K, remittantur igitur duo toni ab eo quod est B, ad
let-be K let-be-slackened therefore two tones from that which is B to

id quod est E, et sit E 373248. Rursus ab eo quod est E
that which is E and let-be E 373248 again from that which is E
intendo diatessaron ad id quod est F 497664. Quoniam igitur
I-stretch-up a-diatessaron to that which is F 497664 since therefore
inter E atque B duo sunt toni, inter E atque F diatessaron,
between E and B two are tones between E and F a-diatessaron
inter B igitur atque F minus semitonium reperitur.
between B therefore and F the-lesser semitone is-found
Sublatis enim a diatessaron consonantia duobus tonis,
(with)-taken-away for from the-diatessaron consonance two tones
fit reliquum semitonium minus, quod in primis numeris
there-becomes the-remaining semitone lesser which in the-first numbers
constare praedixi 256 et 243. Quos eosdem numeros si
to-stand I-said-before 256 and 243 which same numbers if
millies noningenties quadragies quaterque multiplices B
a-thousand-times nine-hundred-times forty-times and-four-times you-multiply the-B
atque F, numeros explicabis. Quos necessarie est eamdem
and F numbers you-will-unfold (that)-these necessarily it-is the-same
proportionem superius dictis numeris continere, qui uno atque
proportion with-the-above said numbers to-contain which by-one and
eodem numero, id est 1944 pariter multiplicati crevere. Item
the-same number that is 1944 equally multiplied have-grown likewise
ab eo quod est F intendo diatessaron, scilicet ad G, et sit G
from that which is F I-stretch-up a-diatessaron namely to G and let-be G
663552. Rursus ab eodem G remitto ad P duos tonos, et fit
663552 again from the-same G I-slacken to P two tones and becomes
P 524288. Quod P necessarie est, ut eumdem sonum quem C
P 524288 which P necessarily it-is that the-same sound as the-C
numerus exhibeat. Ad aequalitatem namque ejus tali ratione
number may-show-forth to equality for-indeed of-it by-such reckoning
progressus est. Etenim ea quae est A C diapason consonantia,
advanced it-has for-indeed that which is A C the-diapason consonance

quae constat quinque tonis ac duobus semitoniis minoribus, ab
which consists of-five tones and two semitones lesser by
6 tonis commate superatur. Ab eodem igitur A termino numerus
6 tones by-a-comma is-surpassed from the-same therefore A term the-number
P, quinque tonis ac semitoniis duobus recessit hoc modo.
P by-five tones and semitones two has-receded in-this manner
Ab eo quod est A usque ad id quod est B quinque nimirum
from that which is A up to that which is B five assuredly
colliguntur toni; ab eo autem quod est B usque ad id quod
are-gathered tones from that moreover which is B up to that which
est F minus esse semitonium pernotatur, F vero atque P,
is F the-lesser to-be semitone is-noted F but and P
idem rursus semitonium minus includunt. A igitur usque ad
the-same again semitone lesser enclose A therefore up to
P quinque tonos ac duo semitonia minora produxit. Jure
P five tones and two semitones lesser has-brought-forth by-right
igitur P atque C eisdem numeris conscribuntur. Sed quoniam
therefore P and C with-the-same numbers are-written but since
inter F atque C semitonium minus est, videamus haec
between F and C the-semitone lesser is let-us-see these-(things)
quae sit eorum differentia, ut eam commati comparemus.
what may-be their difference that it to-the-comma we-may-compare
Est autem eorum differentia 26624, et sit haec M. Igitur K
is moreover their difference 26624 and let-be this M therefore K
commatis differentia est, M autem semitonii minoris. Si igitur
the-comma's difference is M moreover the-semitone's lesser if therefore
K numerum tertio auxerimus, fiet numerus
the-K number threefold we-shall-have-increased there-will-become the-number
21459. Et sit hic L. Si vero quater eumdem numerum K
21459 and let-be this L if but four-times the-same number K
multiplicare voluerimus, fient 28612, et sit hoc N.
to-multiply we-shall-have-wished they-will-become 28612 and let-be this N

Igitur M major quidem est ab L; idem autem M minor
therefore M greater indeed is than L the-same moreover M lesser
est ab N. Sed N quater aucto commate succrevit; L
is than N but N four-times increased by-the-comma has-grown L
autem tertio, M vero semitonii minoris obtinet differentiam.
moreover threefold M but the-semitone's lesser holds difference
Jure igitur dictum est minus semitonium minus
by-right therefore said it-has-been (that)-the-lesser semitone less
quidem esse quam quatuor commata, majus vero quam tria.
indeed to-be than four commas greater but than three

Apotome majorem esse quam 4 commata, minorem quam 5. Tonum majorem quam 8, minorem quam 9

III.15 · THE APOTOME AND THE TONE IN COMMAS

The same reckoning bounds the apotome. With B at 472392, C at 497664, and D at 531441, the apotome stands between C and D with difference 33777, while the comma's difference is 7153; five commas make 35765, which is more, and four make 28612, which is less. The apotome is therefore greater than four commas and less than five — and since apotome and lesser semitone together make a tone, the tone is greater than eight commas and less than nine.

Eadem hac ratione et semitonium majus quod apotomen
by-the-same this reckoning also the-semitone greater which the-apotome
dici supra retulimus, quot commatum sit possumus
to-be-called above we-related of-how-many commas it-may-be we-are-able
invenire hoc modo: Sit A 262144. Quinque vero ab eo
to-find in-this manner let-be A 262144 by-five but from it
distans tonis sit B 472392. Sex vero distans tonis ab eo
distant tones let-be B 472392 by-six but distant tones from that
quod est A, sit D, scilicet 531441, inter B igitur atque D
which is A let-be D namely 531441 between B therefore and D

tonus est; B vero ab eo quod est C distat per semitonium
a-tone is B but from that which is C stands-apart by the-semitone
minus, et sit C 497664. Relinquitur ergo inter C atque
lesser and let-be C 497664 there-remains thus between C and
D apotome proportio. Nam cum sit tonus B D, ex eo si
D the-apotome proportion for since is a-tone B D from it if
auferas B C semitonium minus, C D relinquitur majus,
you-take-away B C the-semitone lesser C D there-remains the-greater
quod apotomen esse supra retulimus, inter D igitur atque C
which the-apotome to-be above we-related between D therefore and C
est differentia 33777; haec autem sit E, sed erat, commatis
is the-difference 33777 this moreover let-be E but was the-comma's
differentia 7153, haec sit F. Si igitur F, id quod est comma,
difference 7153 this let-be F if therefore F that which is the-comma
quinquies multiplicem, fient mihi 35765, et sit hoc G.
five-times I-multiply they-will-become for-me 35765 and let-be this G
Si vero idem F quater multiplicem, fit H numerus
if but the-same F four-times I-multiply becomes the-H number
qui est 28612. G igitur ab eo quod est E majus est, H
which is 28612 G therefore than that which is E greater is H
minus. Sed G quinquies auctum est comma. H vero quater,
less but G five-times increased is a-comma H but four-times
E autem apotomes differentia est. Jure igitur dictum est
E moreover the-apotome's difference is by-right therefore said it-has-been
apotomen minorem quidem esse quam quinque commata,
(that)-the-apotome lesser indeed to-be than five commas
majorem vero quam quatuor; ex hoc igitur comprobatur
greater but than four from this therefore is-proved
tonum majorem quidem esse quam sunt octo commata,
(that)-the-tone greater indeed to-be than are eight commas
minorem vero quam novem. Nam si minus semitonium,
lesser but than nine for if the-lesser semitone

majus quidem est quam tria commata, minus vero quam
greater indeed is than three commas lesser but than
quatuor, apotome autem major quidem est quam quatuor
four the-apotome moreover greater indeed is than four
commata, minor vero quam quinque. Junctum semitonium
commas lesser but than five joined the-semitone
minus semitonio majori, quod est apotome, erit omne majus
lesser to-the-semitone greater which is the-apotome will-be the-whole greater
quidem 8 commatibus, minus vero quam 9; sed apotome
indeed than-8 commas lesser but than 9 but the-apotome
atque semitonium minus unum efficiunt tonum. Tonus igitur
and the-semitone lesser one make tone the-tone therefore
major quidem est 8 commatibus, minor vero 9.
greater indeed is than-8 commas lesser but than-9

Superius dictorum per numeros demonstratio

III.16 · THE DEMONSTRATION IN NUMBERS

Boethius then demonstrates it on the tone directly: the tone's own difference between 472392 and 531441 is 59049, and nine commas make 64377 while eight make 57224, so the tone is less than nine commas and greater than eight. Subjoined numbers likewise show the greater semitone exceeding the lesser by a comma and the tone exceeding two lesser semitones by a comma — 7153 in each case. The division of the monochord remains, and he defers it to the following book.

Sed quanquam per hanc rationem demonstratum est
but although through this reckoning demonstrated it-has-been
quemadmodum tonus commatibus comparetur, non est tamen
how the-tone to-the-commas is-compared not it-is however
quasi segnibus delassandum, quo minus per se hanc
as-if by-the-sluggish to-be-wearied whereby the-less through itself this

contra commata comparationem retinere tonus ipse monstretur.
against the-commas comparison to-retain the-tone itself may-be-shown
Sit igitur A quidem 262144, B autem quinque ab eo distans
let-be therefore A indeed 262144 B moreover by-five from it distant
tonis 472392. C vero diapason ad id quod est A continens
tones 472392 C but the-diapason to that which is A containing
symphoniam, scilicet in numeris 524288; D autem ab eo
symphony namely in numbers 524288 D moreover from that
quod est A, sex totos differens tonos 531441. D igitur
which is A by-six whole differing tones 531441 D therefore
ab eo quod est C distat commate, sex toni scilicet ab
from that which is C stands-apart by-a-comma six tones namely from
diapason consonantia. Id autem sit E 7153. D autem ab eo
the-diapason consonance that moreover let-be E 7153 D moreover from that
quod est B tono integerrimo distat, sex toni scilicet a
which is B by-a-tone most-entire stands-apart six tones namely from
quinque tonis, id autem sit F 59049. Si igitur E novies
five tones that moreover let-be F 59049 if therefore E nine-times
auxero, fiet mihi H 64377. Sin vero octies,
I-shall-have-increased there-will-become for-me H 64377 but-if indeed eight-times
fient 57224, id sit G. Sed H quidem F numero
they-will-become 57224 that let-be G but H indeed to-the-F number
comparatus superat, G vero superatur, et est F toni
compared surpasses G but is-surpassed and is F the-tone's
differentia. H autem novies multiplicatum comma, G vero
difference H moreover nine-times the-multiplied comma G but
octies. Demonstratus est igitur tonus minor quidem
eight-times demonstrated has-been therefore (that)-the-tone lesser indeed
novem esse commatibus, eisdem vero octo commatibus major.
than-nine to-be commas than-the-same but eight commas greater

Ita his praemissis licet majus semitonium minore
so with-these premised although the-greater semitone from-the-lesser
semitonio commate distare monstratum sit, tamen
semitone by-a-comma to-stand-apart shown it-may-have-been yet
idem quoque per se et per subjectos numeros tali
the-same too through itself and through subjoined numbers by-such
ratione probabitur. Sit A numerus 497664. Ab eo vero
reckoning will-be-proved let-be the-A number 497664 from it but
semitonium minus distans sit B numerus, qui jam supra
by-the-semitone lesser distant let-be the-B number which already above
quoque descriptus est, 524288. Apotome vero distat ab
too described has-been 524288 by-the-apotome but stands-apart from
eo quod est A is numerus, qui colligitur unitatibus 531441,
that which is A that number which is-gathered in-units 531441
et sit hoc C. Quoniam igitur A B minus semitonium, A
and let-be this C since therefore A B the-lesser semitone A
C majus, differentia ejus quod est B ab eo quod est C
C the-greater the-difference of-that which is B from that which is C
perquirenda est. Ea est 7153, id sit D. Sed hic numerus
to-be-sought-out is it is 7153 that let-be D but this number
dudum comma monstrabat; inter majus igitur semitonium
just-now the-comma was-showing between the-greater therefore semitone
ac minus comma differentiam facit. Rursus demonstrandum
and the-lesser the-comma difference makes again to-be-demonstrated
propono tonum duobus semitoniis minoribus solo
I-propose (that)-the-tone than-two semitones lesser by-a-sole
commate esse majorem. Sit A numerus 472392. Ab hoc
comma to-be greater let-be the-A number 472392 from this
intendatur tonus 531441, et sit hoc D. Ab eo vero quod est
let-be-stretched a-tone 531441 and let-be this D from that but which is
A intendatur semitonium minus, quod est B, ac sit B 497664.
A let-be-stretched the-semitone lesser which is B and let-be B 497664

Item ab eo quod est B semitonium aliud intendatur minus,
likewise from that which is B a-semitone another let-be-stretched lesser
quod est C, et sit C 524288. Quoniam igitur A D tonus
which is C and let-be C 524288 since therefore A D a-tone
est, A C vero duo continent minora semitonia: videamus quae
is A C but two contain lesser semitones let-us-see what
sit differentia inter C atque D numeros constituta. Est
may-be the-difference between the-C and D numbers established it-is
autem E, scilicet unitatibus 7153. Demonstratum est igitur
moreover E namely in-units 7153 demonstrated it-has-been therefore
tonum duobus semitoniis minoribus commate esse majorem.
(that)-the-tone than-two semitones lesser by-a-comma to-be greater
Sed quoniam omnia jam quae probanda promisimus propria
but since all-(things) now which to-be-proved we-promised by-their-proper
ratione monstrata sunt, nunc quod superest musicae
reckoning shown have-been now what remains for-the-musical
institutioni, regularis monochordi facienda est partitio.
instruction of-the-regular monochord to-be-made is the-partition
Quam rem quoniam longior tractatus extendit, in posterioris
which matter since a-longer treatment stretches-out into the-later
commentarii disputationem censuimus transferendam.
commentary's discussion we-have-judged to-be-transferred

Vocum differentias in quantitate consistere

IV.1 · PITCH DIFFERENCES ARE QUANTITIES

Boethius restates the ground of pitch from the earlier books: without motion there is no beat, and without a beat no sound; slow motion makes a low sound and swift motion a high one. Every sound is bound together from its parts by proportion, and proportions are considered chiefly in numbers — multiple, superparticular, or superpartient. Notes struck together that blend into one sweet sound are consonant; those that do not blend are dissonant.

Etsi omnia quae demonstranda erant superioris libri
although all-things which to-be-demonstrated were of-the-former book
tractatione digessimus, non poenitet tamen rursus eadem
by-the-treatment we-have-set-forth not it-repents(-us) yet again the-same-things
breviter memoria recolligenda praestare cum quadam diversitate
briefly by-memory to-be-regathered to-present with a-certain diversity
tractatus, ut his rursus ad memoriam redeuntibus ad regulae
of-treatment that these again to memory returning to of-the-rule
divisionem, quo tota tendit intentio, veniamus. Si foret
the-division whither the-whole tends (our-)intent we-may-come if there-were
rerum omnium quies, nullus auditum sonus feriret. Id autem
of-things all rest no the-hearing sound would-strike this moreover
fieret, quoniam cessantibus cunctis motibus nullae inter
would-come-about since ceasing (with-)all motions no among
se res pulsum cierent: ut igitur sit vox, pulsu est
themselves things a-beat would-stir that therefore there-be a-voice of-a-beat there-is
opus. Sed sit pulsus, motus necesse est antecedat.
need but (that)-there-be a-beat (that)-motion necessary it-is should-precede
Ut ergo sit vox, motum esse necesse est. Sed omnis
that then there-be a-voice motion to-be necessary it-is but every

motus habet in se tum velocitatem, tum etiam tarditatem. Si
motion has in itself now swiftness now also slowness if
igitur sit tardus in pellendo motus, gravior redditur sonus.
therefore be slow in striking the-motion lower is-rendered the-sound
Nam ut tarditas proxima stationis est, ita gravitas contigua
for as slowness nearest of-standstill is so lowness (is-)bordering
taciturnitati. Velox vero motus acutam voculam praestat.
on-silence swift but motion a-high little-voice yields
Praeterea, quae gravis est, intensione crescit ad
moreover (that-voice)-which low is by-tightening grows toward
medium. Quae vero acuta, remissione decrescit ad medium.
the-middle (that)-which but high by-slackening grows-less toward the-middle
Unde fit ut omnis sonus quasi ex quibusdam
whence it-comes that every sound as-it-were from certain
partibus compositus esse videatur. Omnis autem partium
parts composed to-be may-seem every moreover of-parts
conjunctio quadam proportione committitur. Sonorum igitur
joining by-a-certain proportion is-bound-together of-sounds therefore
conjunctio proportionibus constituta est. Proportiones autem
the-joining by-proportions established is proportions moreover
principaliter in numeris considerantur, proportio vero simplex
chiefly in numbers are-considered a-proportion but simple
numerorum vel in multiplicibus, vel in superparticularibus,
of-numbers either in multiples or in superparticulars
vel in superpartientibus invenitur. Secundum vero multiplices
or in superpartients is-found according-to but multiple
proportiones, vel superparticulares, consonae vel dissonae
proportions or superparticular consonant or dissonant
voces exaudiuntur. Consonae quidem sunt quae simul
notes are-heard consonant indeed are (those-)which together
pulsae suavem permixtum inter se conjungunt sonum;
struck a-sweet blended between themselves join sound

dissonae vero quae simul pulsae non reddunt suavem
dissonant but (those-)which together struck not render a-sweet
neque permixtum sonum. His igitur ita praedictis, de
nor blended sound these therefore thus said-before about
proportionibus pauca dicamus.
proportions a-few-things let-us-say

Diversae de intervallis speculationes

IV.2 · THEOREMS ON INTERVALS

Boethius proves a chain of theorems on intervals, each in letters and then in numbers. A multiple doubled stays multiple, and an interval whose double is multiple is itself multiple; no mean falls proportionally within a superparticular; a non-multiple doubled is neither multiple nor superparticular. Sesquialter and sesquitertian make the double, double and sesquialter the triple, a sesquialter less a sesquitertian leaves the sesquioctave, and six sesquioctaves, 262144 to 531441, exceed one double.

Si intervallum multiplex binario multiplicetur, id quod fit
if an-interval multiple by-the-two be-multiplied that which arises
ex hac multiplicatione intervallum, multiplex est. Sit
from this multiplication an-interval multiple is let-there-be
multiplex intervallum B C, et B multiplex ejus quod est C,
a-multiple interval B C and B a-multiple of-that which is C
et fiat ut est C ad B, ita B ad D. Quoniam igitur B
and let-it-become as is C to B so B to D since therefore B
multiplex est, ejus quod est C metitur C terminus, id quod
a-multiple is of-that which is C measures C the-term that which
est B, vel bis, vel tertio, vel deinceps, et est ut C ad B, ita
is B either twice or thrice or so-forth and it-is as C to B so
B ad D. Metitur igitur B terminus id quod est D. Quocirca
B to D measures therefore B the-term that which is D wherefore

quia C terminus id quod est B metitur, metietur etiam
because C the-term that which is B measures it-will-measure also
D. Multiplex est igitur D ejus quod est C, et est C D
D a-multiple is therefore D of-that which is C and is C D
intervallum, effectum ex composito bis copulatoque sibimet
the-interval made from compounded twice and-coupled to-itself
et per binarium multiplicato B C intervallo. In numeris
and by the-two multiplied B C interval in numbers
quoque idem probatur. Sit enim B ad C duplum, ut
too the-same is-proved let-be for B to C double as
binarius ad unitatem, et fiat ut C ad B, ita B ad D.
the-two to the-unit and let-it-become as C to B so B to D
Erit igitur D quaternarius. Multiplex est autem B ad C,
will-be therefore D the-four a-multiple is moreover B to C
id est binarius ad unitatem. Multiplex est D ad B, id est
that is the-two to the-unit a-multiple is D to B that is
quaternarius ad binarium. Multiplex est igitur D quaternarius
the-four to the-two a-multiple is therefore D the-four
ad C unitatem. Est autem quadruplus quaternarius unitatis,
to C the-unit (it-)is moreover quadruple the-four of-the-unit
et binario multiplicata medietas quod est intervallum B
and by-the-two multiplied the-mean which is the-interval B
C. Si intervallum binario multiplicatum multiplex effecerit,
C if an-interval by-the-two multiplied a-multiple shall-have-made
intervallum ipsum quoque multiplex erit. Sit intervallum
the-interval itself too multiple will-be let-there-be an-interval
B C, et fiat ut C ad B, ita B ad D, et D sit ad C
B C and let-it-become as C to B so B to D and D let-be to C
multiplex. Dico quia B ejus quod est C multiplex est. Quoniam
a-multiple I-say that B of-that which is C a-multiple is since
enim D ejus quod est C multiplex est, metietur C id quod
for D of-that which is C a-multiple is will-measure C that which

est D, metietur et B. Ostensum est vero quoniam si
is D it-will-measure also B shown it-has-been but that if
sint termini proportionaliter constituti, cum primus fuerit
there-be terms proportionally established when the-first shall-have-been
ultimo comparatus, si primus ultimum fuerit mensus,
to-the-last compared if the-first the-last shall-have measured
metietur et medium. C igitur metietur id quod est B.
it-will-measure also the-middle C therefore will-measure that which is B
Multiplex est igitur B ejus quod est C. Id rursus ex
a-multiple is therefore B of-that which is C this again from
numeris, sit C unitas, D vero ex duplicata proportione
numbers let-be C the-unit D but from the-doubled proportion
B C quaternarius, et est multiplex ejus quod est C. Est
B C the-four and is a-multiple of-that which is C (it-)is
enim quadruplus. Quoniam igitur quadruplus ex duplicata
for quadruple since therefore the-quadruple from the-doubled
B C proportione generatur, B C proportio dimidium
B C proportion is-begotten the-B C proportion the-half
ejus erit, igitur B C proportio dupla est. Sed duplum
of-it will-be therefore the-B C proportion double is but the-double
multiplex est: erit igitur B C proportio multiplex.
multiple is will-be therefore the-B C proportion multiple
Superparticularis intervalli medius numerus, neque unus,
of-a-superparticular interval a-middle number neither one
neque plures proportionaliter intervenient. Sit enim B C
nor more proportionally will-come-between let-be for the-B C
proportio superparticularis, et in eadem proportione minimi
proportion superparticular and in the-same proportion the-least
sint D F et G. Quoniam D F et G minimi sunt in eadem
let-be D F and G since D F and G the-least are in the-same
proportione, sunt ejusdem proportionis primi. Quocirca eos
proportion they-are of-the-same proportion prime wherefore them

unitas metietur. Auferatur igitur G ab D F et relinquatur
the-unit will-measure let-be-taken-away therefore G from D F and let-be-left
D, hic est igitur utrorumque mensura communis: haec igitur
D this is therefore of-both the-measure common this therefore
erit unitas. Quocirca nullus inter F D atque G incidet
will-be the-unit wherefore no between F D and G will-fall
numerus, qui sit ab F D quidem minor, major vero ab G.
number which be than F D indeed less greater but than G
Sola enim interest unitas, quanti vero in superparticularibus
alone for lies-between the-unit how-many but in superparticular
proportionibus proportionaliter inter ejusdem proportionis
proportions proportionally between of-the-same proportion
minimos intercident, tot etiam inter caeteros ejusdem
the-least will-fall-between so-many also between the-rest of-the-same
proportionis incident. Sed nullus inter F D atque G minimos
proportion will-fall but none between F D and G the-least
ejusdem proportionis intervenire potest. Nullus igitur inter B
of-the-same proportion to-come-between can none therefore between B
atque C proportionaliter cadet, et in numeris sit quaelibet
and C proportionally will-fall and in numbers let-there-be whatsoever
superparticularis proportio, ut sesquialtera; hi vero sunt 10
superparticular proportion as the-sesquialter these but are 10
et 15, in eadem vero proportione minimi 2 et 3, aufero
and 15 in the-same but proportion the-least 2 and 3 I-take-away
de 3 binarium; fit reliqua unitas, eadem utrosque metitur.
from 3 the-two becomes remaining the-unit the-same both measures
Nullus erit igitur inter binarium ternariumque numerus,
none will-be therefore between the-two and-the-three a-number
qui sit binario major, minor vero ternario. Alioquin
which be than-the-two greater less but than-the-three otherwise
unitas dividetur, quod est inconveniens. Quare ne inter 10
the-unit will-be-divided which is unfitting wherefore not between 10

quidem atque 15 quisquam invenietur numerus qui talem
even and 15 any will-be-found number which such
ad 10 obtineat proportionem qualem ad eum tenent 15. Si
to 10 may-hold a-proportion as to it hold 15 if
intervallum non multiplex binario multiplicetur, id quod fit
an-interval not multiple by-the-two be-multiplied that which arises
ex hac multiplicatione nec multiplex est nec superparticulare.
from this multiplication neither multiple is nor superparticular
Sit enim intervallum non multiplex B C, et fiat
let-there-be for an-interval not multiple B C and let-it-become
ut C ad B, sic B ad D. Dico quoniam D ejus quod est
as C to B so B to D I-say that D of-that which is
C, neque multiplex est, neque superparticularis. Sit enim,
C neither a-multiple is nor superparticular let-it-be for
si fieri potest, primum D ejus quod est C multiplex, et
if to-be-done it-can first D of-that which is C a-multiple and
quoniam cognitum est, si intervallum binario multiplicatum
since known it-is if an-interval by-the-two multiplied
sit, et multiplex intervallum creatum, id quod multiplicatum
be and a-multiple interval created that which multiplied
est bis intervallum esse multiplex. Erit igitur B C multiplex,
is twice the-interval to-be multiple will-be therefore B C multiple
sed non est positum. Non igitur erit D ejus quod est
but not it-is laid-down not therefore will-be D of-that which is
C multiplex, nec vero superparticulare. Nam superparticularis
C a-multiple nor indeed superparticular for of-a-superparticular
proportionis medius proportionaliter terminus nullus intervenit.
of-the-proportion a-middle proportionally term no comes-between
Inter D vero et C est proportionaliter terminus constitutus,
between D but and C is proportionally a-term established
id est B. Nam ut est C ad B, ita B ad D; impossibile igitur
that is B for as is C to B so B to D impossible therefore

erit D ejus quod est C, vel multiplicem esse, vel
it-will-be (that-)D of-that which is C either a-multiple to-be or
superparticularem, quod oportebat ostendere. Et in numeris
superparticular which it-behooved to-show and in numbers
sit non multiplex intervallum 6 ad 4, fiatque ut
let-there-be a-not multiple interval 6 to 4 and-let-it-become as
sunt 4 ad 6, ita sex ad alium quemlibet numerum.
are 4 to 6 so six to another whatever number
Hic erit igitur novenarius, qui quaternarii neque multiplex
this will-be therefore the-nine which of-the-four neither a-multiple
neque superparticularis est. Si intervallum binario multiplicetur,
nor superparticular is if an-interval by-the-two be-multiplied
atque id quod ex ea multiplicatione creabitur multiplex
and that which from that multiplication will-be-created multiple
non fit, ipsum quoque non erit multiplex. Sit enim
not becomes itself too not will-be multiple let-there-be for
intervallum B C, fiatque ut C ad B, ita B ad D, et
an-interval B C and-let-it-become as C to B so B to D and
non fit D ejus quod est C multiplex. Dico quoniam nec
not becomes D of-that which is C a-multiple I-say that neither
B ejus quod est C erit multiplex. Si enim est et D ejus
B of-that which is C will-be a-multiple if for it-is also D of-that
quod est C, est multiplex. At non est, non erit igitur B
which is C is a-multiple but not it-is not will-be therefore B
ejus quod est C multiplex. Duplex intervallum ex duobus
of-that which is C a-multiple the-double interval from two
maximis superparticularibus, conjungitur sesquialtero et
the-greatest superparticulars is-joined-together the-sesquialter and
sesquitertio. Sit enim A quidem ejus quod est B sesquialter,
the-sesquitertian let-be for A indeed of-that which is B sesquialter
B vero ejus quod est C sesquitertius. Dico quoniam A ejus
B but of-that which is C sesquitertian I-say that A of-that

quod est C duplex est. Quoniam igitur sesquialter est A ejus
which is C double is since therefore sesquialter is A of-that
quod est B, igitur A habet in se totum B ejusque dimidium.
which is B therefore A has in itself the-whole B and-of-it the-half
Duo igitur A aequi sunt tribus B. Rursus quoniam B ejus quod
two therefore A equal are to-three B again since B of-that which
est C sesquitertius est, B igitur habet C, et ejus tertiam partem.
is C sesquitertian is B therefore has C and of-it the-third part
Tres igitur B aequi sunt ad quatuor C. Tres autem B aequi
three therefore B equal are to four C three moreover B equal
erant duobus A. Duo igitur A aequi sunt ad quatuor C; unus
were to-two A two therefore A equal are to four C one
igitur A aequus est duobus C. Duplex erit igitur A ejus qui
therefore A equal is to-two C double will-be therefore A of-that which
est C, et in numeris. Sit enim sesquialter 12 ad 8, sesquitertius
is C and in numbers let-be for the-sesquialter 12 to 8 the-sesquitertian
vero 8 ad 6: ergo 12 ad 6 duplices sunt. Ex duplici intervallo
but 8 to 6 therefore 12 to 6 double are from the-double interval
atque sesquialtero triplex nascitur intervallum. Sit enim A
and the-sesquialter triple is-born an-interval let-be for A
ejus quod est B duplex, B autem ejus quod est C sesquialter.
of-that which is B double B moreover of-that which is C sesquialter
Dico quoniam A ejus quod est C triplex est. Nam quoniam
I-say that A of-that which is C triple is for since
A ejus quod est B duplex est, A igitur aequus est duobus B.
A of-that which is B double is A therefore equal is to-two B
Rursus quoniam B ejus quod est C sesquialter est, B igitur
again since B of-that which is C sesquialter is B therefore
habet in se totum C et ejus dimidiam partem. Duo igitur
has in itself the-whole C and of-it the-half part two therefore
B aequi sunt tribus C. Sed duo B aequi erant uni A, et
B equal are to-three C but two B equal were to-one A and

unus igitur A aequus est tribus C; igitur A uno C triplex
one therefore A equal is to-three C therefore A of-one C triple
est. Et in numeris sit duplex quidem senarius ternario,
is and in numbers let-be double indeed the-six of-the-three
sesquialter vero ternarius, binario. Senarius igitur triplex est
sesquialter but the-three of-the-two the-six therefore triple is
binario. Si sesquialtero intervallo sesquitertium demptum
of-the-two if from-the-sesquialter interval the-sesquitertian taken-away
fuerit intervallum, erit quod relinquitur sesquioctavum.
shall-have-been an-interval will-be (that-)which is-left sesquioctave
Sit enim A quidem ejus quod est B sesquialter, at vero
let-be for A indeed of-that which is B sesquialter but indeed
C ejus quod est B sesquitertius: dico quoniam A ejus
C of-that which is B sesquitertian I-say that A of-that
quod est C sesquioctavus est. Quoniam enim A ejus quod
which is C sesquioctave is since for A of-that which
est B sesquialter est, A igitur habet in se B et ejus
is B sesquialter is A therefore has in itself B and of-it
dimidiam partem. Octo igitur A aequi sunt ad duodecim
the-half part eight therefore A equal are to twelve
B. Rursus quoniam C ejus quod est B sesquitertius est,
B again since C of-that which is B sesquitertian is
C igitur habet in se B et tertiam ejus partem.
C therefore has in itself B and the-third of-it part
Novem igitur C aequi sunt ad duodecim B. Duodecim B aequi
nine therefore C equal are to twelve B twelve B equal
erant ad octo A; et octo igitur A aequi sunt ad novem C. Igitur
were to eight A and eight therefore A equal are to nine C therefore
A aequus est ei quod est C, et octavae ejus parti. A igitur
A equal is to-that which is C and to-the-eighth of-it part A therefore
ejus quod est C sesquioctavus est. Et in numeris sesquialterum
of-that which is C sesquioctave is and in numbers the-sesquialter

quidem intervallum sit novenarius ad senarium, sesquitertium
indeed interval let-be the-nine to the-six the-sesquitertian
vero octonarius ad senarium. Novem igitur ad octo sesquioctava
but the-eight to the-six nine therefore to eight the-sesquioctave
proportio est. Sex proportiones sesquioctavae majores sunt
proportion is six proportions sesquioctave greater are
uno duplici intervallo. Sit enim quidem numerus A,
than-one double interval let-there-be for indeed a-number A
hujus autem sit sesquioctavus B, hujus sesquioctavus C,
of-this moreover let-be the-sesquioctave B of-this the-sesquioctave C
et hujus sesquioctavus D, et hujus sesquioctavus F, ejusque
and of-this the-sesquioctave D and of-this the-sesquioctave F and-of-it
sesquioctavus G, atque hujus sesquioctavus K. Id autem
the-sesquioctave G and of-this the-sesquioctave K this moreover
fiat secundum descriptum in Arithmetica modum, et sint
let-be-done according-to the-described in the-Arithmetic manner and let-be
numeri A B C D F G K, et sit A 262144, hujus autem
the-numbers A B C D F G K and let-be A 262144 of-this moreover
sesquioctavus qui est B 294912, hujus autem sesquioctavus qui
the-sesquioctave which is B 294912 of-this moreover the-sesquioctave which
est C 331776, hujus autem sesquioctavus qui est D 373248,
is C 331776 of-this moreover the-sesquioctave which is D 373248
hujus autem sesquioctavus qui est F 419904, hujus autem
of-this moreover the-sesquioctave which is F 419904 of-this moreover
sesquioctavus qui est G 472392, hujus autem sesquioctavus
the-sesquioctave which is G 472392 of-this moreover the-sesquioctave
qui est K 531441. Et sunt 531441 quod est K, plus
which is K 531441 and are 531441 (that-)which is K more
quam duplices ad 262144 quod est A. Sex igitur sesquioctava
than double to 262144 which is A six therefore sesquioctave
proportiones ampliores sunt uno duplici intervallo.
proportions ampler are than-one double interval

Musicarum per Graecas ac Latinas litteras notarum nuncupatio

IV.3 · THE NOTATION OF THE NOTES

Before dividing the string, Boethius sets out the notation the old musicians devised so that each string's name can be read from a little sign instead of being written in full — a melody could even be recorded this way above a verse. He gives the Lydian mode's signs through the three genera in two rows, the upper for the words and the lower for the percussion, describing each sign as a Greek or Latin letter turned round, lying down, incomplete, or carrying a stroke.

Restat quoniam sumus nervum secundum praedictas
it-remains since we-are the-string according-to the-aforesaid
consonantias per regulam divisuri, quoniamque necessarios
consonances by the-rule about-to-divide and-since the-necessary
sonos tribus generibus cantilenae exhibebit ista partitio,
sounds for-the-three genera of-song will-exhibit that partition
musicas interim notas apponere, ut cum divisam lineam
the-musical meanwhile signs to-set-down that when the-divided line
iisdem notulis signaverimus, quod unicuique sit nomen,
with-the-same little-signs we-shall-have-marked what to-each be the-name
facillime possit agnosci. Veteres enim musici propter
most-easily it-can to-be-recognized the-old for musicians for
compendium scriptionis, ne integra semper nomina necesse
the-shortening of-writing lest complete always the-names necessary
esset apponere, excogitavere notulas quasdam quibus
it-should-be to-set-down devised little-signs certain by-which
nervorum vocabula notarentur, easque per genera modosque
of-the-strings the-names might-be-marked and-them by genera and-modes
divisere, simul etiam hac brevitate captantes, ut si quando
they-divided at-once also at-this brevity aiming that if ever
melos aliquod musicus voluisset ascribere, super versum
a-melody some a-musician should-have-wished to-write-above over a-verse

rhythmica metri compositione distentum, has sonorum notulas
by-the-rhythmic of-meter composition stretched-out these of-the-sounds little-signs
ascriberet, ita miro modo reperientes, ut non tantum
he-might-write-above so in-wondrous manner discovering that not only
carminum verba, quae litteris explicarentur, sed melos
of-songs the-words which by-letters were-unfolded but the-melody
quoque ipsum, quod his notulis signaretur, in memoriam
too itself which by-these little-signs was-marked into memory
posteritatemque duraret. Sed ex his omnibus modis unum
and-posterity might-endure but from these all modes one
interim Lydium ejusque notulas per tria genera disponamus,
meanwhile the-Lydian and-its little-signs through three genera let-us-set-out
in reliquis modis idem facere in tempus aliud differentes.
in the-remaining modes the-same to-do to time another deferring
Sane si quando dispositionem notarum Graecarum litterarum
truly if ever the-arrangement of-the-signs of-Greek letters
nuncupatione descripsero, lector nulla novitate turbetur.
by-the-naming I-shall-have-described let-the-reader by-no novelty be-troubled
Graecis enim litteris nunc in quamlibet partem imminutis,
with-Greek for letters now into whatever part lessened
nunc etiam inflexis, tota haec notarum descriptio constituta
now also bent whole this of-the-signs diagram established
est. Nos vero cavemus aliquid ab antiquitatis auctoritate
is we but avoid anything from of-antiquity the-authority
transvertere. Erunt igitur priores ac superiores notulae
to-turn-aside will-be therefore the-former and upper little-signs
dictionis, id est verborum, secundae vero atque inferiores
of-the-diction that is of-the-words the-second but and lower
percussionis. Proslambanomenos, qui acquisitus dici
of-the-percussion proslambanomenos which the-acquired to-be-called
potest, Z non integrum et tau jacens. Hypate hypaton,
can Z not complete and a-tau lying-down hypate hypaton

quae est principalis principalium, L conversum et rectum.
which is the-principal of-principals L turned-round and upright
Parhypate hypaton, id est subprincipalis principalium, B non
parhypate hypaton that is the-subprincipal of-principals B not
integrum, et supinum B. Hypaton enharmonios, quae est
complete and upside-down B hypaton enharmonios which is
principalium enharmonios, V supinum, et Γ conversum, retro
of-principals the-enharmonic V upside-down and Γ turned-round behind
habens virgulam. Hypate chromaticae, quae est principalium
having a-stroke hypate chromatice which is of-principals
chromatica, V supinum habens lineam et conversum
the-chromatic V upside-down having a-line and (a-V)-turned-round
duas habens lineas. Hypaton diatonos, quae est principalium
two having lines hypaton diatonos which is of-principals
extenta, Φ Graecum et digammon. Hypate meson, quae
the-stretched Φ Greek and a-digamma hypate meson which
est principalis mediarum, C et Γ.
is the-principal of-the-middles C and Γ
Parhypate meson, quae est subprincipalis mediarum, P
parhypate meson which is the-subprincipal of-the-middles P
et C supinum. Meson enharmonios, quae est mediarum
and C upside-down meson enharmonios which is of-the-middles
enharmonios, Π Graecum et C conversum. Meson chromaticae,
the-enharmonic Π Greek and C turned-round meson chromatice
quae est mediarum chromatica, Π Graecum habens virgulam,
which is of-the-middles the-chromatic Π Greek having a-stroke
et C conversum per medium habens virgulam. Meson
and C turned-round through the-middle having a-stroke meson
diatonos, quae est mediarum extenta, M Graecum et Π
diatonos which is of-the-middles the-stretched M Greek and Π
Graecum diductum. Mese, quae est media, I et Λ jacens.
Greek drawn-apart mese which is the-middle I and Λ lying-down

Trite synemmenon, quae est tertia conjunctarum, Φ et Λ
trite synemmenon which is the-third of-the-joined Φ and Λ
supinum. Synemmenon enharmonios, quae est conjunctarum
upside-down synemmenon enharmonios which is of-the-joined
enharmonios, H Graecum et Λ jacens conversum per
the-enharmonic H Greek and Λ lying-down turned-round through
medium habens virgulam. Synemmenon chromaticae, quae
the-middle having a-stroke synemmenon chromatice which
est conjunctarum chromatica, H Graecum habens virgulam,
is of-the-joined the-chromatic H Greek having a-stroke
et Λ conversum habens virgulam. Synemmenon diatonos,
and Λ turned-round having a-stroke synemmenon diatonos
quae est conjunctarum extenta, L et N. Nete synemmenon,
which is of-the-joined the-stretched L and N nete synemmenon
quae est ultima conjunctarum, M quadratum supinum et Z.
which is the-last of-the-joined M square upside-down and Z
Paramese, quae est sub media, Z et Π Graecum jacens.
paramese which is below the-middle Z and Π Greek lying-down
Trite diezeugmenon, quae tertia divisarum est, E quadratum
trite diezeugmenon which the-third of-the-divided is E square
et Π Graecum supinum. Diezeugmenon enharmonios, quae
and Π Greek upside-down diezeugmenon enharmonios which
est divisarum enharmonios, Δ et Π Graecum jacens
is of-the-divided the-enharmonic Δ and Π Greek lying-down
conversum. Diezeugmenon chromaticae, quae est divisarum
turned-round diezeugmenon chromatice which is of-the-divided
chromatica, Δ habens virgulam et Π Graecum jacens
the-chromatic Δ having a-stroke and Π Greek lying-down
conversum, habens lineam angularem. Diezeugmenon diatonos,
turned-round having a-line angular diezeugmenon diatonos
quae est divisarum extenta, Π quadratum supinum. Nete
which is of-the-divided the-stretched Π square upside-down nete

diezeugmenon, quae est ultima divisarum, Φ jacens et
diezeugmenon which is the-last of-the-divided Φ lying-down and
N inversum diductum. Trite hyperboleon, quae est tertia
N inverted drawn-apart trite hyperboleon which is the-third
excellentium, semi A dextrum deorsum respiciens et semi A
of-the-excelling half A right downward looking and half A
sinistrum sursum respiciens. Hyperboleon enharmonios, quae
left upward looking hyperboleon enharmonios which
est excellentium enharmonios, T supinum et semi A dextrum
is of-the-excelling the-enharmonic T upside-down and half A right
supinum. Hyperboleon chromaticae, quae est excellentium
upside-down hyperboleon chromatice which is of-the-excelling
chromatica, T supinum habens lineam et semi A dextrum
the-chromatic T upside-down having a-line and half A right
supinum habens retro lineam. Hyperboleon diatonos, quae
upside-down having behind a-line hyperboleon diatonos which
est excellentium extenta, M Graecum habens acutam et Π
is of-the-excelling the-stretched M Greek having an-acute and Π
diductum habens acutam. Nete hyperboleon, quae est ultima
drawn-apart having an-acute nete hyperboleon which is the-last
excellentium, I habens acutam, et Λ jacens habens acutam.
of-the-excelling I having an-acute and Λ lying-down having an-acute

Monochordi regularis partitio in genere diatonico

IV.4 · DIVIDING THE MONOCHORD IN THE DIATONIC GENUS

Boethius begins the regular division of the monochord. A longer string or a greater number sounds lower, so here, unlike in the earlier books, the greater numbers go to the lower notes; a rule equal to the string carries the cuts onto it. Cutting at halves, quarters, thirds, and ninths, he fixes proslambanomenos, mese, nete hyperboleon, lichanos hypaton, hypate hypaton, and hypate meson, then paramese, the netai, and paranete hyperboleon, checking the consonances.

Sed jam tempus est ad regularis monochordi divisionem
but now time it-is to of-the-regular monochord the-division
venire. De qua re illud est praedicendum, quod sive in
to-come about which matter this is to-be-said-beforehand that whether in
mensura nervi, sive in numeris, atque in eorum proportione
the-measure of-a-string or in numbers and in their proportion
statuatur describenda divisio, majus spatium chordae et
be-established to-be-described division the-greater space of-the-string and
major numeri multitudo sonos graviores efficiet. At si
the-greater of-the-number multitude sounds lower will-make but if
fuerit nervi longitudo contractior, et in numeris
shall-have-been of-the-string the-length more-contracted and in numbers
aeque non multa pluralitas, acutiores voces edi necesse
likewise not great the-plurality higher voices to-be-uttered necessary
est. Atque ex hac comparatione, quantum unaquaeque
it-is and from this comparison by-how-much each-(string)
fuerit vel longior vel plurium numerorum, aliaque
shall-have-been either longer or of-more numbers and-another
vel contractior, vel paucioribus signata numeris, tanto
either more-contracted or with-fewer marked numbers by-so-much
vel gravior vel acutior invenitur. Nec lectorem res illa
either lower or higher it-is-found nor the-reader thing that

conturbet, quod intendentes saepe spatia proportionum
let-it-disturb (the-fact)-that (in-)raising often the-spaces of-proportions
numero majore signavimus, remittentes vero minore, cum
with-a-number greater we-marked (in-)slackening but with-a-lesser although
intensio acumen faciat, remissio gravitatem. Illic enim tantum
raising highness makes slackening lowness there for only
proportionum spatia signabamus, nihil de gravitatis aut
of-proportions the-spaces we-were-marking nothing about of-lowness or
acuminis proprietate laborantes, atque ideo et in acumen
of-highness the-property laboring and therefore both into highness
majoribus numeris intendimus, et minoribus in gravitatem
with-greater numbers we-stretched-up and with-lesser into lowness
saepe remisimus. Hic vero ubi chordarum spatia sonosque
often we-slackened here but where of-strings the-spaces and-sounds
metiemur, naturam rerum sequi necesse est, majorique
we-shall-measure the-nature of-things to-follow necessary it-is and-to-the-greater
longitudini chordarum ex qua gravitas existit, ampliores,
length of-strings from which lowness arises ampler-(numbers)
minori vero ex qua vocis acumen nascitur, dare breviores.
to-the-lesser but from which of-the-voice highness is-born to-give shorter-(ones)
Sit chorda intensa A B, huic aequa sit regula quae
let-there-be a-string stretched A B to-this equal let-be a-rule which
propositis partitionibus dividatur, ut, ea regula chorda
with-the-proposed partitions may-be-divided so-that that rule to-the-string
apposita, eaedem divisiones in nervi longitudine signentur
applied the-same divisions on of-the-string the-length may-be-marked
quas ante assignaveramus in regula. Nos vero hanc dividimus
which before we-had-assigned on the-rule we but this-(rule) divide
ita, quasi ipsam chordam et non regulam partiamur. Dividatur
so as-if the-very string and not the-rule we-should-part let-be-divided
igitur A B in quatuor partes, per tria puncta quae sunt C
therefore A B into four parts through three points which are C

D E. Erit igitur tota quidem A B dupla ab his quae sunt
D E will-be therefore the-whole indeed A B double from these which are
D B, A D; sigillatim vero A D, D B duplae sunt ab his
D B A D severally but A D D B double are from these
quae sunt A C, C D, D E, E B. Erit igitur A B quidem
which are A C C D D E E B will-be therefore A B indeed
gravissima, id est proslambanomenos, D B autem mese. Est
lowest that is proslambanomenos D B moreover mese it-is
enim dimidia totius, et sicut A B ab ea quae est B D dupla
for the-half of-the-whole and as A B from that which is B D double
est spatio, ita B D ab ea quae est A B, dupla est acumine.
is in-space so B D from that which is A B double is in-highness
Nam, ut superius dictum est, spatii et acuminis semper
for as above said it-has-been of-space and of-highness always
ordo conversus est. Nam tanto est chorda major in
the-order inverted is for by-so-much is a-string greater in
acumine, quanto fuerit minor in spatio, quocirca
highness by-how-much it-shall-have-been lesser in space wherefore
erit et E B nete hyperboleon quoniam E B ejus quae est
will-be also E B nete hyperboleon since E B of-that which is
D B, dimidia quidem in quantitate, dupla vero in acumine.
D B the-half indeed in quantity double but in highness
Rursus quoniam eadem E B ejus quae est A B quarta pars
again since the-same E B of-that which is A B the-fourth part
est in spatio, quadrupla erit ab eadem in acumine. Erit
is in space quadruple will-be from the-same in highness will-be
igitur (ut dictum est) nete hyperboleon dupla in acumine
therefore as said it-has-been nete hyperboleon double in highness
ab ea quae est mese. Mese autem dupla in acumine
from that which is mese mese moreover double in highness
ab ea quae est proslambanomenos. Nete vero hyperboleon
from that which is proslambanomenos nete but hyperboleon

quadrupla in acumine ab ea quae est proslambanomenos;
quadruple in highness from that which is proslambanomenos
consonabit igitur proslambanomenos ad mesen diapason,
will-sound-consonant therefore proslambanomenos to mese the-diapason
mese ad neten hyperboleon diapason, proslambanomenos ad
mese to nete hyperboleon the-diapason proslambanomenos to
neten hyperboleon bis diapason. Rursus quoniam aequae
nete hyperboleon twice the-diapason again since equal
sunt partes A C, C D, D E, E B.
are the-parts A C C D D E E B
Est autem A B quatuor earumdem partium, C B autem trium,
is moreover A B of-four of-the-same parts C B moreover of-three
A B sesquitertia est ab ea quae est C B. Rursus quia trium
A B sesquitertian is from that which is C B again because of-three
est aequalium partium C B, sed D B duarum, erit igitur C
it-is of-equal parts C B but D B of-two will-be therefore C
B sesquialtera ejus quae est D B. Rursus quoniam C B est
B sesquialter of-that which is D B again since C B is
trium partium aequalium, qualis est una E B, tripla igitur
of-three parts equal such-as is one-(part) E B triple therefore
est C B ab ea quae est E B; erit igitur C B lychanos
is C B from that which is E B will-be therefore C B lichanos
hypaton diatonos, consonabitque proslambanomenos quidem
hypaton diatonos and-will-sound-consonant proslambanomenos indeed
ad lychanon hypaton diatonon diatessaron consonantiam.
to lichanos hypaton diatonos the-diatessaron consonance
Eadem vero lychanos hypaton diatonos consonabit ad
the-same but lichanos hypaton diatonos will-sound-consonant to
mesen consonantiam diapente. Eademque lychanos diatonos
mese the-consonance diapente and-the-same lichanos diatonos
consonabit ad neten hyperboleon, diapason et diapente.
will-sound-consonant to nete hyperboleon the-diapason and diapente

Rursus si de tota A B nonam partem auferam, eam quae
again if from the-whole A B the-ninth part I-take-away that which
est A F, erunt partes octo F B. Erit igitur F B hypate
is A F there-will-be parts eight F B will-be therefore F B hypate
hypaton, ad quam sesquioctavam contineat proportionem A
hypaton to which the-sesquioctave may-hold proportion A
B, id est proslambanomenos, in musica vero tonum.
B that is proslambanomenos in music but a-tone
Quanto spatium majus erit, tanto major gravitas; ubi vero
by-how-much the-space greater will-be so-much-the greater lowness where but
minus, ibi major acuties. Dum ergo A B in quadruplo spatio
less there the-greater highness while therefore A B in quadruple space
excedit E B, erit recte proslambanomenos in quadruplo gravior,
it-exceeds E B will-be rightly proslambanomenos in quadruple lower
et E B acutior in quadruplo, et eadem chorda A B ad F B est
and E B higher in quadruple and the-same string A B to F B is
sesquioctava spatio, quare tono ab F B vincitur; erit ergo
sesquioctave in-space wherefore by-a-tone by F B it-is-surpassed will-be therefore
hypate hypaton. Eadem rursus ad C B sesquitertia est spatio,
hypate hypaton the-same again to C B sesquitertian is in-space
erit ergo C B lychanos hypaton diatonos. Vincetque C B in
will-be therefore C B lichanos hypaton diatonos and-will-surpass C B in
acumine ipsum A B consonantia diatessaron. Insuper idem A
highness the-very A B by-the-consonance diatessaron moreover the-same A
B spatio duplum est ad D B; erit idcirco acumine duplum
B in-space double is to D B will-be on-that-account in-highness double
ipsum D B ad A B. Et sic E B neten hyperboleon constituet,
the-very D B to A B and thus E B nete hyperboleon will-constitute
quod doctus lector facile conspiciet dum acutius contemplatur.
which the-learned reader easily will-perceive while more-keenly he-contemplates

Superior vero descriptio inferiora signa quae continet, ejus
the-upper but diagram the-lower signs which it-contains of-that
sunt descriptionis, ubi chordis notulas apposuimus, quoniam
are of-diagram where to-the-strings little-signs we-set since
earum nomina longum fuit ascribere. Item si A B tribus
their names long it-was to-write-beside likewise if A B into-three
incisionibus partiamur, erit pars tertia A G. Duae
sections we-should-part will-be the-part third A G two-(parts)
igitur ejusdem erunt G B. Consonabit igitur A B
therefore of-the-same will-be G B will-sound-consonant therefore A B
proslambanomenos ad G B, quae est hypate meson, diapente
proslambanomenos to G B which is hypate meson the-diapente
consonantiam in proportione sesquialtera constitutam. C B
consonance in proportion sesquialter established C B
autem ad G B erit sesquioctava, et continebit tonum, idque
moreover to G B will-be sesquioctave and will-hold a-tone and-that
ordine cadit. Nam lychanos hypaton diatonos, id est C B,
in-order falls for lichanos hypaton diatonos that is C B
ad eam quae est hypate meson, id est G B, continet tonum.
to that which is hypate meson that is G B holds a-tone
Rursus A B quidem proslambanomenos ad C B lychanon
again A B indeed proslambanomenos to C B lichanos
hypaton diatonos consonantiam habet diatessaron, A B autem
hypaton diatonos the-consonance has diatessaron A B moreover
proslambanomenos ad G B hypate meson habet consonantiam
proslambanomenos to G B hypate meson has the-consonance
diapente. Item C B ad D B, id est lychanos hypaton diatonos
diapente likewise C B to D B that is lichanos hypaton diatonos
ad mesen habet consonantiam diapente, G B autem ad D B, id
to mese has the-consonance diapente G B moreover to D B that
est hypate meson ad mesen, habet consonantiam diatessaron.
is hypate meson to mese has the-consonance diatessaron

Lychanos autem hypaton, id est C B, ad hypaten meson
lichanos moreover hypaton that is C B to hypate meson
comparata, id est G B, distabit tono. Si autem ejus
compared that is G B it-will-be-distant by-a-tone if moreover of-that
quae est C B quartam partem sumpsero, erit C K. Igitur
which is C B the-fourth part I-shall-have-taken will-be C K therefore
C B ad K B obtinet sesquitertiam proportionem, K B autem
C B to K B holds the-sesquitertian proportion K B moreover
ab ea quae est D B sesquioctava proportione distabit.
from that which is D B by-the-sesquioctave proportion will-be-distant
Erit igitur K B quidem lychanos diatonos meson, et erit
will-be therefore K B indeed lichanos diatonos meson and will-be
C B, id est lychanos hypaton diatonos, ad K B, id est
C B that is lichanos hypaton diatonos to K B that is
lychanos diatonon meson diatessaron consonantiam continens.
lichanos diatonos meson the-diatessaron consonance holding
Rursus si ejus quae est D B nonam partem sumpsero,
again if of-that which is D B the-ninth part I-shall-have-taken
erit mihi D L. Igitur L B erit paramese. Si autem ejus
will-be for-me D L therefore L B will-be paramese if moreover of-that
quae est D B quartam partem sumpsero, erit D M. Igitur
which is D B the-fourth part I-shall-have-taken will-be D M therefore
M B erit nete synemmenon. Si autem ejus quae est D B
M B will-be nete synemmenon if moreover of-that which is D B
tertiam partem sumpsero, erit D N. Igitur N B erit nete
the-third part I-shall-have-taken will-be D N therefore N B will-be nete
diezeugmenon. Si autem K B in duas partes aequas fuerit
diezeugmenon if moreover K B into two parts equal shall-have-been
divisa, erit K X, eritque X B paranete hyperboleon.
divided will-be K X and-will-be X B paranete hyperboleon

Monochordi netarum hyperboleon per tria genera partitio

IV.5 · THE HYPERBOLEON TETRACHORD IN THREE GENERA

Boethius divides the hyperboleon tetrachord through all three genera, doubling the letters (AA, BB) when the alphabet runs out. From proslambanomenos at 9216 and nete hyperboleon at 2304 he computes the diatonic paranete at 2592 and trite at 2916, the chromatic paranete at 2736, and, taking the diatonic trite (2916) as the enharmonic paranete, splits the semitone down to nete diezeugmenon at 3072 into two dieses, the enharmonic trite at 2994.

Nunc igitur diatonici generis descriptio facta est in eo
now therefore of-the-diatonic genus the-diagram made has-been in that
scilicet modo qui est simplicior ac princeps, quem Lydium
namely mode which is simpler and the-chief which Lydian
nuncupamus. De quibus modis nunc disserendum non est; ut
we-name about which modes now to-be-discoursed not it-is so-that
vero per tria genera currat mixta descriptio, et omnibus
but through the-three genera may-run the-mixed diagram and to-all
propria numerorum pluralitas apponatur ad conservandas scilicet
its-proper of-numbers plurality may-be-applied for preserving namely
proportiones, vel tonorum atque dieseon, excogitatus est
the-proportions whether of-tones and of-dieses devised has-been
numerus qui haec omnia posset explere, ut maximus
a-number which these all might-be-able to-fill-out so-that the-greatest
quidem ad proslambanomenon describatur, qui sit 9216;
indeed at proslambanomenos may-be-written which may-be 9216
minimus vero 2304. Reliquorum vero sonorum proportiones
the-least but 2304 of-the-remaining but sounds the-proportions
in horum medietate texentur. Sane ab inferiore procedimus
in of-these the-mean will-be-woven assuredly from the-lower we-proceed
omniumque nomina chordarum non solum nominibus, verum
and-of-all the-names of-strings not only by-names but

etiam oppositis litteris demonstramus. Sed ita ut quoniam
also by-set-against letters we-show but so that since
trium generum est facienda partitio, nervorumque modus
of-three genera is to-be-made the-partition and-of-strings the-measure
litterarum excedit numerum, ubi defecerint litterae, easdem
of-letters exceeds the-number when shall-have-failed the-letters the-same
geminemus rursus hoc modo, ut quando ad Z fuerit
let-us-double again in-this way that when to Z it-shall-have-been
usque perventum, ita describamus reliquos nervos: bis A
all-the-way arrived thus we-may-write the-remaining strings twice A
id est AA, et bis B, id est BB, et bis C, id est CC.
that is AA and twice B that is BB and twice C that is CC
Sit igitur primus quidem numerus maximusque qui
let-there-be therefore the-first indeed number and-greatest which
proslambanomenon obtineat locum 9216. Sitque totius
of-proslambanomenos may-hold the-place 9216 and-let-be of-the-whole
chordae modum ab eo quod est A usque ad id quod est
string the-measure from that which is A up to that which is
LL, hanc, id est A, proslambanomenon 9216 divido, dimidiam
LL this that is A proslambanomenos 9216 I-divide (its)-half
ad O, ut sit tota A dupla ad ea quae est O. Item
at O so-that may-be the-whole A double to that which is O likewise
O sit dupla ab ea quae est LL. Erit igitur A quidem
O let-be double from that which is LL will-be therefore A indeed
proslambanomenos, O autem mese, et LL nete hyperboleon:
proslambanomenos O moreover mese and LL nete hyperboleon
habebit igitur A quidem 9216, O vero horum dimidium, id est
will-have therefore A indeed 9216 O but of-these the-half that is
4608, ut mese ad proslambanomenon diapason consonantia
4608 so-that mese to proslambanomenos the-diapason by-consonance
conveniat. Ea vero quae est LL dimidium est meses, ut sit
may-accord that but which is LL the-half is of-mese so-that may-be

proslambanomenos ab ea quae est nete hyperboleon quadrupla,
proslambanomenos from that which is nete hyperboleon quadruple
et bis diapason ad eam consonet symphoniam, sitque
and twice the-diapason to it may-sound-consonant the-symphony and-let-be
LL 2304. Si igitur ex 2304 octavam abstulero partem,
LL 2304 if therefore from 2304 the-eighth I-shall-have-taken-away part
id est 288, eisdemque adjecero, fient mihi 2592,
that is 288 and-to-the-same I-shall-have-added they-will-become for-me 2592
eritque NN 2592 quae sit paranete hyperboleon ad neten
and-will-be NN 2592 which let-be paranete hyperboleon to nete
hyperboleon obtinens distantiam tonum. Rursus ejus quae est
hyperboleon holding the-distance a-tone again of-that which is
NN, id est 2592, aufero octavam quae est 324, eamque eis
NN that is 2592 I-take-away the-eighth which is 324 and-it to-those
quorum est octava subjungo, erunt 2916, fietque mihi
of-which it-is the-eighth I-subjoin they-will-be 2916 and-will-become for-me
FF trite hyperboleon diatonos in diatonico scilicet genere 2916
FF trite hyperboleon diatonos in the-diatonic namely genus 2916
tonum quidem distans ab ea quae est paranete hyperboleon,
a-tone indeed distant from that which is paranete hyperboleon
ditonum vero ab ea quae est nete hyperboleon, eadem vero
a-ditone but from that which is nete hyperboleon the-same but
FF erit in chromatico genere trite hyperboleon chromatica.
FF will-be in the-chromatic genus trite hyperboleon chromatica
In enharmonio vero paranete hyperboleon enharmonios,
in the-enharmonic but paranete hyperboleon enharmonios
quod facilius agnoscetur cur eveniat, cum trium
which more-easily will-be-recognized why it-comes-about when of-the-three
generum tria prima tetrachorda a nete hyperboleon
genera the-three first tetrachords from nete hyperboleon
inchoantia descripserimus. Quoniam vero si a sesquitertia
beginning we-shall-have-described since but if from the-sesquitertian

proportione duas sesquioctavas abstulero, relinquitur
proportion two sesquioctaves I-shall-have-taken-away there-remains
mihi semitonium minus, sumo tertiam ejus quae est LL, id
for-me a-semitone lesser I-take the-third of-that which is LL that
est neten hyperboleon, et sunt 768. Hos eisdem adjicio,
is nete hyperboleon and they-are 768 these to-the-same I-add
fient mihi 3072. Quorum est DD nete diezeugmenon
they-will-become for-me 3072 of-which is DD nete diezeugmenon
continens ad triten hyperboleon semitonium minus. Nam
holding to trite hyperboleon the-semitone lesser for
quoniam nete diezeugmenon ad neten hyperboleon diatessaron
since nete diezeugmenon to nete hyperboleon the-diatessaron
continet consonantiam, trite autem hyperboleon diatonos ab
holds consonance trite moreover hyperboleon diatonos from
ea ditonum distat, relinquitur spatium quod est inter neten
it a-ditone is-distant there-remains a-space which is between nete
diezeugmenon et triten hyperboleon semitonii minoris.
diezeugmenon and trite hyperboleon of-the-semitone lesser
Quoniam igitur tetrachordum hyperboleon diatonici generis
since therefore the-tetrachord hyperboleon of-the-diatonic genus
explevimus, nunc chromatici et enharmonii tetrachorda
we-have-filled-out now of-the-chromatic and of-the-enharmonic the-tetrachords
supplenda sunt hoc modo: quoniam enim paranete
to-be-supplied are in-this way since for paranete
hyperboleon ad neten hyperboleon in diatonico quidem
hyperboleon to nete hyperboleon in the-diatonic indeed
genere tono distat, in chromatico vero tribus semitoniis,
genus by-a-tone is-distant in the-chromatic but by-three semitones
in enharmonio vero duobus tonis.
in the-enharmonic but by-two tones

Si distantiam paranetes hyperboleon et netes hyperboleon
if the-distance of-paranete hyperboleon and of-nete hyperboleon
diatonici generis sumpserimus, ejusque dimidium paranete
of-the-diatonic genus we-shall-have-taken and-its half to-paranete
hyperboleon, quae est diatonici generis, apponamus, habebimus
hyperboleon which is of-the-diatonic genus we-should-apply we-shall-have
numerum tribus semitoniis ab hyperboleon nete distantem,
a-number by-three semitones from hyperboleon nete distant
et erit haec in chromatico genere paranete hyperboleon.
and will-be this-(note) in the-chromatic genus paranete hyperboleon
Aufero igitur de 2592, id est de paranete hyperboleon
I-take-away therefore from 2592 that is from paranete hyperboleon
diatonici generis 2304, id est nete hyperboleon, relinquuntur
of-the-diatonic genus 2304 that is nete hyperboleon there-remain
mihi 288, hos divido, erunt mihi 144, eosdem adjiciam 2592,
for-me 288 these I-divide they-will-be for-me 144 the-same I-shall-add to-2592
id est paranete hyperboleon diatonici generis, et erunt
that is paranete hyperboleon of-the-diatonic genus and they-will-be
mihi 2736: haec erit paranete hyperboleon chromatica. Rursus
for-me 2736 this will-be paranete hyperboleon chromatica again
quoniam trite hyperboleon, vel diatonica, vel chromatica,
since trite hyperboleon whether diatonica or chromatica
duos tonos distat a nete hyperboleon, et in enharmonio
two tones is-distant from nete hyperboleon and in the-enharmonic
genere paranete hyperboleon duobus tonis distat ab ea quae
genus paranete hyperboleon by-two tones is-distant from that which
est nete hyperboleon, eadem erit in enharmonico genere
is nete hyperboleon the-same will-be in the-enharmonic genus
paranete hyperboleon, quae est in diatonico vel chromatico trite
paranete hyperboleon which is in the-diatonic or chromatic trite
hyperboleon. Sed quoniam trite hyperboleon diatonici generis
hyperboleon but since trite hyperboleon of-the-diatonic genus

et chromatici ad neten diezeugmenon minus semitonium
and of-the-chromatic to nete diezeugmenon the-lesser semitone
servant, constat autem tetrachordum enharmonii generis ex
they-keep consists moreover the-tetrachord of-the-enharmonic genus of
duobus integris tonis, et diesi, ac diesi, quae sunt dimidia
two whole tones and a-diesis and a-diesis which are the-half
spatia semitonii minoris, distantiam eam quae est inter neten
spaces of-the-semitone lesser the-distance that which is between nete
diezeugmenon et paranetem hyperboleon enharmonium sumo;
diezeugmenon and paranete hyperboleon enharmonios I-take
sed quoniam nete diezeugmenon est 3072, paranete autem
but since nete diezeugmenon is 3072 paranete moreover
hyperboleon enharmonios 2916, horum distantia erit 156,
hyperboleon enharmonios 2916 of-these the-distance will-be 156
horum sumo dimidiam partem, qui sunt 78. Hos adjicio 2916,
of-these I-take the-half part which are 78 these I-add to-2916
fient 2994, haec erit EE trite hyperboleon enharmonios;
they-will-become 2994 this will-be EE trite hyperboleon enharmonios
descriptum est igitur secundum tria genera tetrachordum
described has-been therefore according-to the-three genera the-tetrachord
quod est hyperboleon, cujus formam subter adjecimus.
which is hyperboleon whose form below we-have-added

Ratio superius digestae descriptionis

IV.6 · THE REASONING OF THE DIVISION

Boethius explains the reasoning behind the division just made. Every tetrachord renders the diatessaron, which is two tones and a lesser semitone; the diatonic marks its steps T, T, and S. The chromatic gathers three semitones into one step (TTT), splitting a tone only roughly in half, since a tone cannot be cut into two equal parts, while the enharmonic takes two whole tones and parts the leftover semitone into two dieses (D); the same pattern will serve for the tetrachords still to come.

Tria igitur tetrachorda tali nobis ratione descripta sunt.
three therefore tetrachords by-such by-us reckoning described have-been
Tetrachordum enim omne diatessaron refert consonantiam.
a-tetrachord for every the-diatessaron renders consonance
Igitur nete hyperboleon, et nete diezeugmenon in tribus
therefore nete hyperboleon and nete diezeugmenon in the-three
generibus, id est, vel in diatono, vel in chromate, vel in
genera that is whether in the-diatonic or in the-chroma or in
enharmonio diatessaron continent symphoniam. Diatessaron
the-enharmonic the-diatessaron they-hold symphony the-diatessaron
autem consonantia constat duobus tonis et semitonio minore.
moreover consonance consists of-two tones and a-semitone lesser
Id hoc modo per tria genera in supradictis tetrachordis
this in-this way through the-three genera in the-above-said tetrachords
divisum est. In diatonico enim genere quod est primum
divided has-been in the-diatonic for genus which is first
paranete hyperboleon, id est 2592, ad neten hyperboleon,
paranete hyperboleon that is 2592 to nete hyperboleon
id est 2304, obtinet distantiam tonum, quod tali notula
that is 2304 holds the-distance a-tone which with-such a-little-sign
inscripsimus T. Rursus trite hyperboleon diatonici generis,
we-have-inscribed T again trite hyperboleon of-the-diatonic genus

quae est 2916, ad paraneten hyperboleon diatonici generis,
which is 2916 to paranete hyperboleon of-the-diatonic genus
quae est 2592, rursus obtinet differentiam quam simili
which is 2592 again holds the-difference which with-a-like
nota insignivimus T. Nete autem diezeugmenon ad triten
sign we-have-signed T nete moreover diezeugmenon to trite
hyperboleon, id est 3072 ad 2916, semitonium refert, quod
hyperboleon that is 3072 to 2916 a-semitone renders which
tali notula signavimus S; hoc est totum spatium netes
with-such a-little-sign we-have-marked S this is the-whole space of-nete
diezeugmenon, et netes hyperboleon duorum tonorum ac
diezeugmenon and of-nete hyperboleon of-two tones and
semitonii.
of-a-semitone
Sed idem duo toni ac semitonium in chromatico genere
but the-same two tones and the-semitone in the-chromatic genus
hac ratione divisi sunt. Secundum enim genus, quod
by-this reckoning divided have-been the-second for genus which
est chromaticum, hoc modo descriptum est: paranete
is the-chromatic in-this way described has-been paranete
enim chromatice hyperboleon, quae est 2736, ad neten
for chromatice hyperboleon which is 2736 to nete
hyperboleon, quae est 2304 comparata, continet spatium
hyperboleon which is 2304 compared holds the-space
paranetes hyperboleon diatonici generis ad neten hyperboleon
of-paranete hyperboleon of-the-diatonic genus to nete hyperboleon
quod est unus tonus, id est, duo semitonia majus ac
which is one tone that is two semitones the-greater and
minus, et divisum rursus spatium paranetes hyperboleon
the-lesser and divided again the-space of-paranete hyperboleon
diatonici, ad neten hyperboleon. Ita enim factum est,
of-the-diatonic to nete hyperboleon so for done it-has-been

qui est dimidius tonus, sed non integre, quia (ut supra
which is the-half tone but not entirely because as above
uberrime monstratum est) non potest tonus in duo
most-fully shown it-has-been not is-able a-tone into two
aequa partiri. Consignabimus igitur hoc spatium trium
equal-(parts) to-be-parted we-shall-mark therefore this space of-three
semitoniorum id est toni ac semitonii, hoc modo.
semitones that is of-a-tone and of-a-semitone in-this way
TTT Rursus paranete hyperboleon chromatica ad triten
TTT again paranete hyperboleon chromatica to trite
hyperboleon retinet partem toni, id est semitonium, quod
hyperboleon retains a-part of-the-tone that is the-semitone which
reliquum fuit ex duobus tonis qui continentur inter triten
left-over was from the-two tones which are-contained between trite
hyperboleon diatonicam, et neten hyperboleon.
hyperboleon diatonica and nete hyperboleon
Subtractis vero quatuor semitoniis, reliquum ex toto
having-been-subtracted but the-four semitones the-remaining from the-whole
tetrachordo spatium semitonii est, quod continetur inter
tetrachord space of-a-semitone is which is-contained between
neten diezeugmenon, et triten hyperboleon. Constat igitur
nete diezeugmenon and trite hyperboleon consists therefore
et hoc tetrachordum ex duobus tonis ac semitonio,
also this tetrachord of two tones and a-semitone
divisum in uno quidem spatio tribus semitoniis. In duobus
divided in one indeed space by-three semitones in two
autem spatiis, duobus semitoniis. Tria vero spatia nervis
moreover spaces by-two semitones three but spaces by-strings
quatuor continentur. In enharmonio vero genere summa
four are-contained in the-enharmonic but genus utmost
est id pernoscendi facilitas, ab ea enim quae est nete
is it of-learning-thoroughly the-ease from that for which is nete

hyperboleon, id est 2304, paranete hyperboleon enharmonios,
hyperboleon that is 2304 paranete hyperboleon enharmonios
id est 2916, duos tonos integros distat, quos hoc modo
that is 2916 two tones whole is-distant which in-this way
notabimus T T. Relinquitur igitur ex totius tetrachordi
we-shall-note T T there-remains therefore from of-the-whole tetrachord
duobus tonis ac semitonio unum quidem semitonium quod
the-two tones and the-semitone one indeed semitone which
continetur inter neten diezeugmenon et paraneten hyperboleon
is-contained between nete diezeugmenon and paranete hyperboleon
enharmonion. Quod scilicet divisimus in duas dieses trite
enharmonios which namely we-have-divided into two dieses trite
hyperboleon enharmonion, media interjecta, spatiumque
hyperboleon enharmonios in-the-middle interposed and-the-space
dieseos hoc modo signavimus D; ita igitur nobis hyperboleon
of-a-diesis in-this way we-have-marked D thus therefore by-us the-hyperboleon
tetrachordum descriptum est. Quo peracto ad
tetrachord described has-been this having-been-accomplished to
diezeugmenon tetrachordum veniamus. Nec immorandum
the-diezeugmenon tetrachord let-us-come nor to-be-lingered
est eisdem commemorationibus in caeteris. Cum ab hac
it-is in-the-same rehearsals in the-rest since from this
descriptione etiam in aliis sumi possit exemplum.
diagram also in the-others to-be-taken can the-pattern

Monochordi netarum diezeugmenon per tria genera partitio

IV.7 · THE DIEZEUGMENON TETRACHORD

Boethius divides the diezeugmenon tetrachord. From nete diezeugmenon at 3072 he derives mese at 4608 and paramese at 4096, then the diatonic paranete at 3456, the chromatic paranete at 3648, and the enharmonic trite at 3992, splitting the semitone against paramese into two dieses. He joins this tetrachord's diagram onto the hyperboleon one already drawn, so that the form of the whole disposition rises little by little.

Netes igitur diezeugmenon, quae est 3072, si dimidium
of-nete therefore diezeugmenon which is 3072 if the-half
sumam, erunt 1536. Qui eisdem additi fiunt 4608
I-shall-take there-will-be 1536 which to-the-same added they-become 4608
quae est mese, quam O littera designavimus; quod si
which is mese which O by-the-letter we-have-marked but if
eisdem netes diezeugmenon, id est DD, scilicet 3072,
from-the-same of-nete diezeugmenon that is DD namely 3072
auferam tertiam partem, erunt 1024. Qui eisdem
I-shall-take-away the-third part there-will-be 1024 which to-the-same
conjuncti facient 4096 quae vocabitur paramese, X littera
joined they-will-make 4096 which will-be-called paramese X by-the-letter
subnotata. Nete igitur diezeugmenon, id est 3072, ad
marked-beneath nete therefore diezeugmenon that is 3072 to
mesen, id est 4608, quoniam in sesquialtera comparatione
mese that is 4608 since in sesquialter comparison
consistit, diapente consonabit symphoniam. Eadem vero
it-stands the-diapente will-sound-together the-symphony the-same but
nete diezeugmenon, id est 3072, ad paramesen, quae ad eam
nete diezeugmenon that is 3072 to paramese which to it
in sesquitertia proportione composita est, diatessaron retinet
in sesquitertian proportion placed-together is the-diatessaron it-retains

consonantiam. Si igitur ab ea quae est nete diezeugmenon
consonance if therefore from it which is nete diezeugmenon
3072 octavam auferam partem, id est 384, eisque
3072 the-eighth I-shall-take-away part that is 384 and-to-the-same
adjiciam, fient 3456, paranete diezeugmenon diatonos,
I-shall-add they-will-become 3456 paranete diezeugmenon diatonos
CC litteris pernotata, ad neten diezeugmenon obtinens tonum.
CC by-letters marked to nete diezeugmenon holding a-tone
Ab hac vero si octavam auferam partem, id est de
from this but if the-eighth I-shall-take-away part that is from
3456, quae est 432, eosque eidem adjungam, erunt 3888.
3456 which is 432 and-these to-the-same I-shall-add there-will-be 3888
Eritque ea Y trite diezeugmenon diatonos. Sed quoniam
and-there-will-be it Y trite diezeugmenon diatonos but since
nete diezeugmenon ad paramesen sesquitertiam obtinebat
nete diezeugmenon to paramese the-sesquitertian was-holding
proportionem, trite autem diezeugmenon diatonos a nete
proportion trite moreover diezeugmenon diatonos from nete
diezeugmenon duos tonos abest, continebitur inter triten
diezeugmenon two tones is-distant there-will-be-contained between trite
diezeugmenon et paramesen semitonium minus. Diatonicum
diezeugmenon and paramese the-semitone lesser the-diatonic
igitur genus, in hoc quoque tetrachordo ac pentachordo ita
therefore genus in this too tetrachord and pentachord so
expletum est, ut tetrachordi quidem ejus, quod est netes
filled-out is that of-the-tetrachord indeed of-that which is of-nete
diezeugmenon ad paramesen, diatessaron consonantia sit.
diezeugmenon to paramese the-diatessaron consonance may-be
Pentachordi vero ejus quod est netes diezeugmenon ad
of-the-pentachord but of-that which is of-nete diezeugmenon to
mesen, diapente sit consonantia. Enharmonium vero atque
mese the-diapente may-be consonance the-enharmonic but and

chromaticum genus hac ratione texemus: Sumo distantiam
chromatic genus by-this reckoning we-shall-weave I-take the-distance
netes et paranetes diezeugmenon diatoni, id est 3072 et
of-nete and of-paranete diezeugmenon diatonos that is 3072 and
3456, est eorum differentia 384; hanc divido, erunt 192;
3456 is of-them the-difference 384 this I-divide there-will-be 192
hanc si sumam et ei quae est paranete diezeugmenon
this if I-shall-take and to-that which is paranete diezeugmenon
diatonos adjungam, id est 3456, fient 3648, haec
diatonos I-shall-add that is 3456 they-will-become 3648 this
erit paranete diezeugmenon chromatica, BB geminatis litteris
will-be paranete diezeugmenon chromatica BB by-twinned letters
adnotata distans nete diezeugmenon tono et semitonio,
noted distant from-nete diezeugmenon by-a-tone and by-a-semitone
id est tribus semitoniis, continens ad triten diezeugmenon
that is by-three semitones containing to trite diezeugmenon
dudum quidem diatonicam.
formerly indeed diatonic
Nunc vero chromaticam, id est 3888, semitonium reliquum
now but chromatic that is 3888 the-semitone the-remaining
ab eo tono, quod divisum est inter paraneten diatonon
from that tone which divided was between paranete diatonos
diezeugmenon, et triten diatonon diezeugmenon, et sit
diezeugmenon and trite diatonos diezeugmenon and let-be
reliquum aliud ex tetrachordo semitonium inter triten
remaining another from the-tetrachord semitone between trite
diezeugmenon chromaticam et paramesen, quod scilicet
diezeugmenon chromatica and paramese which namely
ex diatessaron consonantia relinquitur, ea quae est inter
from the-diatessaron consonance is-left that which is between
neten diezeugmenon et paramesen subtractis duobus tonis,
nete diezeugmenon and paramese subtracted the-two tones

quos nete diezeugmenon chromatica, et trite diezeugmenon
which nete diezeugmenon chromatica and trite diezeugmenon
chromatica continebant. Quae autem in diatonico genere
chromatica were-containing what moreover in the-diatonic genus
diatonica est, in chromatico autem trite diezeugmenon
diatonica is in the-chromatic moreover trite diezeugmenon
chromatica, ea in enharmonio genere paranete diezeugmenon
chromatica it in the-enharmonic genus paranete diezeugmenon
enharmonios dicitur, integros enim duos tonos distat
enharmonios is-called whole for two tones it-is-distant
ab ea quae est nete diezeugmenon et notetur AA.
from that which is nete diezeugmenon and let-it-be-noted AA
Et inter neten diezeugmenon et paraneten enharmonion
and between nete diezeugmenon and paranete enharmonios
diezeugmenon nulla interest chorda, atque ideo paranetes
diezeugmenon no lies-between string and therefore of-paranete
vocabulo nuncupatur. Semitonium vero quod est inter
by-the-name it-is-called the-semitone but which is between
paraneten enharmonion diezeugmenon et paramesen, id est
paranete enharmonios diezeugmenon and paramese that is
inter AA et X hac ratione partimur, ut fiant duae
between AA and X by-this reckoning we-divide that there-may-become two
dieses. Sumo differentiam paranetes enharmonii diezeugmenon
dieses I-take the-difference of-paranete enharmonios diezeugmenon
et parameses, id est 3888 et 4096, ea est 208, hanc divido,
and of-paramese that is 3888 and 4096 it is 208 this I-divide
fient 104, hos appono 3888, fient 3992. Ea erit
they-will-become 104 these I-set-to 3888 they-will-become 3992 it will-be
trite diezeugmenon enharmonios Z littera pernotata. Hujus
trite diezeugmenon enharmonios Z by-the-letter marked of-this
igitur tetrachordi per tria genera descriptionem subter
therefore of-the-tetrachord through the-three genera the-diagram below

adjeci, superiusque dispositum hyperboleon tetrachordum
I-have-added and-above the-disposed hyperboleon tetrachord
aggregavi, uti esset utrorumque una descriptio, et
I-have-joined-on so-that there-might-be of-both one diagram and
paulatim juncta dispositionis totius forma consurgeret.
little-by-little joined of-the-disposition whole the-form might-rise-up

Monochordi netarum synemmenon per tria genera partitio

IV.8 · THE SYNEMMENON TETRACHORD

Boethius divides the synemmenon tetrachord, so called because it joins directly onto mese. Taking one tone away from the pentachord between nete diezeugmenon and mese lets a new tetrachord hang from mese: nete synemmenon at 3456, paranete at 3888, and trite at 4374 in the diatonic. The chromatic paranete then falls at 4104 and the enharmonic trite at 4491. He adds this diagram to the hyperboleon and diezeugmenon ones.

Duo quidem tetrachorda quae sibimet quidem conjuncta
two indeed tetrachords which to-each-other indeed joined
sunt, a mese vero disjuncta, trium generum superior
are from mese but disjoined of-the-three genera the-above
descriptio, quemadmodum locarentur ostendit. Nunc ad aliud
diagram in-what-manner they-were-placed has-shown now to another
tetrachordum veniendum est, quod synemmenon vocatur,
tetrachord to-be-come is which synemmenon is-called
quod junctum est ei quae est mese. Quoniam enim inter
because joined it-is to-that which is mese since for between
neten diezeugmenon et mesen diapente consonantiam esse
nete diezeugmenon and mese the-diapente consonance to-be
praediximus, est autem diapente consonantia trium tonorum
we-have-said-before is moreover the-diapente consonance of-three tones

ac semitonii. Tres vero sunt toni in hoc pentachordo,
and of-a-semitone three but are the-tones in this pentachord
quorum unus quidem netes diezeugmenon ad paraneten
of-which one indeed of-nete diezeugmenon to paranete
diezeugmenon diatonon, paranetes diezeugmenon diatoni ad
diezeugmenon diatonos of-paranete diezeugmenon diatonos to
triten diezeugmenon diatonon. Tertius autem parameses
trite diezeugmenon diatonos the-third moreover of-paramese
ad mesen, reliquumque semitonium trites diezeugmenon
to mese and-the-remaining semitone of-trite diezeugmenon
diatoni ad paramesen, quoniamque netes diezeugmenon et
diatonos to paramese and-since of-nete diezeugmenon and
parameses tetrachordum ab ea quae est mese, eo tono
of-paramese the-tetrachord from that which is mese by-that tone
disjunctum est, quod est inter paramesen ac mesen. Si ex
disjoined is which is between paramese and mese if from
eo pentachordo, quod est a nete diezeugmenon ad mesen,
that pentachord which is from nete diezeugmenon to mese
unum abstulerimus tonum, eum scilicet qui continetur
one we-shall-have-taken-away tone that-one namely which is-contained
inter neten diezeugmenon et paraneten diezeugmenon
between nete diezeugmenon and paranete diezeugmenon
diatonon, poterimus aliud tetrachordum ad mesen jungere,
diatonos we-shall-be-able another tetrachord to mese to-join
ut fiat synemmenon.
that it-may-become synemmenon
quod est conjunctum hoc modo. Nam quoniam paranetes
which is joined in-this way for since of-paranete
diezeugmenon diatoni, quae est CC, numerus est 3456, horum
diezeugmenon diatonos which is CC the-number is 3456 of-these
tertia eisdem addita faciet mesen. Hic ergo numerus in
the-third to-the-same added will-make mese this therefore number in

diezeugmenon tetrachordo CC litteris adnotatus, tono distabat
the-diezeugmenon tetrachord CC by-letters noted by-a-tone was-distant
a nete diezeugmenon in genere diatonico, et paranete
from nete diezeugmenon in the-genus diatonic and paranete
diezeugmenon diatonos vocabatur, in synemmenon autem
diezeugmenon diatonos was-called in the-synemmenon moreover
tetrachordo, id est, conjunctarum sit nete synemmenon in
tetrachord that is of-the-joined-(strings) let-be nete synemmenon in
tribus generibus constituta V littera pernotata, et ab ea
the-three genera established V by-the-letter marked and from it
octava pars auferatur quae est 432 eisque apponatur,
the-eighth part let-be-taken-away which is 432 and-to-the-same let-be-set
fient 3888 quae est paranete synemmenon et T
they-will-become 3888 which is paranete synemmenon and T
littera pernotata. Hujus pars sumatur octava ea quae est
by-the-letter marked of-this a-part let-be-taken the-eighth that which is
486; haec summa si eisdem, quorum octava est, aggregetur,
486 this sum if to-the-same of-which the-eighth it-is be-joined
fient 4374, quae est trite synemmenon diatonos, id
they-will-become 4374 which is trite synemmenon diatonos that
est, Q. Sed quoniam nete synemmenon ad mesen, id
is Q but since nete synemmenon to mese that
est 3456 ad 4608, sesquitertiam obtinet proportionem quae
is 3456 to 4608 the-sesquitertian holds proportion which
est diatessaron, trite autem synemmenon, id est 4374 ad
is the-diatessaron trite moreover synemmenon that is 4374 to
3456, duorum tonorum obtinet proportionem, relinquitur
3456 of-two tones it-holds the-proportion there-remains
trites synemmenon diatoni ad mesen proportio semitonii, et
of-trite synemmenon diatonos to mese the-proportion of-a-semitone and

conjunctum est hoc tetrachordum cum mese, atque ideo
joined is this the-tetrachord with mese and therefore
synemmenon quasi continuum et conjunctum vocatur.
synemmenon as-if continuous and joined it-is-called
Et diatonici quidem generis hoc modo est facta proportio.
and of-the-diatonic indeed genus in-this way is made the-proportion
Chromatici vero talis divisio est: sumo netes synemmenon
of-the-chromatic but such the-division is I-take of-nete synemmenon
et paranetes synemmenon diatonici, id est 3456 et 3888
and of-paranete synemmenon of-the-diatonic that is 3456 and 3888
differentiam, ea est 432; hanc si divido ut semitonium fiat,
the-difference it is 432 this if I-divide that a-semitone may-become
fiunt 216; hanc adjicio ad 3888 ut tria semitonia fiant,
they-become 216 this I-add to 3888 that three semitones may-become
erunt 4104, quae est paranete synemmenon chromatica, cui
there-will-be 4104 which is paranete synemmenon chromatica to-which
littera S superapposita est. Ab hac igitur, id est, paranete
the-letter S set-above is from this therefore that is paranete
synemmenon chromatica, ad triten synemmenon prius quidem
synemmenon chromatica to trite synemmenon before indeed
diatonicam, nunc vero chromaticam semitonium est. A qua
diatonic now but chromatic a-semitone is from which
trite synemmenon chromatica usque mesen aliud semitonium
trite synemmenon chromatica up-to mese another semitone
reperitur. Sed quoniam a nete synemmenon usque ad triten
is-found but since from nete synemmenon all-the-way to trite
synemmenon diatonicam vel chromaticam duo toni sunt, quae
synemmenon diatonic or chromatic two tones are which
est in diatonico vel chromatico generibus trite synemmenon
is in the-diatonic or the-chromatic genera trite synemmenon
diatonos vel chromatica, eadem in genere enharmonio paranete
diatonos or chromatica the-same in the-genus enharmonic paranete

synemmenon enharmonios est, quae sit R, a qua usque
synemmenon enharmonios is which let-be R from which all-the-way
ad mesen semitonium est, hoc partior in duas dieses hoc
to mese a-semitone is this I-part into two dieses in-this
modo: Sumo differentiam paranetes synemmenon enharmonii
way I-take the-difference of-paranete synemmenon enharmonios
et meses, id est 4374 et 4608, facit 234;
and of-mese that is 4374 and 4608 it-makes 234
hanc divido, fient 117; hanc adjicio paranete synemmenon
this I-divide they-will-become 117 this I-add to-paranete synemmenon
enharmonios, id est 4374, fient 4491, quae P littera
enharmonios that is 4374 they-will-become 4491 which P by-the-letter
pernotatur, et sit ea trite synemmenon enharmonios.
is-marked and let-be it trite synemmenon enharmonios
Eritque semitonium quod continetur inter paraneten
and-there-will-be the-semitone which is-contained between paranete
synemmenon enharmonion et mesen, id est inter 4374 et
synemmenon enharmonios and mese that is between 4374 and
4608, divisum per triten synemmenon enharmonios eam,
4608 divided through trite synemmenon enharmonios that-one
scilicet quae est 4491. Quocirca hujus quoque tetrachordi
namely which is 4491 wherefore of-this too tetrachord
expedita ratio est. Nunc autem facienda est descriptio,
unfolded the-reckoning is now moreover to-be-made is the-diagram
juncta tamen cum caeteris tetrachordis, id est hyperboleon
joined however with the-rest-of-the tetrachords that is hyperboleon
ac diezeugmenon, ut paulatim fiat descriptionis rata
and diezeugmenon that little-by-little may-come-about of-the-diagram a-sure
progressio.
progression

Monochordi meson per tria genera partitio

IV.9 · THE MESON TETRACHORD

Boethius divides the meson tetrachord through all three genera. From mese at 4608 he derives hypate meson at 6144, then the diatonic lichanos at 5184 and parhypate at 5832, the chromatic lichanos at 5472, and the enharmonic parhypate at 5988, which splits the remaining semitone into two dieses. He asks that the finished tetrachord be added to the diagrams already drawn.

Ex his igitur quae praedicta sunt, in caeteris non arbitror
from these therefore which said-before have-been in the-rest not I-think
diutius laborandum esse. Ad horum enim exemplar etiam
longer to-be-labored to-be after of-these for the-pattern also
reliqua tetrachorda meson atque hypaton texenda sunt.
the-remaining tetrachords meson and hypaton to-be-woven are
Ac primum quidem diatonici generis meson tetrachordon
and first indeed of-the-diatonic genus the-meson tetrachord
hoc ordine describemus. Meses enim quae est O 4608 sumo
in-this order we-shall-describe of-mese for which is O 4608 I-take
tertiam partem, ea est 1536; hanc eidem copulo, fient
the-third part it is 1536 this to-the-same I-join they-will-become
6144, ea sit H hypate meson diatessaron ad mesen continens
6144 it let-be H hypate meson the-diatessaron to mese holding
consonantiam, haec duobus tonis ac semitonio ita dividitur:
consonance this in-two tones and a-semitone thus is-divided
Sumo enim meses, id est 4608, octavam partem, quae est
I-take for of-mese that is 4608 the-eighth part which is
576; hanc eidem adjungo, fient 5184, ea est lichanos
576 this to-the-same I-add they-will-become 5184 it is lichanos
meson diatonos, id est M. Cujus interim pars sumitur octava,
meson diatonos that is M of-which meanwhile a-part is-taken the-eighth

ea est 648; hanc eidem adjungo, fient 5832. Ea sit I
it is 648 this to-the-same I-add they-will-become 5832 it let-be I
parhypate meson diatonos tonum obtinens ad lichanon meson
parhypate meson diatonos a-tone holding to lichanos meson
diatonon, duobus autem tonis distans a mese. Relinquitur
diatonos by-two moreover tones distant from mese there-remains
igitur semitonium inter hypaten meson diatonon et parhypaten
therefore the-semitone between hypate meson diatonos and parhypate
meson diatonon constitutum, id est inter 6144 et 5832.
meson diatonos set that is between 6144 and 5832
Idem vero tetrachordum meses atque hypates meson in
the-same but tetrachord of-mese and of-hypate meson in
chromatico genere tali ratione partimur. Sumo meses
the-chromatic genus by-such reckoning we-divide I-take of-mese
differentiam ad lichanon meson diatonon, id est 4608 ad 5184,
the-difference to lichanos meson diatonos that is 4608 to 5184
ea est 576; hanc dimidiam partior, fient 288; eadem
it is 576 this in-half I-part they-will-become 288 the-same
adjicio numero majori, id est 5184, fient 5472, quae
I-add to-the-number greater that is 5184 they-will-become 5472 which
fit N lichanos meson chromatice. Relinquuntur igitur duo
becomes N lichanos meson chromatice there-remain therefore two
semitonia, inter lichanon meson chromaticen et parhypaten
semitones between lichanos meson chromatice and parhypate
meson chromaticen, id est inter 5472 et 5832, et aliud inter
meson chromatice that is between 5472 and 5832 and the-other between
parhypate meson chromaticen et hypaten meson, id est inter
parhypate meson chromatice and hypate meson that is between
5832 et 6144. Enharmonium vero genus hoc modo dividimus,
5832 and 6144 the-enharmonic but genus in-this way we-divide
quoniam ea quae erat parhypate meson diatonos, vel ea quae
since that which was parhypate meson diatonos or that which

erat parhypate meson chromatice, duos tonos distabat a mese
was parhypate meson chromatice two tones was-distant from mese
obtinens numerum 5832, ea in enharmonio genere erit lichanos
holding the-number 5832 it in the-enharmonic genus will-be lichanos
meson enharmonios, L littera pernotata, duos nihilominus
meson enharmonios L by-the-letter marked two nonetheless
ad mesen obtinens tonos. Reliquum igitur semitonium quod
to mese holding tones the-remaining therefore semitone which
est inter lichanos meson enharmonion et hypaten meson,
is between lichanos meson enharmonios and hypate meson
id est inter 5832 et 6144 hoc modo dividimus.
that is between 5832 and 6144 in-this way we-divide
Aufero differentiam 5832 ad 6144, ea est 312: hanc dimidiam
I-take-away the-difference 5832 to 6144 it is 312 this in-half
partior, fient 156; hanc ad 5832 jungo, fient
I-part they-will-become 156 this to 5832 I-join they-will-become
5988; et haec sit K parhypate meson enharmonios. Duae
5988 and this let-be K parhypate meson enharmonios two
vero sunt dieses inter lichanon meson enharmonion et
but are the-dieses between lichanos meson enharmonios and
parhypate meson enharmonion, id est inter 5832 et 5988,
parhypate meson enharmonios that is between 5832 and 5988
et inter parhypaten meson enharmonion et hypaten meson,
and between parhypate meson enharmonios and hypate meson
id est inter 5988 et 6144. Divisum est igitur meson
that is between 5988 and 6144 divided is therefore the-meson
tetrachordon. Quod ita in descriptione ponatur, ut superius
tetrachord which so in the-diagram let-be-placed that above
descriptis tetrachordis aggregetur.
to-the-described tetrachords it-be-joined

Monochordi hypaton per tria genera partitio, et totius dispositio descriptionis

IV.10 · THE HYPATON TETRACHORD, AND THE WHOLE DIAGRAM

Boethius divides the last tetrachord, the hypaton, through the three genera. From hypate meson at 6144 he derives proslambanomenos at 9216 and hypate hypaton at 8192, then the diatonic lichanos at 6912 and parhypate at 7776, the chromatic lichanos at 7296, and the enharmonic parhypate at 7984. Joined to the tetrachords already set out, this completes the whole diagram of the regular monochord divided through all the genera.

Nunc vero hypaton tetrachordum per tria genera
now but the-hypaton tetrachord through the-three genera
dividendum est. Sumo hypates meson, id est 6144: dimidiam
to-be-divided is I-take of-hypate meson that is 6144 the-half
partem, quae sit 3072, hanc eidem si adjecero, fient
part which let-be 3072 this to-the-same if I-shall-have-added they-will-become
9216, quae est proslambanomenos ad hypaten meson diapente
9216 which is proslambanomenos to hypate meson the-diapente
consonantiam servans. Ejusdem autem hypates meson a
consonance keeping of-the-same moreover of-hypate meson from
6144, si auferam tertiam partem quae est 2048 eidemque
6144 if I-shall-take-away the-third part which is 2048 and-to-the-same
adjecero, fient 8192. B hypate hypaton. Igitur
I-shall-have-added they-will-become 8192 B hypate hypaton therefore
hypaten meson ad proslambanomenon diapente est consonantia,
hypate meson to proslambanomenos the-diapente is consonance
ad hypaten vero hypaton diatessaron. Ab hac igitur hypate
to hypate but hypaton the-diatessaron from this therefore hypate
meson, id est 6144, pars auferatur octava, erunt 768;
meson that is 6144 a-part let-be-taken-away the-eighth there-will-be 768
hanc eisdem si quis adjungat, fient 6912. Quae est E
this to-the-same if anyone should-add they-will-become 6912 which is E

lichanos hypaton diatonos ad hypaten meson toni obtinens
lichanos hypaton diatonos to hypate meson of-a-tone holding
proportionem. Rursus de 6912 pars auferatur octava,
the-proportion again from 6912 a-part let-be-taken-away the-eighth
erunt 864; hoc si eisdem copuletur, fient 7776, C
there-will-be 864 this if to-the-same be-joined they-will-become 7776 C
parhypate hypaton diatonos, ad lichanon hypaton diatonon
parhypate hypaton diatonos to lichanos hypaton diatonos
toni, ad hypaten meson duorum tonorum distantiam servans.
of-a-tone to hypate meson of-two tones the-distance keeping
Relinquitur igitur semitonium inter parhypaten hypaton
there-remains therefore the-semitone between parhypate hypaton
diatonon et hypaten hypaton, id est inter 7776 et 8192, et
diatonos and hypate hypaton that is between 7776 and 8192 and
diatonici quidem hypaton tale tetrachordum est, chromaticum
of-the-diatonic indeed the-hypaton such tetrachord is the-chromatic
vero tali ratione dividimus. Sumo enim differentiam hypates
but by-such reckoning we-divide I-take for the-difference of-hypate
meson, et ejus quae est lichanos hypaton diatonos, id est 6144
meson and of-that which is lichanos hypaton diatonos that is 6144
et 6912, ea est 768; hanc dimidiam partior ut duo efficiam
and 6912 it is 768 this in-half I-part that two I-may-make-out
semitonia, fient 384: hanc adjicio 6912 ut tria semitonia
semitones they-will-become 384 this I-add to-6912 that three semitones
fiant, erunt 7296: haec erit F lichanos hypaton chromatice
may-become there-will-be 7296 this will-be F lichanos hypaton chromatice
ab ea quae est hypate meson tribus semitoniis distans.
from that which is hypate meson by-three semitones distant
Relinquuntur ergo duo semitonia, unum quidem inter lichanos
there-remain therefore two semitones one indeed between lichanos
hypaton chromaticen et parhypaten hypaton chromaticen, id
hypaton chromatice and parhypate hypaton chromatice that

est inter 7296 et 7776. Aliud vero inter parhypaten hypaton
is between 7296 and 7776 the-other but between parhypate hypaton
chromatice et hypaten hypaton, id est inter 7776 et 8192.
chromatice and hypate hypaton that is between 7776 and 8192
Restat enharmonium genus, cujus ad superius exemplar
there-remains the-enharmonic genus of-which after the-above pattern
talis divisio est. Quoniam enim parhypate hypaton diatonos vel
such the-division is since for parhypate hypaton diatonos or
parhypate hypaton chromatice, quae 7776 unitatibus insignita
parhypate hypaton chromatice which with-7776 units marked
est, duobus tonis distat ab ea quae est hypate meson, eadem
is by-two tones is-distant from that which is hypate meson the-same
erit in genere enharmonio lichanos hypaton enharmonios,
will-be in the-genus enharmonic lichanos hypaton enharmonios
quae ab hypate meson duobus integris differat tonis.
which from hypate meson by-two whole may-differ tones
Restat igitur ex diatessaron consonantia semitonium
there-remains therefore from the-diatessaron consonance a-semitone
quod est inter lichanos hypaton enharmonion et hypaten
which is between lichanos hypaton enharmonios and hypate
hypaton, id est inter 7776 et 8192, hoc in duas dieses
hypaton that is between 7776 and 8192 this into two dieses
ita dividimus. Sumo differentiam ejus quae est lichanos
so we-divide I-take the-difference of-that which is lichanos
hypaton enharmonios, et hypates hypaton, id est 7776 et
hypaton enharmonios and of-hypate hypaton that is 7776 and
8192, ea est 416; hujus dimidiam sumo, sunt 208; hanc
8192 it is 416 of-this the-half I-take they-are 208 this
adjicio 7776, fient 7984 quae sit D parhypate hypaton
I-add to-7776 they-will-become 7984 which let-be D parhypate hypaton
enharmonios. Sunt igitur duae dieses: una quidem quae est
enharmonios there-are therefore two dieses one indeed which is

inter lichanon hypaton enharmonion et parhypaten hypaton
between lichanos hypaton enharmonios and parhypate hypaton
enharmonion, id est inter 7776, et 7984; altera vero
enharmonios that is between 7776 and 7984 the-other but
quae est inter parhypaten hypaton enharmonion, et hypaten
which is between parhypate hypaton enharmonios and hypate
hypaton, id est inter 7984 et 8192. Tonus vero ultimus inter
hypaton that is between 7984 and 8192 the-tone but last between
proslambanomenon et hypaten hypaton, id est inter 9216
proslambanomenos and hypate hypaton that is between 9216
et 8192, continetur. Divisum est igitur hypaton tetrachordum
and 8192 is-contained divided is therefore the-hypaton tetrachord
secundum tria genera diatonicum chromaticum enharmonion.
according-to the-three genera the-diatonic the-chromatic the-enharmonic
Quod si superioribus tetrachordis hyperboleon diezeugmenon
which if to-the-above tetrachords hyperboleon diezeugmenon
synemmenon meson adjungatur, fit integra perfectaque
synemmenon meson it-be-joined there-comes-about the-whole and-perfect
descriptio divisi per omnia monochordi regularis.
diagram of-the-divided through all-(genera) of-monochord regular

Ratio superius dispositae descriptionis

IV.11 · THE REASONING OF THE DIAGRAM

Boethius reads the consonances off the finished diagram: proslambanomenos to mese and mese to nete hyperboleon each hold the diapason, proslambanomenos to nete hyperboleon the double diapason, and the diatessaron binds hypate hypaton to hypate meson, hypate meson to mese, and the like pairs above. He then lists the five tetrachords — hypaton, meson, synemmenon, diezeugmenon, hyperboleon — naming the first and last string of each.

In superiore igitur forma obtinet quidem consonantiam
in the-above therefore form holds indeed the-consonance
diapason proslambanomenos ad mesen, mese autem ad neten
diapason proslambanomenos to mese mese moreover to nete
hyperboleon. Bis diapason autem proslambanomenos ad neten
hyperboleon twice the-diapason moreover proslambanomenos to nete
hyperboleon. Diatessaron autem consonantiam servat hypate
hyperboleon the-diatessaron moreover consonance keeps hypate
hypaton ad hypaten meson. Hypate meson ad mesen, mese
hypaton to hypate meson hypate meson to mese mese
ad neten synemmenon, paramese ad neten diezeugmenon, nete
to nete synemmenon paramese to nete diezeugmenon nete
diezeugmenon ad neten hyperboleon. Atque hoc ita fit,
diezeugmenon to nete hyperboleon and this so comes-about
ut his consonantiis integra tetrachorda numeremus. Atque
that by-these consonances whole tetrachords we-may-number and
ut clarius omnis in hac forma respiciatur ordo nervorum,
that more-clearly the-whole in this form may-be-viewed order of-the-strings
secundum tria genera 5 tantum notantur esse tetrachorda.
according-to the-three genera 5 only there-are-noted to-be tetrachords
Primum atque gravissimum hypaton, cujus est princeps
the-first and lowest hypaton of-which is the-chief

hypate hypaton, ultima autem hypate meson, secundum
hypate hypaton the-last moreover hypate meson the-second
vero meson cujus est princeps hypate meson, extrema vero
but meson of-which is the-chief hypate meson the-outmost but
mese. Tertium synemmenon, cujus est princeps mese, finalis
mese the-third synemmenon of-which is the-chief mese the-final
nete synemmenon. Quartum diezeugmenon, cujus est prima
nete synemmenon the-fourth diezeugmenon of-which is the-first
paramese, nete vero diezeugmenon extrema. Quintum vero est
paramese nete but diezeugmenon the-outmost the-fifth but is
hyperboleon cujus est quidem princeps nete diezeugmenon,
hyperboleon of-which is indeed the-chief nete diezeugmenon
ad neten vero hyperboleon terminatur extrema.
at nete but hyperboleon is-bounded the-outmost

De stantibus et mobilibus vocibus

IV.12 · STANDING AND MOVABLE NOTES

Boethius sorts the notes into three classes. Proslambanomenos, the two hypatai, mese, nete synemmenon, paramese, and the two remaining netai are wholly immovable, keeping the same names and places in all three genera. The lichanoi and paranetai are wholly movable, changing with each genus; the parhypatai and tritai are neither, since they stay the same in the diatonic and chromatic but change in the enharmonic, where they pass over into lichanoi and paranetai.

Harum vero omnium vocum partim sunt in totum
of-these but all notes partly they-are in whole
immobiles, partim in totum mobiles, partim vero nec in
immovable partly in whole movable partly but neither in
totum immobiles, nec in totum mobiles. In totum immobiles
whole immovable nor in whole movable in whole immovable

sunt proslambanomenos, hypate hypaton, hypate meson,
are proslambanomenos hypate hypaton hypate meson
mese, nete synemmenon, paramese, nete diezeugmenon, nete
mese nete synemmenon paramese nete diezeugmenon nete
hyperboleon. Idcirco quoniam in omnibus tribus generibus
hyperboleon for-this-reason since in all three genera
eaedem sunt, nec nomina nec loca permutantes, sive
the-same they-are neither names nor places changing whether
pentachorda, sive tetrachorda contineant. Pentachorda quidem
pentachords or tetrachords they-may-contain pentachords indeed
ut proslambanomenos ad hypaten meson, et mese ad neten
as proslambanomenos to hypate meson and mese to nete
diezeugmenon. Tetrachorda vero ut hypate hypaton ad hypaten
diezeugmenon tetrachords but as hypate hypaton to hypate
meson, hypate meson ad mesen. Mobiles vero sunt quae
meson hypate meson to mese movable but are those-which
secundum singula genera permutantur hoc modo ut paranete
according-to single genera are-changed in-this way as paranete
et lichanos diatonici et chromatici, trite et parhypate
and lichanos of-the-diatonic and of-the-chromatic trite and parhypate
enharmonici. Alia est enim paranete hyperboleon diatonos,
of-the-enharmonic one is for paranete hyperboleon diatonos
alia paranete hyperboleon chromatica, alia trite enharmonios.
another paranete hyperboleon chromatica another trite enharmonios
Diversae sunt etiam paranete diezeugmenon diatonos, atque
different are also paranete diezeugmenon diatonos and
chromatica. Nec est eadem quae in generibus caeteris
chromatica nor is the-same that-which in the-genera other
trite diezeugmenon enharmonios, neque eaedem sunt paranete
trite diezeugmenon enharmonios nor the-same are paranete

synemmenon diatonos et chromatica et trite synemmenon
synemmenon diatonos and chromatica and trite synemmenon
enharmonios, his qui sunt in reliquis generibus trite.
enharmonios to-these which are in the-remaining genera trite
Distant etiam lichanos meson diatonos et lichanos meson
they-differ also lichanos meson diatonos and lichanos meson
chromatice, et parhypate meson enharmonios nulli aliorum
chromatice and parhypate meson enharmonios to-none of-the-other
generum parhypate similis invenitur. Nec eosdem locos
genera to-the-parhypate similar is-found nor the-same places
ac numeros servant lichanos hypaton diatonos et lichanos
and numbers they-keep lichanos hypaton diatonos and lichanos
hypaton chromatice. Nam parhypate hypaton enharmonios
hypaton chromatice for parhypate hypaton enharmonios
aliorum generum parhypate reperitur esse dissimilis. Non in
of-the-other genera to-the-parhypate is-found to-be unlike not in
totum vero immobiles aut mobiles sunt, quae in duobus
whole but immovable or movable are those-which in two
quidem generibus manent, id est chromatico et diatonico,
indeed genera remain that is the-chromatic and the-diatonic
sed in enharmonio permutantur. Id autem sic consideratur
but in the-enharmonic are-changed this moreover thus is-considered
trite hyperboleon diatonos, et trite hyperboleon chromatice
trite hyperboleon diatonos and trite hyperboleon chromatice
eadem in superiori forma descripta est, hisdem numeris
the-same in the-above form described is with-the-same numbers
2916. At vero cum enharmonium genus aspicimus, triten aliam
2916 but indeed when the-enharmonic genus we-behold trite another
reperimus, id est 2994. Quae igitur vox duobus fuit generibus
we-find that is 2994 which therefore note to-two was genera
communis, eadem in tertio permutata est. Idem est et in
common the-same in the-third changed is the-same is also in

diezeugmenon tetrachordo. Nam trite diezeugmenon diatonos
the-diezeugmenon tetrachord for trite diezeugmenon diatonos
et trite diezeugmenon chromatica eaedem sunt, sibique
and trite diezeugmenon chromatica the-same are and-with-each-other
consentiunt, trite autem diezeugmenon enharmonios a
agree trite moreover diezeugmenon enharmonios from
superiore distat, in synemmenis etiam idem est.
the-above is-distant in the-synemmena also the-same it-is
Trite enim synemmenon diatonos et trite synemmenon
trite for synemmenon diatonos and trite synemmenon
chromatice eaedem sunt. Sed trite synemmenon enharmonios
chromatice the-same are but trite synemmenon enharmonios
est diversa. Item parhypate meson diatonos, et parhypate
is different likewise parhypate meson diatonos and parhypate
meson chromatica eaedem notantur. Sed in enharmonio
meson chromatica the-same are-noted but in the-enharmonic
genere sicut superius trite ita hic parhypate juxta hypaten
genus just-as above trite so here parhypate next-to the-hypate
meson quidem inveniuntur, vi autem ac soni acumine
meson indeed they-are-found in-force moreover and of-sound in-highness
diversae sunt caeteris. Rursus parhypate hypaton diatonos et
different they-are from-the-rest again parhypate hypaton diatonos and
parhypate hypaton chromatica eadem est. Sed non eadem est
parhypate hypaton chromatica the-same is but not the-same is
cum in enharmonio genere quaeritur. Sed ut harum non plena
when in the-enharmonic genus it-is-sought but that of-these not the-full
mutabilitas clarius colliquescat ad hyperboleon tetrachordum
changeability more-clearly may-grow-clear to the-hyperboleon tetrachord
redeamus, in hoc igitur quae in diatonico atque in chromatico
let-us-return in this therefore that-which in the-diatonic and in the-chromatic
genere trite hyperboleon est, eadem mutatur in enharmonio,
genus trite hyperboleon is the-same is-changed in the-enharmonic

et fit paranete. Item quae trite diezeugmenon in
and becomes paranete likewise that-which trite diezeugmenon in
diatonico vel chromatico genere vocabatur, paranete in
the-diatonic or chromatic genus was-called paranete in
enharmonio dicitur quae trite synemmenon in chromatico
the-enharmonic is-called that-which trite synemmenon in the-chromatic
vel diatonico fuit, in enharmonio in paraneten transit.
or the-diatonic was in the-enharmonic into paranete passes-over
Quae vero parhypate meson in chromatico vel diatonico
that-which but parhypate meson in the-chromatic or the-diatonic
videbatur, eadem lichanos meson in enharmonio reperitur.
was-seen the-same lichanos meson in the-enharmonic is-found
Quae autem parhypate hypaton, vel in diatonico vel
that-which moreover parhypate hypaton either in the-diatonic or
in chromatico dicebatur, lichanos hypaton in enharmonio
in the-chromatic was-called lichanos hypaton in the-enharmonic
nuncupatur. Sunt igitur quidem immobiles, proslambanomenos,
is-named there-are therefore indeed immovable proslambanomenos
hypate hypaton, hypate meson, mese, nete synemmenon,
hypate hypaton hypate meson mese nete synemmenon
paramese, nete diezeugmenon, nete hyperboleon; mobiles vero,
paramese nete diezeugmenon nete hyperboleon movable but
quas lichanos, vel paranetas, vel diatonicas, vel chromaticas,
those-which lichanoi or paranetai or diatonic or chromatic
vel enharmonicas nominamus. Non in totum mobiles aut
or enharmonic we-name not in whole movable or
immobiles, quas parhypatas tritas in diatonico vel chromate,
immovable those-which parhypatai tritai in the-diatonic or the-chroma
lichanos autem vel paranetas in enharmonico genere dicimus.
lichanoi moreover or paranetai in the-enharmonic genus we-call

De consonantiarum speciebus

IV.13 · THE SPECIES OF THE CONSONANCES

Boethius defines the species of the primary consonances — diapason, diapente, diatessaron — on fourteen strings lettered A to O. The diatessaron has three species, the diapente four, and the diapason seven, always one fewer than the number of notes. He then asks which species are bounded by fixed notes: for the diatessaron only the first species from each fixed starting point, for the diapente two, and for the diapason three, the rest being closed by movable notes.

Nunc de speciebus primarum consonantiarum tractandum
now about the-species of-the-primary consonances to-be-treated
est; primae autem consonantiae sunt diapason, diapente,
it-is the-primary moreover consonances are diapason diapente
diatessaron. Species autem est quaedam positio propriam habens
diatessaron a-species moreover is a-certain placing (its)-own having
formam secundum unum quodque genus in uniuscujusque
form according-to one each genus in of-each
proportionis consonantiam facientis terminis constituta, ut in
proportion a-consonance making the-terms established as in
diatonico genere. Nam si diezeugmenon tetrachordum inter
the-diatonic genus for if the-diezeugmenon tetrachord between
hyperboleon tetrachordum meseque ponamus, subtracto
the-hyperboleon tetrachord and-mese we-should-place (with)-removed
scilicet synemmenon tetrachordo, erunt 15 nervi. At si
namely the-synemmenon tetrachord there-will-be 15 strings but if
ab his proslambanomenos detrahatur, erunt quatuordecim.
from these proslambanomenos be-drawn-away there-will-be fourteen
Hi ergo disponantur hoc modo. Sit A hypate hypaton, B
these then let-be-arranged in-this way let-be A hypate hypaton B
parhypate hypaton, C lichanos hypaton, D hypate meson, E
parhypate hypaton C lichanos hypaton D hypate meson E

parhypate meson, F lichanos meson, G mese, H paramese,
parhypate meson F lichanos meson G mese H paramese
I trite diezeugmenon, K paranete diezeugmenon, L nete
I trite diezeugmenon K paranete diezeugmenon L nete
diezeugmenon, M trite hyperboleon, N paranete hyperboleon,
diezeugmenon M trite hyperboleon N paranete hyperboleon
O nete hyperboleon. Ab hypate igitur ad paramesen diapason
O nete hyperboleon from hypate therefore to paramese the-diapason
consonantia est. A mese vero ad hypaten meson diatessaron,
consonance is from mese but to hypate meson a-diatessaron
ab eadem vero mese ad lichanon hypaton diapente. Erit
from the-same but mese to lichanos hypaton a-diapente will-be
igitur diapason quidem octo chordarum.
therefore the-diapason indeed of-eight strings
Diatessaron vero quatuor, diapente autem quinque. Ac per hoc
the-diatessaron but of-four the-diapente moreover of-five and through this
habebit diatessaron quidem species tres, diapente autem species
will-have the-diatessaron indeed species three the-diapente moreover species
quatuor, diapason vero species septem. Semperque una minus
four the-diapason but species seven and-always one fewer
species erit quam fuerint voces, ut enim a mese caeteras
species will-be than shall-have-been the-voices as for from mese the-rest
ordinamus diatessaron consonantiae species tres sunt, hoc
we-order of-the-diatessaron consonance species three are in-this
modo: una quidem species erit ab G ad D; secunda vero ab
way one indeed species will-be from G to D the-second but from
F ad C; tertia vero ab E ad B, et huc usque diatessaron
F to C the-third but from E to B and to-here all-the-way of-the-diatessaron
species progrediuntur. Idcirco quia huc usque species
the-species proceed for-this-reason because to-here all-the-way a-species
binos continet nervos ejusdem diatessaron, ut G D quidem
two-apiece contains strings of-the-same diatessaron as G D indeed

eos qui sunt F E contineat, F C eos qui sunt E D, et E B eos
those which are F E may-contain F C those which are E D and E B those
contineat qui sunt D C; si vero his adjecero diatessaron D
may-contain which are D C if but to-these I-shall-have-added the-diatessaron D
A, diversa non erit ab ea quae est G D, unum enim solum
A different not it-will-be from that which is G D one for only
G D consonantiae nervum continebit, id est D solum.
of-the-G D consonance a-string it-will-contain that is D alone
Excessit igitur G D consonantiam, atque ideo
it-has-gone-beyond therefore the-G D consonance and therefore
diatessaron tres species habere perhibetur. Et in caeteris quidem
the-diatessaron three species to-have is-held and in the-rest indeed
consonantiis idem est. Diapente autem erunt species
consonances the-same it-is of-the-diapente moreover there-will-be species
quatuor hoc modo: Una quidem est H ad D; alia vero ab eo
four in-this way one indeed is H to D another but from that
quod est G ad C; alia ab eo quod est F ad B; alia autem ab
which is G to C another from that which is F to B another moreover from
eo quod est E ad A. Diapason vero consonantiae 7 erunt
that which is E to A of-the-diapason but consonance 7 there-will-be
species, hoc modo: prima ab eo quod est O ad G secunda ab
species in-this way the-first from that which is O to G the-second from
eo quod est N ad F; tertia ab eo quod est M ad E; quarta ab
that which is N to F the-third from that which is M to E the-fourth from
eo quod est L ad D; quinta ab eo quod est K ad C; sexta ab
that which is L to D the-fifth from that which is K to C the-sixth from
eo quod est I ad B; septima ab eo quod est H ad A. Liquet
that which is I to B the-seventh from that which is H to A it-is-clear
igitur ex his quae dicta sunt, diatessaron consonantiam
therefore from these which said have-been (that)-the-diatessaron consonance
semel tantum in immobilibus ac statutis vocibus contineri.
once only in immobile and fixed voices to-be-contained

Nam si ab hypate hypaton incipiam, erit A D, id est
for if from hypate hypaton I-shall-begin it-will-be A D that is
ab hypate hypaton in meson hypaten ea quae est in ordine
from hypate hypaton into meson hypate that which is in order
prima. Nam caeterae non statutis vocibus terminantur, ut B
first for the-rest not by-fixed voices are-bounded as B
E C F. Nam parhypate hypaton et parhypate meson, lichanos
E C F for parhypate hypaton and parhypate meson lichanos
hypaton et lichanos meson mobiles esse monstratae sunt.
hypaton and lichanos meson movable to-be shown have-been
Quod si rursus ab hypate meson diatessaron consonantiam
but if again from hypate meson the-diatessaron consonance
inchoemus, erit species diatessaron statutis vocibus
we-should-begin there-will-be a-species of-diatessaron by-fixed voices
terminata G D, ea quae est prima, id est ab hypate
bounded G D that which is first that is from hypate
meson in mesen. Reliquae minime, ut E H et F I. Nam
meson into mese the-remaining not-at-all as E H and F I for
parhypate meson et lichanos meson et trite diezeugmenon
parhypate meson and lichanos meson and trite diezeugmenon
non probantur immobiles. Rursus si eamdem diatessaron
not are-proved immobile again if the-same diatessaron
paramese suscipiat ordiendam, erit ea quoque quae
paramese should-take-up to-be-begun there-will-be that too which
statutis coerceatur sonis, diatessaron species H L, id est
by-fixed is-bounded sounds a-diatessaron species H L that is
a paramese in nete diezeugmenon, quae est prima. Nam
from paramese into nete diezeugmenon which is first for
caeterae quae sunt I M et K N mobilibus terminantur sonis.
the-rest which are I M and K N by-movable are-bounded sounds

Nam triten diezeugmenon, et paraneten diezeugmenon,
for trite diezeugmenon and paranete diezeugmenon
et triten hyperboleon, et paraneten hyperboleon mobiles
and trite hyperboleon and paranete hyperboleon movable
voces esse praediximus. Item diapente consonantia duas
voices to-be we-have-said-before likewise the-diapente consonance two
tantummodo species tenet, quae statutis vocibus includuntur.
only species holds which by-fixed voices are-enclosed
Ut si ab hypate meson ordiamur, una quidem est D H,
as if from hypate meson we-should-begin one indeed is D H
id est ab hypate meson in paramesen, ea quae est prima.
that is from hypate meson into paramese that which is first
Altera vero G L, id est a mese in neten diezeugmenon,
the-other but G L that is from mese into nete diezeugmenon
haec vero est secunda. Reliquae vero, id est E I et F
this but is the-second the-remaining but that is E I and F
K, minime statutis vocibus clausae sunt. Nam parhypate,
K not-at-all by-fixed voices closed are for parhypate
et lichanos, et trite, et paranete instabiles approbantur.
and lichanos and trite and paranete unstable are-proved
Similis autem ratio erit si a nete diezeugmenon in
similar moreover the-reckoning will-be if from nete diezeugmenon into
graviorem partem, id est ad mesen, consonantiae hujus
the-lower part that is to mese of-the-consonance of-this
species considerentur, eisdem enim immobilibus vocibus, quae
the-species be-considered by-the-same for immobile voices which
superius dictae sunt, continebuntur. Sive autem ab hypate
above said have-been they-will-be-contained whether moreover from hypate
meson, seu a mese, seu a paramese, sive etiam a nete
meson or from mese or from paramese or even from nete
hyperboleon consonantias ad graviorem partem ducamus.
hyperboleon the-consonances to the-lower part we-should-lead

Duarum quae statutis vocibus coerceantur, non poterit
of-the-two which by-fixed voices are-bounded not it-will-be-able
esse destructio. Diapason vero consonantiae sive ab hypate
to-be a-destruction of-the-diapason but consonance whether from hypate
hypaton in paramesen, sive a nete hyperboleon in mesen
hypaton into paramese or from nete hyperboleon into mese
ordo sumatur, tres tantummodo species obtinebit, quae
the-order be-taken three only species it-will-obtain which
immobilibus vocibus coerceantur. Nam ab hypate hypaton
by-immobile voices are-bounded for from hypate hypaton
ordientibus una est A H, ea quae est prima ab hypate
(for-those)-beginning one is A H that which is first from hypate
hypaton in paramesen. Altera D L ea quae quarta
hypaton into paramese the-other D L that which (is)-the-fourth
ab hypate meson in neten diezeugmenon. Dehinc G O
from hypate meson into nete diezeugmenon thereafter G O
haec septima est, id est a mese in neten hyperboleon.
this the-seventh is that is from mese into nete hyperboleon
Reliquarum vero specierum voces extimae nullo modo
of-the-remaining but species the-voices outermost in-no way
immobilibus vocibus constitutae sunt. Nam parhypate, et
by-immobile voices established are for parhypate and
lichanos, et trite, paranete (ut supra quoque dictum est)
lichanos and trite paranete as above also said it-has-been
immobiles non sunt. Similiter autem et per easdem voces si
immobile not are similarly moreover also through the-same voices if
ab hyperboleon nete ordiamur, specierum ordo contexitur,
from hyperboleon nete we-should-begin of-the-species the-order is-woven
quorum omnium intelligentiam subjecta descriptio docet.
of-which all the-understanding the-subjoined diagram teaches

De modorum exordiis, in quo dispositio notarum per singulos modos ac voces

Boethius derives the modes from the species of the diapason: a mode, also called trope or tone, is a whole constitution of notes raised or lowered as one body. Tightening every string of the Hypodorian by a tone yields the Hypophrygian, another tone the Hypolydian, and a semitone more the Dorian, and so on through the seven modes to the Mixolydian. Because the ancients stamped each mode's notes with different marks, a diagram of the marks must come first.

Ex diapason igitur consonantiae speciebus existunt qui
out-of of-the-diapason therefore consonance the-species arise (those)-which
appellantur modi, quos eosdem tropos vel tonos nominant.
are-called modes which the-same tropes or tones they-name
Sunt autem tropi constitutiones in totis vocum ordinibus,
there-are moreover tropes constitutions in whole of-voices orders
vel gravitate, vel acumine differentes, constitutio vero est
either in-lowness or in-highness differing a-constitution but is
plenum veluti modulationis corpus ex consonantiarum
a-full as-it-were of-melody body out-of of-consonances
conjunctione consistens, quale est, vel diapason, vel diapason
the-joining consisting such-as is either diapason or diapason
et diatessaron, vel bis diapason. Est enim diapason
and diatessaron or twice diapason is for the-diapason
constitutio, a proslambanomenos in mesen caeteris quae
a-constitution from proslambanomenos into mese with-the-rest which
sunt mediae vocibus annumeratis, vel a mese rursus in
are middle voices counted-in or from mese again into
neten hyperboleon cum vocibus interjectis, vel ab hypate
nete hyperboleon with the-voices interposed or from hypate
meson in neten diezeugmenon, cum his quas extremae voces
meson into nete diezeugmenon with these which the-outermost voices

medias claudunt. Diapason et diatessaron vero constitutio ea
as-middle enclose the-diapason and diatessaron but constitution that
est quae a proslambanomenos in neten synemmenon, cum
is which from proslambanomenos into nete synemmenon with
his quae mediae sunt interjectae constat. Bis diapason autem
these which middle are interposed consists twice diapason moreover
a proslambanomenon in neten hyperboleon, cum his quae in
from proslambanomenos into nete hyperboleon with these which in
medio sunt interpositae consideratur. Has igitur constitutiones
the-middle are placed-between is-considered these therefore constitutions
si quis totas faciat acutiores vel in gravius totas remittat,
if anyone whole should-make higher or into the-lower whole should-let-down
secundum supradictas diapason consonantiae species efficiet
according-to the-aforesaid of-the-diapason consonance species he-will-produce
modos septem, quorum nomina sunt haec: hypodorius,
modes seven whose names are these hypodorius
hypophrygius, hypolydius, dorius, phrygius, lydius, mixolydius;
hypophrygius hypolydius dorius phrygius lydius mixolydius
horum vero sic ordo procedit. Sit in diatonico genere vocum
of-these but thus the-order proceeds let-be in the-diatonic genus of-voices
ordo dispositus a proslambanomene in neten hyperboleon,
an-order arranged from proslambanomenos into nete hyperboleon
atque hic sit hypodorius modus. Si quis proslambanomenon
and this let-be the-hypodorius mode if anyone proslambanomenos
in acumen intendat tono, hypatenque hypaton eodem tono
into highness should-tighten by-a-tone and-hypate hypaton by-the-same tone
attenuet, caeterasque phthongorum omnes faciat acutiores,
should-sharpen and-the-rest of-the-notes all should-make higher
acutior totus ordo proveniet, quam fuit prius quam toni
higher the-whole order will-come-forth than it-was before (that) of-the-tone
susciperet intensionem. Erit igitur tota constitutio acutior
it-received the-tightening will-be therefore the-whole constitution higher

effecta hypophrygius modus, quod si in hypophrygio toni
made the-hypophrygius mode but if in the-hypophrygius of-a-tone
rursum intensionem voces acceperint, hypolydii modulatio
again the-tightening the-voices shall-have-received of-the-hypolydius the-melody
nascetur. At si hypolydium quis semitonio intendat, dorium
will-be-born but if the-hypolydius anyone by-a-semitone should-tighten the-dorius
faciet, et in aliis quidem similis est in acumen
he-will-make and in the-others indeed similar is into highness
intensionemque processus. Quorum, non ut intelligentia
and-tightening the-advance of-which not that by-understanding
solummodo ratio comprehendatur, verum oculis quoque
only the-reckoning be-grasped but by-the-eyes too
forma possit agnosci, ab antiquis tradita musicis
the-form may-be-able to-be-recognized by the-ancient handed-down musicians
descriptio supponenda est; sed quoniam per singulos modos
a-diagram to-be-placed-below is but since through the-several modes
a veteribus musicis unaquaeque vox diversis notulis insignita
by the-old musicians each voice with-diverse little-signs stamped
est, descriptio prius notularum videtur esse ponenda,
has-been a-diagram first of-the-little-signs seems to-be to-be-placed
ut his primum per se cognitis in modorum
that (with)-these first through themselves known in of-the-modes
descriptione facilis possit esse dispectio.
the-diagram easy may-be-able to-be the-discerning

Descriptio continens modorum ordinem ac differentias

IV.15 · THE ORDER OF THE MODES

Boethius explains the diagram he has set out: it holds the strings' names with the marks set alongside, and the mode — Lydian, Phrygian, Dorian — that each column of marks belongs to. Since the modes are found in the species of the diapason, he describes them in the diatonic genus only, so that their order falls under the eye and does not delay the understanding.

Superior igitur descriptio chordarum nomina tenet ascripta,
the-above therefore diagram of-the-strings the-names holds written-beside
notulas vero juxta positas, et quae cujusque sit modi,
the-little-signs but alongside placed and what of-each may-be mode
sive lydii, sive phrygii, sive dorii vocabulorum signat
whether of-lydius or of-phrygius or of-dorius of-the-names marks
adjectio. Sed quoniam hos modo diximus in speciebus
the-addition but since these just-now we-have-said in the-species
diapason consonantiae reperiri, age eosdem in diatonico
of-the-diapason consonance to-be-found come the-same in the-diatonic
tantum genere describamus, ut qui eorum ordo sit, sub
only genus let-us-describe that what of-them the-order may-be under
aspectu cadens intelligentiam non moretur.
the-sight falling the-understanding not may-delay

De superius disposita modorum descriptione

IV.16 · READING THE DIAGRAM OF THE MODES

Boethius teaches how to read the diagram of the modes, in which an eighth mode, the Hypermixolydian, stands above the seven. A whole compartment between two marks means a tone, a bare line a semitone, and the mese decides which of two orders is the lower. Working up from the Hypodorian he measures mese against mese — tone, tone, semitone, and so on — so that the Dorian's mese stands a diatessaron from the Hypodorian's and the Mixolydian's, and a diapente from the Hypermixolydian's.

Septem quidem praediximus esse modos, sed nihil videatur
seven indeed we-have-said-before to-be the-modes but nothing let-it-seem
incongruum quod octavus super annexus est. Hujus enim
incongruous that an-eighth above annexed has-been of-this for
adjectionis rationem paulo posterius eloquemur. Nunc illud
addition the-reason by-a-little later we-shall-declare now this
est considerandum quod hae paginulae, quas inter se
is to-be-considered that these little-compartments which between themselves
rectus linearum ordo distinguit, aliae quidem habent notulas
the-straight of-lines order separates some indeed have little-signs
musicas, aliae vero minime, velut in eo modo qui scribitur
musical others but not-at-all as in that mode which is-written
hypermixolydius. Prima quidem paginula M tertia Φ
hypermixolydius the-first indeed little-compartment M the-third Φ
litteris adnotatur. Secunda a notula vacat. In hac
with-the-letters is-noted the-second of a-little-sign is-free in this
igitur intercapedine notularum tonus interesse monstratur.
therefore the-gap of-the-little-signs a-tone to-lie-between is-shown
Quod vero Φ tertiae atque / quartae paginae notam
(the-fact)-that but Φ of-the-third and / of-the-fourth compartment the-sign
non paginula dividit, sed versus recto ordine diductus,
not a-little-compartment divides but a-line in-straight course drawn

semitonium eas differre pronuntiat, quod probatur hoc
by-a-semitone (that)-they to-differ declares which is-proved in-this
modo: nam si proslambanomenos est M, Φ hypate hypaton,
way for if proslambanomenos is M Φ hypate hypaton
/ parhypate hypaton, necesse est inter proslambanomenon
/ parhypate hypaton necessary it-is between proslambanomenos
quod est M et inter hypaten hypaton, quod est Φ
which is M and between hypate hypaton which is Φ
toni esse distantiam, inter hypaten autem hypaton quod
of-a-tone to-be the-distance between hypate moreover hypaton which
est Φ et parhypaten hypaton, quod est /, semitonii
is Φ and parhypate hypaton which is / of-a-semitone
differentiam contineri jamque hoc regulariter in cunctis
the-difference to-be-contained and-now this as-a-rule in all
est considerandum, ut si vocum notulas integra pagina
is to-be-considered that if of-voices the-little-signs a-whole compartment
disgregaverit, toni inter eas sciamus esse distantiam.
shall-have-parted of-a-tone between them we-should-know to-be the-distance
Sin versus notulas, ac non pagina distinguit, semitonii
but-if a-line the-little-signs and not a-compartment separates of-a-semitone
non ignoremus esse distantiam. His igitur ita praemissis,
not let-us-be-ignorant to-be the-distance (with)-these therefore thus premised
si duo ordines in bis diapason consonantia constituti
if two orders in the-twice diapason consonance established
sibi invicem comparentur, ut qui ordo sit gravior
to-themselves in-turn be-compared that which order may-be lower
possit agnosci, si proslambanomenos proslambanomene
it-may-be-able to-be-recognized if proslambanomenos than-proslambanomenos
fuerit gravior vel quaelibet alia vox ejusdem loci voce
shall-have-been lower or any other voice of-the-same place than-the-voice
gravior pernotetur, in eodem scilicet genere constituta, totum
lower be-noted in the-same namely genus established the-whole

quoque ordinem necesse est esse graviorem. Tamen id melius
too order necessary it-is to-be lower yet this better
sumetur ad mediam quae est mese. Duorum enim ordinum
will-be-taken at the-middle which is mese of-two for orders
bis diapason consonantium cujus mese fuerit gravior,
twice diapason sounding-together of-which mese shall-have-been lower
ejusdem totus ordo quoque gravior erit. Nam caeterae
of-the-same the-whole order too lower will-be for the-rest
singulae singulis comparatae nihilominus graviores invenientur,
one-by-one with-each compared nonetheless lower will-be-found
itaque si media ab alia media tono aut acutior videatur,
and-so if the-middle than another middle by-a-tone either higher should-seem
aut gravior, omnes quoque nervi si in eodem genere sint,
or lower all too the-strings if in the-same genus they-be
singuli sibimet comparati tono acutiores aut graviores
each to-themselves compared by-a-tone higher or lower
esse videbuntur. Quatuor autem mediis si prima ad quartam
to-be will-seem with-four moreover middles if the-first to the-fourth
diatessaron distantiam servet, prima vero a secunda tono
a-diatessaron distance should-keep the-first but from the-second by-a-tone
differat, secunda quoque a tertia eodem differat tono,
should-differ the-second too from the-third by-the-same should-differ tone
tertia ad quartam semitonii faciet differentiam hoc modo:
the-third to the-fourth of-a-semitone will-make the-difference in-this way
sint 4 mediae A B C D, et A ei quae est D comparata
let-be 4 middles A B C D and A to-that which is D compared
servet ad eam sesquitertiam proportionem, quae est diatessaron.
let-it-keep to it the-sesquitertian proportion which is diatessaron
Item A a B distet tono, B a C distet tono,
likewise A from B let-stand-apart by-a-tone B from C let-stand-apart by-a-tone
relinquitur ut C ad D semitonii distantiam servet. Et si
it-remains that C to D of-a-semitone the-distance keep and if

quinque sint mediae, eodem modo. Si enim prima a
five there-be middles in-the-same way if for the-first from
quinta sesquialtera disteterit proportione, primaque a
the-fifth by-the-sesquialter shall-have-stood-apart proportion and-the-first from
secunda, ac secunda a tertia, quartaque a quinta, singulis
the-second and the-second from the-third and-the-fourth from the-fifth by-single
disteterint tonis, tertia ad quartam semitonii faciet
shall-have-stood-apart tones the-third to the-fourth of-a-semitone will-make
differentiam. Item quaecunque mediae aliorum modorum
the-difference likewise whichever middles of-other modes
proslambanomenos accedunt hae graviores modos operantur.
the-proslambanomenoi approach these lower modes work-out
Quae netis, illae acutiores efficiunt. Quoniam igitur
(those)-which the-netes those higher make since therefore
in superiore pagina descriptis modis partem sinistram
in the-above page with-the-described modes the-part left
legentis proslambanomeni primi tenent. Dextera vero legentis
of-the-reader the-proslambanomenoi first hold the-right but of-the-reader
extremis clauditur netis, erit omnibus quidem acutior modis,
by-the-outermost is-closed netes will-be than-all indeed higher modes
qui inscribitur hypermixolydius, omnibus vero gravior
(that)-which is-inscribed hypermixolydius than-all but lower
is qui hypodorius. Nos vero a gravissimo hypodorio
that-one which hypodorius we but from the-lowest hypodorius
inchoantes, caeteros quam inter se habent differentiam
beginning the-rest what between themselves they-have difference
designabimus. Namque in hypodorio modo mese quae est ω
we-shall-mark-out for in the-hypodorius mode mese which is ω
ab ea mese quae est in modo hypophrygio tono distabit.
from that mese which is in the-mode hypophrygius by-a-tone will-stand-apart

Quod in hoc facile perspicietur, si quis ad mesen hypophrygii,
which in this easily will-be-seen if anyone to mese of-hypophrygius
quae est Φ, ejusdem hypophrygii ω comparet, quae est
which is Φ of-the-same hypophrygius ω should-compare (that)-which is
hypodorii quidem mese, in hypophrygio autem lichanos meson.
of-hypodorius indeed mese in the-hypophrygius moreover lichanos meson
Nam Φ atque ω tono differunt, quod pagina interjecta
for Φ and ω by-a-tone they-differ which the-compartment interposed
demonstrat. Item mese hypolydii ab ea quae est mese
shows likewise mese of-hypolydius from that which is mese
hypophrygii toni differentiam facit. Namque C, quae est mese
of-hypophrygius of-a-tone the-difference makes for C which is mese
hypolydii, tono distat a Φ, quae est hypolydii quidem
of-hypolydius by-a-tone stands-apart from Φ which is of-hypolydius indeed
lichanos meson, in hypophrygio autem mese. Item mese
lichanos meson in the-hypophrygius moreover mese likewise mese
hypolydii, quae est C, ab ea quae est mese dorii semitonio
of-hypolydius which is C from that which is mese of-dorius by-a-semitone
distat. Quod hinc poterit agnosci, quoniam ordinem
stands-apart which from-here it-will-be-able to-be-recognized since the-order
sursum prodeuntem ejus meses, quae est hypolydii, atque eum
upward going-forth of-that mese which is of-hypolydius and that
ordinem in sursum prodeuntem ejus meses quae est dorii,
order in upward going-forth of-that mese which is of-dorius
unus versiculus, non pagina distinguit. Quo fit
one little-line not a-compartment separates whereby it-comes-about
ut mese hypodorii ab ea mese quae est dorii, integra
that mese of-hypodorius from that mese which is of-dorius by-a-whole
diatessaron consonantia distet, idque probatur hoc modo:
diatessaron consonance stands-apart and-that is-proved in-this way
nam quae est mese ω in hypodorio, eadem est ω in dorio;
for (that)-which is mese ω in the-hypodorius the-same is ω in the-dorius

hypate meson est ab ea quae est mese, quovis modo vel genere
hypate meson is from that which is mese in-any mode or genus
diatessaron consonantia differens. Item mese dorii quae est
by-the-diatessaron consonance differing likewise mese of-dorius which is
Π ab ea mese quae est phrygii, id est M, distat tono.
Π from that mese which is of-phrygius that is M stands-apart by-a-tone
Nam quae est mese in dorio, eadem in phrygio lichanos
for (that)-which is mese in the-dorius the-same in the-phrygius lichanos
meson. Rursus mese phrygii quae est M, ab ea mese quae est
meson again mese of-phrygius which is M from that mese which is
lydii, id est I, distat tono. Nam quae in phrygio est
of-lydius that is I stands-apart by-a-tone for (that)-which in the-phrygius is
M mese, in lydio est lichanos meson. Rursus mese lydii
M mese in the-lydius is lichanos meson again mese of-the-lydius
modi ab ea mese quae est myxolydii, id est H, semitonio
mode from that mese which is of-mixolydius that is H by-a-semitone
distat. Etenim si ordo qui rectus lydii continet mesen
stands-apart for-indeed if the-order which upright of-lydius contains mese
ei ordini qui rectus mixolydii mesen habet comparatus, non
to-that order which upright of-mixolydius mese has compared not
paginula, sed versu disjungitur. Ea quoque mese quae
by-a-little-compartment but by-a-line is-disjoined that too mese which
est myxolydii H ad eam mesen quae est hypermixolydii, id
is of-mixolydius H to that mese which is of-hypermixolydius that
est Γ, toni differentiam facit; idcirco quoniam H quae in
is Γ of-a-tone the-difference makes for-this-reason because H which in
mixolydio mese est eadem in hypermixolydio lichanos meson est.
the-mixolydius mese is the-same in the-hypermixolydius lichanos meson is
Unde fit ut mese dorii ab ea mese quae est
whence it-comes-about that mese of-dorius from that mese which is
mixolydii diatessaron consonantia distet. Id probatur hoc
of-mixolydius by-the-diatessaron consonance stands-apart this is-proved in-this

modo: nam mese quae est dorii, id est Π, eadem est
way for mese which is of-dorius that is Π the-same is
mixolydii hypate meson, quae ad cujuslibet modi mesen
of-mixolydius hypate meson which to of-any mode mese
diatessaron consonantiam servat. Item mese dorii, id est
the-diatessaron consonance keeps likewise mese of-dorius that is
Π, ad eam mesen quae est hypermixolydii quae est Γ
Π to that mese which is of-hypermixolydius which is Γ
diapente consonantiam servat. Ea enim mese quae est dorii,
the-diapente consonance keeps that for mese which is of-dorius
id est Π, in ordine hypermixolydii lichanos hypaton est;
that is Π in the-order of-hypermixolydius lichanos hypaton is
lichanos autem hypaton ad mesen in diatonico genere in
lichanos moreover hypaton to mese in the-diatonic genus in
quolibet modo si comparetur, diapente consonantia distat.
any mode if it-be-compared by-the-diapente consonance stands-apart
Cur autem octavus modus, qui est hypermixolydius, adjectus
why moreover the-eighth mode which is hypermixolydius added
est, hic patet. Sit bis diapason consonantia haec.
has-been here is-clear let-be the-twice diapason consonance this

Ratio superius dispositae modorum descriptionis

IV.17 · THE REASONING OF THE MODES' DIAGRAM

Boethius states the reasoning of the modes' diagram: the seven species of the diapason, from A–H through G–O, answer to the seven modes. Outside them H–P remains; it is added so that the whole order may be filled up, and this is the eighth mode, which Ptolemy annexed above the rest.

Diapason igitur consonantiam servat A ad id quod est H,
the-diapason therefore consonance keeps A to that which is H
octo enim vocibus continetur: primam igitur dicimus esse
by-eight for voices it-is-contained the-first therefore we-say to-be
speciem diapason eam quae est A H, secundam vero B
species of-diapason that which is A H the-second but B
I, tertiam C K, quartam D L, quintam E M, sextam F
I the-third C K the-fourth D L the-fifth E M the-sixth F
N, septimam G O. Relinquitur igitur extra H P, quae ut
N the-seventh G O there-remains therefore outside H P which that
totus ordo impleatur adjecta est, atque hic est octavus
the-whole order be-filled-up added has-been and this is the-eighth
modus, quem Ptolemaeus super annexuit.
mode which Ptolemy above annexed

Quemadmodum indubitanter musicae consonantiae aure dijudicari possint

IV.18 · JUDGING CONSONANCES BY EAR

Boethius describes a simple instrument for judging the consonances by ear: a measured rule under an even string, with polished hemispheres as bridges and an equal movable one between them. Seven parts with the bridge at the fourth give the sesquitertian diatessaron; five parts split three against two give the diapente, three parts split two against one the diapason, four parts split three against one the triple. Striking the two segments lets the ear weigh consonance and dissonance.

Ut vero indubitanter consonantiarum ratio colligatur, tali
that but undoubtingly of-the-consonances the-reckoning be-gathered by-such
brevissimo ac simplici effici poterit instrumento. Sit
most-brief and simple to-be-effected it-will-be-able an-instrument let-be
igitur regula diligenter extensa A D, cui duo hemispheria,
therefore the-rule carefully stretched-out A D to-which two hemispheres
quas magadas Graeci vocant, insuper apponantur, ita ut ab
which bridges the-Greeks call thereupon be-placed so that from
ea quae est E curvatura ad id quod est B deducta linea
that which is E the-curve to that which is B drawn-down a-line
rectos circa se angulos reddat: item ab ea quae est F
right around itself angles may-render likewise from that which is F
curvatura ad id quod est C punctum deducta linea rectos
the-curve to that which is C the-point drawn-down a-line right
circum se angulos efficiat. Sint vero hae aequaliter undique
around itself angles may-produce let-be but these equally on-every-side
perpolitae ad eosdem usus, sint eisdem aliae aequales
well-polished for the-same uses let-there-be to-these-same others equal
paratae, super has intendatur nervus aequalis undique, is qui
made-ready over these let-be-stretched a-string even on-every-side that which
est A E F D. Si igitur diatessaron consonantiam qualis
is A E F D if therefore the-diatessaron consonance of-what-sort

sit reperire voluero, hoc modo faciam: ab E puncto,
it-be to-find I-shall-have-wished in-this way I-shall-do from the-E point
quo nervus semisphaerium tangit, usque ad F punctum,
where the-string a-hemisphere touches all-the-way to the-F point
quo rursus ab alia parte altero nervus semisphaerio
where again from the-other side to-the-second the-string hemisphere
jungitur, divido spatium quod est E F septem partibus, et
is-joined I-divide the-space which is E F into-seven parts and
ad partem quartam septimarum appono punctum quod est K;
at the-part fourth of-the-sevenths I-place a-point which is K
est igitur E K ad eam quae est K F sesquitertia. Si igitur ad K
is therefore E K to that which is K F sesquitertian if therefore at K
aequum superioribus semisphaeriis apposuero, atque alterutra
an-equal to-the-upper hemispheres I-shall-have-placed and either-part
vicissim E K et K F plectro adhibito pellantur, diatessaron
in-turn E K and K F with-a-plectrum applied be-struck the-diatessaron
distantiam consonabit. Sin vero simul partes easdem E
distance it-will-sound-together but-if but together the-parts the-same E
K et K F utrasque percussero, diatessaron consonantiam
K and K F both I-shall-have-struck the-diatessaron consonance
nosco. Quod si diapente efficere volumus, quinque partibus
I-recognize but if the-diapente to-effect we-wish into-five parts
totam chordam, id est spatium ab E ad F divido, ac
the-whole string that is the-space from E to F I-divide and
tres uni proportioni, duas vero reliquae dabo, atque
three to-one proportion two but to-the-remaining I-shall-give and
ita posito semisphaerio, secundum superius dictum modum
thus with-placed hemisphere according-to the-above said way
consonantias dissonantiasque perpendo; item si diapason
the-consonances and-dissonances I-weigh likewise if the-diapason
consonantiam temptare voluero, totum tribus partibus
consonance to-try I-shall-have-wished the-whole into-three parts

seco, atque in unam duasque distribuens easdem simul vel
I-cut and into one and-two distributing the-same together or
alterutram pulsans, quid consonet, vel quod dissonet
either-one striking what may-sound-together or what may-sound-apart
utraque cognosco. Tripla vero, quae ex permixtis consonantiis
by-both I-learn the-triple but which out-of the-mingled consonances
nascitur, ita redditur, ut si totam in quatuor partium
is-born thus is-rendered that if the-whole into of-four parts
divisionibus partiamus, atque in tres et unam tota nervi
divisions we-part and into three and one the-whole of-the-string
prolixitas dividatur, ita semisphaerium tribus appositum
length be-divided thus a-hemisphere at-the-three placed
triplae proportionis dissonantiam et consonantiam reddat.
of-the-triple proportion the-dissonance and the-consonance may-render

BOOK V

V · PROEM

Boethius states the fifth book's plan: after the regular division of the monochord he will add the points on which the old teachers of music disagree, pass a fine judgment on them all, and supply what the work still lacks. Another division is also possible, he notes, not on one string but on eight or as many as are needed — a kind of cithara — so that the whole reckoning of the proportions lies under the eyes.

Post monochordi regularis divisionem adjicienda arbitror
after of-the-monochord regular the-division to-be-added I-think
esse ea in quibus veteres musicae doctores sententiae
to-be those-things in which the-old of-music teachers of-opinion
diversitate discordant, habendumque de omnibus subtile
by-diversity disagree and-(that)-to-be-had about all a-fine
judicium, atque id quod proposito deest operi, mediocris
judgment and that which to-the-proposed is-lacking work of-middling
doctrinae dispositione supplendum est. Potest enim alia
teaching by-the-arrangement to-be-supplied is there-can for another
quoque esse divisio, in qua non unus tantummodo nervus
also to-be division in which not one only string
assumitur, qui positis proportionibus dividat. Verum octo,
is-taken-up which with-set proportions it-may-divide but eight
atque ejusmodi cithara fiat, aut in pluribus, et quanti
and of-that-sort a-cithara may-be-made or on more and as-many-as
necessarii sunt nervi, tota proportionum ratio quasi
needful are strings the-whole of-the-proportions reckoning as-if
oculis subjecta cernatur.
to-the-eyes laid-under may-be-seen

De vi harmonicae et quae sint ejus instrumenta judicii et quoniam non usque sensibus oporteat credi

V.1 · THE POWER OF HARMONICS

Boethius defines harmonics as the faculty that weighs high and low sounds by sense and reason. Sense grasps things confusedly, as an eye takes a hand-drawn circle for a true one, while reason judges exactly; sense's small errors pile up, so four misjudged steps miss the diapente, and alone it cannot triple a line. Reason must rule the erring sense, which leans on it as on a staff: harmonic judgment has two parts, like adze and compass, sense for the differences, reason for their measure.

Sed de his paulo post loquemur; nunc dicendum est quae
but about these a-little after we-shall-speak now to-be-said it-is what
sit vis harmonicae, de qua tractare instituentes, quatuor
may-be the-power of-harmonics about which to-treat undertaking four
libros implevimus. Naturam vero ejus vimque exprimendam
books we-have-filled the-nature but of-it and-the-power to-be-expressed
in hujus quinti voluminis seriem distulimus. Harmonica est
into of-this fifth volume the-course we-have-put-off harmonics is
facultas differentias acutorum et gravium sonorum sensu ac
a-faculty the-differences of-high and of-low sounds by-sense and
ratione perpendens. Sensus enim ac ratio, quasi quaedam
by-reason weighing sense for and reason as-if certain
facultatis harmonicae instrumenta sunt. Sensus namque
of-the-faculty harmonic instruments are sense for
confusum quiddam ac proxime tale, quale est illud quod
a-confused something and nearly such as is that which
sentit, advertit. Ratio autem dijudicat integritatem atque unas
it-senses it-marks reason however discerns exactness and the-single
persequitur differentias. Itaque sensus invenit quidem confusa
pursues differences and-so sense finds indeed confused-(things)
ac proxima veritati, accipit vero ratione integritatem. Ratio
and near to-truth it-receives but from-reason exactness reason

vero ipsa quidem invenit integritatem, accipit vero a sensu
but itself indeed finds exactness it-receives but from sense
confusam ac proximam verisimilitudinem. Namque sensus
a-confused and approximate truth-likeness for sense
nihil concipit integritatis, sed usque ad proximum venit.
nothing conceives of-exactness but even to the-near it-comes
Ratio vero dijudicat, velut si quis manu circulum scribat,
reason but discerns as if someone by-hand a-circle should-draw
fortasse eum circulum vere oculus arbitretur. Ratio vero nullo
perhaps that circle truly the-eye may-deem reason but in-no
modo esse id quod simulatur intelligit. Hoc vero idcirco
way to-be that which is-imitated understands this but for-this-reason
est, quoniam sensus circa materiam vertitur, speciesque in
is since sense about matter turns and-the-forms in
ea comprehendit, quae ita sunt fluidae atque imperfectae.
it it-grasps which so are fluid and imperfect
Neque determinatae, et usque ad unguem expolitae, sicut est
nor determinate and even to the-nail polished as is
ipsa materia. Quare ipsum quoque confusio sequitur, mentem
itself matter wherefore it-itself too confusion follows the-mind
vero atque rationem, quoniam materia non moratur, species quas
but and reason since matter not delays the-forms which
praevidet, praeter subjecti communionem intuetur, atque ideo
it-foresees apart-from of-the-subject the-fellowship it-beholds and therefore
eam integritas comitatur ac veritas, potiusque in sensu quod
it exactness accompanies and truth and-rather in sense what
peccatur, aut minus est, aut emendat, aut complet. Fortasse
is-erred or less is or it-amends or completes perhaps
autem id quod sensus non integre, sed confuse, atque veritate
however that which sense not exactly but confusedly and than-truth
minus quasi quidam incallidus aestimator agnoscit, in singulis
less as-if some unskilled appraiser recognizes in single-(cases)

minus habeat errati. Collecta vero multiplicantur in
less it-may-have of-error the-gathered-(errors) but are-multiplied into
summam, atque idcirco magnam faciunt differentiam. Nam si
a-sum and therefore great they-make a-difference for if
duas voculas tono sensus distare arbitretur, neque distent,
two little-voices by-a-tone sense to-be-distant should-judge nor they-be-distant
rursusque ab una earum tono putet distare tertiam,
and-again from one of-them by-a-tone should-think to-be-distant a-third
neque sit integra ac vera toni distantia. Item tertiae
nor be exact and true of-the-tone the-distance likewise of-the-third
quartaeque toni sensus differentiam putet, atque in eadem
and-of-the-fourth of-a-tone sense the-difference should-think and in the-same
quoque errent, neque sit differentia toni, ab hac etiam
too they-may-err nor be the-difference of-a-tone from this also
quarta, quintam distare semitonium putet, neque vere, neque
fourth the-fifth to-be-distant a-semitone should-think nor truly nor
integre aestimet, in singulis fortasse minus videatur erratum.
exactly should-reckon in single-(cases) perhaps less may-seem the-error
Quod vero in primo tono sensus reliquit atque id quod in
what but in the-first tone sense left-behind and that which in
secundo et tertio atque in quarto semitonio peccatum est, in
the-second and third and in the-fourth the-semitone erred has-been into
unum congregatum atque collectum efficiet, ut prima vox
one gathered and collected will-bring-about that the-first voice
ad quintam vocem diapente non contineat consonantiam, quod
to the-fifth voice the-diapente not may-contain consonance which
oportebat fieri si tres tonos ac semitonium sensus integre
behooved to-come-about if three tones and a-semitone sense exactly
judicasset. Quod igitur in singulis tonis minus pervidebatur,
had-judged what therefore in the-single tones less was-seen-through
id collectum in consonantiam evidenter apparuit. Atque ut
that collected into the-consonance evidently appeared and that

pervideatur sensum quidem confusa colligere, nullo modo
it-may-be-seen (that)-sense indeed confused-(things) to-gather in-no way
autem ad integritatem rationis ascendere, sic consideremus:
however to the-exactness of-reason to-ascend thus let-us-consider
Datae enim lineae majorem minoremve aliam reperire, nihil
of-a-given for line a-greater or-a-lesser another to-find nothing
est difficile sensui. Proposita vero mensura in tanto majorem,
is difficult for-sense with-proposed but a-measure by so-much a-greater
tantove minorem reperiat, id non faciet sensus prima
or-by-so-much a-lesser (that)-it-find this not will-do of-sense the-first
conceptio, sed solers rationis inventio. Vel si rursus datam
conception but the-clever of-reason discovery or if again a-given
lineam propositum sit vel duplicare, vel dimidiam secare, id
line proposed it-be either to-double or the-half to-cut this
fortasse licet paulo difficilius quam confuse majorem minoremve
perhaps though a-little harder than confusedly a-greater or-a-lesser
reperire, poterit tamen sensus inventione constitui.
to-find it-will-be-able yet of-sense by-the-discovery to-be-settled
Si vero imperetur, ut propositae lineae tripla ponatur, vel
if but it-be-bidden that of-the-proposed line the-triple be-set or
ab ea pars tertia recidatur, vel quadrupla constituatur, vel
from it part a-third be-cut-off or the-quadruple be-established or
pars quarta resecetur nonne impossibile sit sensui, nisi
part a-fourth be-shorn-back would-it-not impossible be for-sense unless
integritas rationis accedat, hoc ideo, quia per processus
the-exactness of-reason come-to-aid this for-this-reason because through advances
quidem rationi locus accrescit, deficit sensui. Si enim octavam
indeed for-reason room grows-on it-fails for-sense if for the-eighth
partem propositae lineae auferre aliquis imperetur, vel
part of-the-proposed line to-take-away someone be-bidden or
ejusdem octuplam dare cogatur, totius quidem dimidiam
of-the-same the-octuple to-give be-forced of-the-whole indeed the-half

sumere compellitur, dimidiaeque dimidiam, ut sit quarta,
to-take he-is-driven and-of-the-half the-half that it-be the-fourth
quartaeque dimidium, ut sit octava. Rursusque totius
and-of-the-fourth the-half that it-be the-eighth and-again of-the-whole
duplam, duplaeque duplam, ut sit quadrupla, quadruplaeque
the-double and-of-the-double the-double that it-be the-quadruple and-of-the-quadruple
duplam, ut sit octupla. Itaque in tanta rerum numerositate
the-double that it-be the-octuple and-so in so-great of-things a-multitude
nihil efficit sensus. Cujus omne judicium subitum, atque in
nothing effects sense whose every judgment (is)-sudden and on
superficie positum integritatem persecutionemque non explicat.
the-surface placed exactness and-the-following-through not it-unfolds
Idcirco non est aurium sensui dandum omne judicium, sed
therefore not is of-the-ears to-the-sense to-be-given every judgment but
exhibenda est etiam ratio quae errantem sensum regat
to-be-brought-forward is also reason which the-erring sense may-rule
ac temperet, qua labens sensus deficiensque veluti baculo
and may-temper on-which slipping sense and-failing as on-a-staff
innitatur. Nam ut singulae artes habent instrumenta quaedam
may-lean for as the-single arts have instruments certain
quibus partim confuse aliquid informent ut asciculum,
with-which partly confusedly something they-may-shape as the-little-adze
partim vero quod est integrum deprehendat, ut circinum.
partly but what is exact it-may-detect as the-compass
Ita etiam harmonica vis habet duas judicii partes:
so also the-harmonic power has two of-judgment parts
unam quidem hujusmodi, per quam sensus comprehendit
one indeed of-this-sort through which sense grasps
subjectarum differentias vocum, aliam vero per quam
of-the-underlying the-differences of-voices the-other but through which
ipsarum differentiarum modum mensuramque considerat.
of-the-very differences the-measure and-the-measurement it-considers

Quid sit harmonica regula, vel quam intentionem harmonicae Pythagorici vel Aristoxenus vel Ptolemaeus esse dixere

V.2 · WHAT THE HARMONIC RULE IS

The harmonic rule is the instrument in which, with reason applied, the differences of sounds are searched out — and the schools disagree about its aim. The Pythagoreans hold that everything must answer to reason, sense giving only seeds of knowledge; Aristoxenus makes reason the companion and second, bounding everything by the judgment of sense. Ptolemy defines the aim as the concord of the two, reason finding proportions that sense does not cry out against, and blames both schools.

Hujusmodi igitur instrumentum in quo rationis adhibito
of-this-sort therefore an-instrument in which of-reason with-applied
modo, sonorum differentiae perquiruntur, vocatur harmonica
the-manner of-sounds the-differences are-searched-out is-called the-harmonic
regula, in qua re multorum doctorum sententiae discordia
rule in which matter of-many teachers of-opinion a-discord
fuit. Quidam enim qui Pythagoricis disciplinis maxime
was certain-(men) for who to-the-Pythagorean teachings most
crediderunt, hanc intentionem harmonicae esse dicebant,
trusted this the-aim of-harmonics to-be said
ut cuncta rationi consentanea sequerentur. Sensum enim
that all-things to-reason agreeable they-might-follow (that)-sense for
dare quaedam quodam modo semina cognitionis, rationem
to-give certain in-a-certain manner seeds of-knowledge (that)-reason
vero perficere. Aristoxenus vero econtrario rationem quidem
but to-perfect Aristoxenus but contrariwise (that)-reason indeed
comitem ac secundarium esse dicebat, cuncta vero sensus
a-companion and secondary to-be said all-things but of-sense
judicio terminari, et ad ejus modulationem consensumque
by-the-judgment to-be-bounded and to its measuring and-assent

esse tenendum. A Ptolemaeo autem quodam modo harmonicae
to-be to-be-held-to by Ptolemy however in-a-certain manner of-harmonics
definitur intentio, ea scilicet ut nihil auribus rationique possit
is-defined the-aim this namely that nothing to-the-ears and-to-reason may-be-able
esse contrarium. Id enim secundum Ptolemaeum harmonicus
to-be contrary this for according-to Ptolemy the-harmonist
videtur intendere, ut id quod sensus judicat ratio quoque
seems to-aim-at that that which sense judges reason too
perpendat, et ita ratio proportiones inveniat ut ne sensus
may-weigh and so reason the-proportions may-find that not sense
reclamet, duorumque horum concordia omnis harmonicae
may-cry-against and-of-the-two of-these by-the-concord the-whole of-harmonics
intentio misceatur. Atque in eo maxime Aristoxenum ac
aim may-be-blended and in this most Aristoxenus and
Pythagoricos reprehendit, quod Aristoxenus nihil rationi, sed
the-Pythagoreans he-blames in-that Aristoxenus nothing to-reason but
tantum sensibus credit, Pythagoricos autem quod minimum
only to-the-senses trusts the-Pythagoreans moreover because least
sensibus, plurimum tamen proportionibus rationis invigilent.
to-the-senses most yet to-the-proportions of-reason they-keep-watch

In quo Aristoxenus vel Pythagorici vel Ptolemaeus gravitatem atque acumen constare posuerint

V.3 · WHERE PITCH RESIDES

All agree that sound is a striking of air, but the schools place the difference of low and high differently. Aristoxenus judges it to lie in quality; the Pythagoreans place it in quantity, and Ptolemy sides with them, holding that denser and finer bodies give out highness, rarer and vaster ones lowness. A string shows it: slackened it becomes as it were rarer and thicker, tightened it is rendered denser and thinner.

Quoniam vero sonum esse omnes consentiunt aeris
since but (that)-sound to-be all agree of-air
percussionem, gravitatis atque acuminis differentiam diversa
a-striking of-lowness and of-highness the-difference by-diverse
ratione ponebant, Aristoxenum secuti et Pythagorici.
reckoning they-placed Aristoxenus (those)-following and the-Pythagoreans
Aristoxenus quippe sonorum differentias secundum gravitatem,
Aristoxenus indeed of-sounds the-differences according-to lowness
atque acumen arbitratur in qualitate consistere; Pythagorici
and highness judges in quality to-consist the-Pythagoreans
vero in quantitate ponebant. Ptolemaeus autem Pythagoricis
but in quantity placed-(it) Ptolemy however to-the-Pythagoreans
propior videtur. Idcirco quoniam ipse quoque gravitatem
nearer seems for-this-reason since be too lowness
atque acumen non in qualitate putat, sed in quantitate
and highness not in quality thinks but in quantity
constitui. Etenim spissiora ac subtiliora corpora acumen,
to-be-settled for-indeed (that)-denser and finer bodies highness
rariora et vastiora edere gravitatem, ut nihil nunc de
rarer and vaster to-give-forth lowness so-that nothing now about
intensionis relaxationisque modo dicatur. Quanquam etiam
of-tightening and-of-slackening the-manner be-said although also

cum relaxatur aliqua quasi sit rarius atque crassius, cum
when is-slackened some-(string) as-if it-be rarer and thicker when
vero intenditur spissius redditur, subtiliusque tenuatur.
but it-is-tightened denser it-is-rendered and-finer is-thinned

De sonorum differentiis Ptolemaei sententia

V.4 · PTOLEMY ON THE DIFFERENCES OF SOUNDS

Boethius reports Ptolemy's division of sounds. Some are unison, giving one sound; of those that differ in pitch, some are continuous, their difference joined by a common bound the way a rainbow's colors pass into one another with no sure edge, while others are parted, silence coming between them. A string twisted while it is struck shows the continuous kind, the sound drawn from low to high without a break.

His igitur ita expeditis, differentias sonorum Ptolemaeus
with-these therefore thus dispatched the-differences of-sounds Ptolemy
dividit hoc modo: Vocum aliae sunt unisonae, aliae minime.
divides in-this manner of-voices some are unison others not-at-all
Unisonae sunt, quarum unus sonus est, vel in gravi, vel
unison-(voices) are whose one sound is whether in the-low or
in acuto. Non unisonae vero; quando alia est gravior, alia
in the-high not unison but when one is lower another
acutior. Harum partim ita sunt, ut earum inter se
higher of-these partly so are that of-them between themselves
differentia communi fine jungatur. Non enim discreta est,
the-difference by-a-common bound may-be-joined not for parted it-is
sed a gravi in acutum ita deducitur, ut continua videatur.
but from the-low into the-high so is-drawn that continuous it-may-seem
Aliae vero sunt non unisonae, quarum differentia silentio
others but are not unison whose difference with-silence
interveniente distinguitur, ut vero voces communi fine
coming-between is-distinguished that but voices by-a-common bound

jungantur, fit hoc modo: Sicut enim cum in nubibus
be-joined it-happens in-this way as for when in the-clouds
arcus aspicitur, ita colores sibimet sunt proximi: ut non
the-bow is-beheld so the-colors to-themselves are near that not
sit certus finis, cum alter ab altero disgregetur. Sed
there-be a-sure bound when one from the-other is-parted but
ita verbi gratia a rubro discedit ad pallidum, ut per
so of-a-word for-the-sake from red it-departs to pale that through
continuam mutationem in sequentem vertatur colorem, nullo
continuous change into the-following it-be-turned color with-no
medio certoque interveniente, qui utrosque distinguat;
middle and-sure-(bound) coming-between which both may-part
ita etiam fieri solet in vocibus, ut si quis percutiat
so also to-happen it-is-wont in voices that if someone should-strike
nervum eumque dum percutit torqueat, evenit ut in
a-string and-it while he-strikes should-twist it-comes-about that in
principio pulsus gravior sit, dum torquetur vero vox illa
the-beginning of-the-stroke lower it-be while it-is-twisted but voice that
tenuetur, continuique fiant gravis vocis sonitus et acutae.
be-thinned and-continuous become of-the-low voice the-soundings and of-the-high

Quae voces enharmoniae sunt aptae

V.5 · NOTES SUITED TO HARMONY

Boethius states which sounds harmonics can treat. Continuous non-unison voices are severed from the harmonic faculty, since they are unlike themselves and sound forth no one thing; parted voices, holding their own places like unmingled colors, are subject to the art. Among these, the ones that can be joined to effect melody are called melodic, and those from which no melody can be made are unmelodic.

Cum igitur non unisonarum vocum aliae sint continuae,
since therefore not of-unison voices some be continuous
aliae disgregatae, continuae quidem tales sunt, ut inter
others parted the-continuous indeed such are that between
se earum differentia communi fine jungatur nec habeat
themselves of-them the-difference by-a-common bound may-be-joined nor may-have
locum designatum vox acuta gravisque quem teneant.
a-place marked-out voice the-high and-the-low which they-may-hold
Discretae vero habent proprios locos, vel uti colores
the-parted but have their-own places even as colors
impermixti, quorum differentia visitur suo quodam loco
unmingled whose difference is-seen in-its certain place
constituta. Continuae quidem non unisonae voces ab
established the-continuous indeed not unison voices from
harmonica facultate separantur. Sunt enim sibi ipsis
the-harmonic faculty are-severed they-are for to-their-own selves
dissimiles, nec unum aliquid personantes. Discretae vero
unlike nor one something sounding-forth the-parted but
voces harmonicae subjiciuntur arti. Potest enim distantium
voices to-the-harmonic are-subjected art there-can for of-distant
sibique dissimilium vocum differentia deprehendi, in
and-to-themselves unlike voices the-difference to-be-detected among

quibus quae junctae efficere melos possint, emmelis
which those-which joined to-effect melody are-able melodic
dicuntur, ecmelis autem quibus junctis effici non potest.
are-called unmelodic however from-which joined to-be-effected not it-can

Quem numerum proportionum Pythagorici statuunt

V.6 · THE PYTHAGOREAN NUMBER OF PROPORTIONS

Boethius turns to the Pythagorean count of the consonances. They judge the diapente and diatessaron simple, join the diapason from them, and admit diapason-plus-diapente in the triple proportion and the double diapason in the quadruple. Diapason-plus-diatessaron they deny to be a consonance, because its proportion of 8 to 3 falls into the multiple superpartient class, while they place all consonance in multiples and superparticulars.

Consonae autem vocantur quae copulatae mixtos suavesque
consonant moreover are-called those-which coupled mingled and-sweet
efficiunt sonos. Dissonae vero quae minime. Et hoc
effect sounds dissonant but those-which not-at-all and this
quidem est Ptolemaei de sonorum differentia judicium. Nunc
indeed is of-Ptolemy about of-sounds the-difference the-judgment now
autem quid a caeteris musicis in consonantiarum positione
however in-what from the-other musicians in of-consonances the-placing
disteterit dicendum videtur: Pythagorici enim consonantias
he-has-differed to-be-said it-seems the-Pythagoreans for the-consonances
diapente ac diatessaron simplices arbitrantur, atque ex his
diapente and diatessaron simple judge and from these
unam diapason consonantiam jungunt; esse etiam diapente ac
the-one diapason consonance they-join to-be also diapente and
diapason, et bis diapason, illam triplicis hanc quadrupli.
diapason and twice diapason that-(one) of-the-triple this-(one) of-the-quadruple

Diapason vero ac diatessaron consonantiam esse non aestimant,
diapason but and diatessaron a-consonance to-be not they-deem
idcirco quoniam non in superparticulari vel multiplici cadit
for-this-reason since not in the-superparticular or multiple it-falls
comparatione, sed in multiplici superpartiente. Est enim haec
comparison but in the-multiple superpartient is for this
proportio vocum ut octo ad 3; si quis enim horum in
the-proportion of-voices as eight to 3 if someone for of-these in
medio quatuor ponat, efficit terminos hos 8 4 3. Quorum
the-middle four should-place he-makes terms these 8 4 3 of-which
octo ad 4 diapason efficiunt consonantiam; 4 ad 3, diatessaron;
eight to 4 the-diapason effect consonance 4 to 3 the-diatessaron
octo vero ad 3, in multiplici superpartiente constituitur.
eight but to 3 in the-multiple superpartient is-established
Quae autem sit multiplex superpartiens comparatio ex
what however may-be the-multiple superpartient comparison from
arithmeticis libris cognoscendum est, et ex his, quae
the-arithmetical books to-be-learned is and from these-(things) which
secundo hujus institutionis libro digessimus. Pythagorici autem
in-the-second of-this institution book we-set-forth the-Pythagoreans moreover
consonantias in multiplicibus ac superparticularibus ponunt,
the-consonances in multiples and superparticulars place
sicut in eodem libro secundo quartoque praedictum est.
as in the-same book second and-the-fourth aforesaid it-has-been
A superpartientibus vero ac multiplicibus superpartientibus
from the-superpartients but and multiple superpartients
consonantiam separant. Quibus autem modis diapason quidem
consonance they-sever in-what however manners the-diapason indeed
duplici, diatessaron vero sesquitertio, ac diapente sesquialtero
to-the-double the-diatessaron but to-the-sesquitertian and the-diapente to-the-sesquialter

jungunt Pythagorici, ex secundo hujus institutionis Musicae
join the-Pythagoreans from the-second of-this institution of-Music
libro quartoque petendum est.
book and-the-fourth to-be-sought is

Quod reprehendat Ptolemaeus Pythagoricos in numero propositionum

V.7 · PTOLEMY AGAINST THE PYTHAGOREANS

Ptolemy blames the Pythagoreans, and the whole demonstration set out in the earlier books, in several ways. The charge is that they join the diatessaron and diapente to the sesquitertian and sesquialter, yet allow no consonance at all to the remaining superparticulars, although these belong to the same genus.

Reprehendit autem Pythagoricos Ptolemaeus, totamque eam
he-blames however the-Pythagoreans Ptolemy and-the-whole that
quam praedictis libris exposuimus demonstrationem pluribus
which in-the-aforesaid books we-set-out demonstration in-several
modis, in quo totum illud etiam quod diatessaron
ways in which the-whole that also (namely)-that the-diatessaron
ac diapente sesquialtero et sesquitertio conjungunt, in
and diapente by-the-sesquialter and by-the-sesquitertian they-join in
reliquis vero superparticularibus, cum ejusdem sint
the-remaining but superparticulars although of-the-same they-be
generis, nullas omnino applicent consonantias.
genus no at-all they-apply consonances

Demonstratio secundum Ptolemaeum diapason et diatessaron consonantiae

V.8 · DIAPASON-PLUS-DIATESSARON AFTER PTOLEMY

Boethius gives Ptolemy's proof that diapason-plus-diatessaron is a symphony. The diapason joins two notes so that they strike the hearing as one string, which the Pythagoreans grant, so any consonance added to it is kept whole. Take the diapason from hypate meson to nete diezeugmenon and add a diatessaron beyond each end: the outer notes, nete hyperboleon and hypate hypaton, each sound a diatessaron with the nearer end and a diatessaron-plus-diapason with the farther.

Probat autem ex diapason et diatessaron quamdam
he-proves however from diapason and diatessaron a-certain
symphoniam fieri hoc modo. Quoniam enim diapason
symphony to-come-about in-this way since for the-diapason
consonantia talem vocis efficit conjunctionem, ut unus
consonance such of-voice effects a-conjunction that one
atque idem nervus esse videatur, idque Pythagorici quoque
and the-same string to-be it-may-seem and-this the-Pythagoreans too
consentiunt; quocirca si qua ei consonantia fuerit addita,
grant wherefore if any to-it consonance shall-have-been added
integra inviolataque servatur. Ita enim diapason consonantiae
whole and-inviolate it-is-kept so for to-the-diapason consonance
additur tanquam uni nervo. Sit igitur diapason consonantia
it-is-added as to-one string let-there-be therefore the-diapason consonance
quae contineatur inter hypaten meson et neten diezeugmenon.
which be-contained between hypate meson and nete diezeugmenon
Utraque haec ita sibi consentit, ac conjungitur sono, ut
each these so to-each-other agrees and is-conjoined in-sound that
una vox quasi unius nervi, non quasi duorum mixta pellat
one voice as-if of-one string not as-if of-two mingled may-strike
auditum. Quamcumque igitur huic diapason consonantiae
the-hearing whatever therefore to-this diapason consonance

consonantiam junxerimus, servatur integra, quia ita jungitur
consonance we-shall-have-joined it-is-kept whole because so it-is-joined
tamquam uni voculae ac nervo. Si igitur hypate meson et
as to-one little-voice and string if therefore to-hypate meson and
neten diezeugmenon duae in acutum diatessaron fuerint
to-nete diezeugmenon two into the-high diatessarons shall-have-been
junctae, sic conjungitur nete quidem diezeugmenon ea quae
joined so is-conjoined to-nete indeed diezeugmenon that which
est nete hyperboleon, hypate autem meson ea quae est
is nete hyperboleon to-hypate however meson that which is
mese, utraque ad utramque consonabit, et mese ad neten
mese each to each will-consonate both mese to nete
diezeugmenon, et eadem mese ad hypaten meson.
diezeugmenon and the-same mese to hypate meson
Item nete hyperboleon ad neten diezeugmenon et ad hypaten
likewise nete hyperboleon to nete diezeugmenon and to hypate
meson. Item si ad graviorem partem utriusque diatessaron
meson likewise if toward the-lower part of-each diatessaron
consonantiae relaxentur, erit ad hypaten quidem meson
consonances be-slackened there-will-be to hypate indeed meson
diatessaron retinens consonantiam hypate hypaton, ad neten
the-diatessaron holding consonance hypate hypaton to nete
autem diezeugmenon paramese. Consonabitque et hypate
however diezeugmenon paramese and-will-consonate both hypate
hypaton ad hypaten meson et ad neten diezeugmenon, et
hypaton to hypate meson and to nete diezeugmenon and
paramese ad neten diezeugmenon et ad hypaten meson.
paramese to nete diezeugmenon and to hypate meson
Sed eo modo ut gravior quae est ad sibi quidem
but in-that manner that the-lower which is to itself indeed
proximam diatessaron retineat consonantiam, ad ulteriorem
the-nearest the-diatessaron may-retain consonance to the-farther

vero diatessaron ac diapason, ut hypate hypaton, ad hypaten
but a-diatessaron and diapason as hypate hypaton to hypate
meson diatessaron, ad neten diezeugmenon diatessaron ac
meson a-diatessaron to nete diezeugmenon a-diatessaron and
diapason. Item nete hyperboleon, quae est acutior, ad sibi
diapason likewise nete hyperboleon which is higher to itself
proximam neten diezeugmenon diatessaron consonantiam, ad
the-nearest nete diezeugmenon a-diatessaron consonance to
hypaten meson diatessaron ac diapason.
hypate meson a-diatessaron and diapason

Quae sit proprietas diapason consonantiae

V.9 · THE PROPERTY OF THE DIAPASON

Boethius explains why this happens: the diapason is almost one voice, and like the number ten it keeps whatever is added to it inviolate — two added to three makes a new five, but two added to ten stays visible in twelve. So the diapason renders no added consonance dissonant: with the diapente it makes the triple, and with the diatessaron a consonance in the multiple superpartient class, the double superbipartient 8 to 3.

Hoc vero idcirco evenire contendit, quoniam diapason
this but for-this-reason to-come-about he-maintains since the-diapason
pene una vocula est, talisque consonantia est, ut unum
almost one little-voice is and-such a-consonance is that one
quodammodo effingat sonum, et sicut denario numero,
in-a-certain-way it-may-fashion sound and as to-the-ten number
qui fuerit additus intra eum positus integer inviolatus
which shall-have-been added within it placed whole inviolate
servatur; cum in caeteris id ita minime eveniat, ita etiam
is-kept while in the-others this thus by-no-means it-happen so also

in hac consonantia. Nam si duo tribus adjicias, quinque
in this consonance for if two to-three you-add five
continuo reddis, et numeri species immutata est. Si vero
straightway you-render and of-the-number the-form altered is if but
eosdem denario addas, duodecim feceris, et binarius
the-same to-ten you-add twelve you-will-have-made and the-two
junctus denario conservatus est. Item ternarius caeterique
joined to-ten preserved is likewise the-three and-the-rest
eodem modo: ita igitur symphonia diapason quamcunque
in-the-same way so therefore the-symphony diapason whatever
aliam susceperit consonantiam servat, nec immutat, nec
other shall-have-received consonance it-keeps nor alters nor
ex consona dissonam reddit. Nam sicut diapente symphonia
from consonant dissonant renders for as the-diapente symphony
juncta diapason consonantiae in tripla scilicet proportione
joined to-the-diapason consonance in the-triple namely proportion
diapason ac diapente consonantiam servat, ita etiam
the-diapason and diapente consonance keeps so also
diatessaron cum sit consonantia juncta cum diapason, aliam
the-diatessaron since it-be a-consonance joined with the-diapason another
consonantiam reddit, et fit secundum Ptolemaeum
consonance renders and there-arises according-to Ptolemy
alterius consonantiae additio ejusque est diapason ac
of-another consonance an-addition and-of-it is the-diapason and
diatessaron in multiplici superpartiente constituta. Estque ea
diatessaron in the-multiple superpartient established and-is that
proportio dupla superbipartiens, ut octo ad tres; habent enim
proportion double superbipartient as eight to three contain for
ternarium octo bis, duasque ejus partes, id est duas unitates.
the-three eight twice and-two of-it parts that is two units

Quibus modis Ptolemaeus consonantias statuat

V.10 · PTOLEMY'S CLASSES OF CONSONANCE

Boethius sets out the classes by which Ptolemy tracks the consonances. Non-unison sounds are equal-sounding, consonant, melodic, dissonant, or unmelodic: the diapason and its double are equal-sounding, making one simple sound from two; the diapente and diatessaron are consonant; melodic intervals, like the tone, fit melody without being consonances. The double proportion, nearest to equality, is fitted to the equal-sounding, and the sesquialter and sesquitertian that divide it to the consonances.

Et de Pythagoricorum quidem opinione Ptolemaeus
and about of-the-Pythagoreans indeed the-opinion Ptolemy
ita dijudicat: quibus vero modis ipse consonantiarum
so judges in-what but manners he-himself of-consonances
proportiones numerosque vestiget, hinc ordiendum est. Voces,
the-proportions and-numbers may-track from-here to-be-begun is voices
inquit, inter se vel unisonae sunt, vel non unisonae.
he-says between themselves either unison are or not unison
Non unisonarum autem vocum aliae quidem sunt aequisonae,
not of-unison however voices some indeed are equal-sounding
aliae consonae, aliae emmelis, aliae dissonae, aliae ecmelis.
others consonant others melodic others dissonant others unmelodic
Et unisonae quidem sunt quae unum atque eumdem
and unison indeed are those-which one and the-same
sigillatim pulsae reddunt sonum. Aequisonae vero quae
singly struck give-back sound equal-sounding but those-which
simul pulsae unum ex duobus atque simplicem quodammodo
together struck one from two and a-simple in-a-certain-way
efficiunt sonum, ut est diapason, eaque duplicata quae est
effect sound as is the-diapason and-it doubled which is
bis diapason. Consonae autem sunt quae compositum
twice diapason consonant however are those-which a-composite

permixtumque suavem tamen efficiunt sonum, ut diapente ac
and-mingled sweet yet effect sound as diapente and
diatessaron. Emmelis autem sunt quaecunque quidem consonae
diatessaron melodic however are whatsoever indeed consonant
non sunt, possunt aptari tamen recte ad melos, ut sunt hae
not are they-can to-be-fitted yet rightly to melody as are these
quae consonantias jungunt. Dissonae vero sunt quae non
which the-consonances join dissonant but are those-which not
permiscent sonos, atque insuaviter feriunt sensum. Ecmelis vero
mingle sounds and unsweetly strike the-sense unmelodic but
quae non recipiuntur in consonantiarum conjunctionem, de
those-which not are-admitted in of-consonances the-conjunction about
quibus paulo posterius in divisione tetrachordorum dicemus.
which a-little later in the-division of-tetrachords we-shall-speak
Quoniam igitur univocis quidem comparationibus proxime
since therefore to-the-one-voiced indeed comparisons most-nearly
sunt aequisonae, necesse est ut aequis numeris ea numerorum
are the-equal-sounding necessary it-is that to-equal numbers that of-numbers
inaequalitas adjungatur, quae est proxima aequis. Est autem
inequality be-adjoined which is nearest to-the-equal is moreover
juxta aequalitatem numerorum ea quae est dupla. Nam et
next-to the-equality of-numbers that which is double for both
prima multiplicitatis species est, et major numerus cum
the-first of-multiplicity form it-is and the-greater number when
minorem numerum supervenit, aequo eum ipsi minori
the-lesser number it-overpasses by-an-equal it to-the-very lesser
transcendit, ut duo unum uno transgrediuntur, qui eidem
it-exceeds as two the-one by-one overstep which to-the-same
unitati aequalis est; jure igitur duplex proportio aequisonis
unit equal is by-right therefore the-double proportion to-the-equal-sounding
aptatur, id est diapason. Bis diapason vero bis duplici,
is-fitted that is the-diapason twice diapason but to-the-twice double

id est quadruplo. Quae autem proportiones dividunt
that is to-the-quadruple what however proportions divide
duplicem proportionem primae ac maximae, his aptandae
the-double proportion the-first and greatest to-these to-be-fitted
sunt consonantiis quae dividunt diapason aequisonantiam.
are to-the-consonances which divide the-diapason equal-sounding-(consonance)
Unde fit ut diapente quidem sesquialterae, diatessaron
whence it-comes that the-diapente indeed to-the-sesquialter the-diatessaron
vero sesquitertiae comparationi copulentur. Junctae vero
but to-the-sesquitertian comparison be-coupled joined but
consonantiae cum aequisonis alias efficiunt consonantias, ut
consonances with the-equal-sounding other effect consonances as
diapente ac diapason in triplo, diatessaron ac diapason in
diapente and diapason in the-triple diatessaron and diapason in
ea proportione quae est octo ad tres. Emmelis autem sunt
that proportion which is eight to three melodic however are
quae diapente ac diatessaron dividunt, ut tonus caeteraeque
those-which the-diapente and diatessaron divide as the-tone and-the-other
proportiones, de quibus paulo posterius in divisione
proportions about which a-little later in the-division
tetrachordorum loquemur, simplices earum scilicet partes.
of-tetrachords we-shall-speak the-simple of-them namely parts

Quae sint aequisonae, vel quae consonae, vel quae emmelis

V.11 · EQUAL-SOUNDING, CONSONANT, MELODIC

Boethius restates the classes: the diapason and double diapason are equal-sounding; the diapente, diatessaron, and their compounds with the diapason are consonances; the tone and the other proportions are melodic. Each class is put together out of the class below it — the equal-sounding out of the consonances, the consonances out of the melodic sounds — and all of them can be found on the instrument described at the end of the fourth book, the string stretched over hemispheres.

Igitur aequisonae quidem sunt diapason ac bis diapason,
therefore equal-sounding indeed are the-diapason and twice diapason
quoniam earum temperamento mixturaque unus atque simplex
since of-them by-the-tempering and-mixture one and simple
quodammodo efficitur sonus. Consonantiae autem sunt, primae
in-a-certain-way is-effected a-sound consonances however are the-first
quidem in superparticularibus sesquialtera et sesquitertia, id
indeed in the-superparticulars the-sesquialter and the-sesquitertian that
est diapente ac diatessaron, et diapason ac diapente, et
is diapente and diatessaron and diapason and diapente and
diapason ac diatessaron, hae sunt compositae atque conjunctae
diapason and diatessaron these are composed and conjoined
ex aequisonis et consonantibus. Emmelis autem reliqui
from the-equal-sounding and the-consonant melodic however the-remaining
inter has poni possunt, ut inter diatessaron ac diapente
between these to-be-placed they-can as between diatessaron and diapente
differentia tonus, jungunturque quodammodo aequisonae
the-difference the-tone and-are-joined in-a-certain-way the-equal-sounding
quidem ex consonantibus, ut diapason ex diatessaron et
indeed from the-consonant as the-diapason from diatessaron and
diapente. Consonantiae autem ex his qui emmelis soni
diapente consonances however from these which melodic sounds

vocantur, ut eadem diapente et diatessaron tonis caeterisque
are-called as the-same diapente and diatessaron from-tones and-the-other
posterius dicendis proportionibus; sed quomodo quidem horum
later to-be-told proportions but in-what-way indeed of-these
omnium proportio colligi possit, ex eo loco sumendum
all the-proportion to-be-gathered can from that place to-be-taken
est quem quarto volumine in fine descripsimus, ubi
it-is which in-the-fourth volume at the-end we-described where
nervus super semisphaeria tendebatur. Ibi enim deprehenditur
a-string over hemispheres was-stretched there for is-detected
aequisonantia, diapason ac bis diapason, et consonantiae
the-equal-sounding diapason and twice diapason and consonances
simplices diapente ac diatessaron, et consonantiae compositae,
simple diapente and diatessaron and consonances composite
diapason ac diapente, et diapason ac diatessaron et qui
diapason and diapente and diapason and diatessaron and those-which
sunt emmelis soni, ut in toni differentia consistentes.
are melodic sounds as in of-the-tone the-difference standing

Quemadmodum Aristoxenus intervallum consideret

V.12 · ARISTOXENUS ON INTERVAL

Boethius briefly opens Aristoxenus' view. Because he yields the whole treatment to the judgment of the ears, Aristoxenus marks the notes themselves with no numbers and measures only the difference between them — improvidently, since he claims to know a difference whose terms he never measures. So he makes the diatessaron two tones and a half, the diapente three and a half, and the diapason six tones, which the earlier volumes proved cannot be.

Quid vero de his, Aristoxenus sentiat breviter aperiendum
what but about these Aristoxenus may-feel briefly to-be-opened
est. Ille enim quoniam minime tractatum rationi constituit,
is he for since not-at-all the-treatment to-reason assigns
sed aurium judicio permittit, idcirco voces ipsas nullis
but of-the-ears to-the-judgment yields therefore voices the-very with-no
numeris notat, ut earum colligat proportiones, sed earum
numbers he-marks that of-them he-may-gather the-proportions but of-them
in medio differentiam sumit, ut speculationem non in ipsis
in the-middle the-difference he-takes so-that the-speculation not in the-very
vocibus, sed in eo quod inter se differunt, collocet, nimis
voices but in that which between themselves they-differ he-may-place too
improvide, qui differentiam se scire arbitretur earum
improvidently (he)-who the-difference himself to-know should-judge of-those
vocum, quarum magnitudinem nullam mensuramve constituat.
voices whose magnitude no or-measure he-establishes
Hic igitur et diatessaron consonantiam duorum tonorum
he therefore both the-diatessaron consonance of-two tones
ac semitonii esse proponit, et diapente trium tonorum ac
and of-a-semitone to-be sets-forth and the-diapente of-three tones and

semitonii, et diapason sex tonorum, quod fieri non
of-a-semitone and the-diapason of-six tones which to-come-about not
posse superioribus voluminibus demonstratum est.
to-be-able in-the-earlier volumes demonstrated has-been

Descriptio octochordi, qua ostenditur diapason consonantiam minorem esse sex tonis

V.13 · THE OCTOCHORD AGAINST SIX TONES

Boethius reports Ptolemy's octochord demonstration that the diapason falls within six tones. Eight strings are stretched, six sesquioctave steps are built up from the first, and a further string is tuned so that the first sounds a true diapason with it. That pair sound the equal-sounding diapason, but the string at six tones is always a little sharper than the diapason string; so six tones exceed the diapason, and Aristoxenus' error is refuted by sense itself.

Docet autem Ptolemaeus per cujusdam octochordi divisionem
he-teaches moreover Ptolemy through of-a-certain octochord the-division
diapason intra sex tonos cadere hoc modo: intendantur
(that)-the-diapason within six tones to-fall in-this manner let-be-stretched
enim octo chordae, id est A B C D E F G H, fiatque
for eight strings that is A B C D E F G H and-let-be-made
sesquioctava A K ejus quae est B L, et B L ejus quae
sesquioctave A K of-that which is B L and B L of-that which
est C M, et C M ejus quae est D N, et D N ejus quae
is C M and C M of-that which is D N and D N of-that which
est E X, et E X ejus quae est F O, et F O ejus quae
is E X and E X of-that which is F O and F O of-that which
est G P. Erunt igitur sex toni. Rursus inter G P et
is G P there-will-be therefore six tones again between G P and
H ducatur medius nervus ad R. Erit igitur A K dupla
H let-be-drawn a-middle string at R it-will-be therefore A K double

ab eo quod est H R. Pulsae igitur simul A K et H R
from that which is H R struck therefore together A K and H R
diapason aequisonantiam consonabunt. Si vero aliquis G
the-diapason equal-sounding-consonance they-will-sound-together if but someone G
P percutiat, semper erit paulo acutior quam est H R.
P should-strike always it-will-be by-a-little higher than is H R
Ac per hoc transcendunt sex toni diapason consonantiam.
and through this surpass (the)-six tones the-diapason consonance
Si enim A K et G P diapason pulsati resonarent, tonorum
if for A K and G P the-diapason struck should-resound of-tones
sex esset diapason consonantia. Si vero his idem A K et G
six would-be the-diapason consonance if but these same A K and G
P non consonantibus A K et H R diapason consonarent,
P not being-consonant A K and H R the-diapason should-sound-together
et H R acutior esset quam G P, diapason consonantia
and H R higher were than G P the-diapason consonance
sex tonos excederet. Nunc vero quia consonantibus A K
six tones would-exceed now truly since being-consonant A K
et H R eadem H R ab ea quae est G P gravior
and H R the-same H R than that which is G P lower
invenitur, non potest dubitari quin sex toni diapason
is-found not can to-be-doubted but-that six tones the-diapason
consonantiam excedant. Atque ita sensu quoque potest
consonance exceed and thus by-sense also can
colligi diapason consonantiam inter sex tonos cadere.
to-be-gathered (that)-the-diapason consonance within six tones to-fall
Sic igitur Aristoxeni error sine dubitatione convincitur.
so therefore of-Aristoxenus the-error without doubt is-refuted

Diatessaron consonantiam tetrachordo contineri

V.14 · THE DIATESSARON IN THE TETRACHORD

Boethius turns to the division of the tetrachords. The diatessaron consonance is effected by four strings, which is why it bears that name. Between the two outer strings sounding the diatessaron, two must be set in the middle, making three proportions with each other and with the extremes.

Nunc de tetrachordorum divisione dicendum est. Etenim
now about of-the-tetrachords the-division to-be-spoken it-is for-indeed
diatessaron consonantia quatuor efficitur nervis; idcirco etiam
the-diatessaron consonance by-four is-effected strings therefore also
diatessaron nuncupatur, ut igitur duobus nervis altrinsecus
diatessaron it-is-named so-that therefore with-two strings on-either-side
positis ad diatessaron symphoniam consonantibus tetrachordum
placed to the-diatessaron symphony sounding-together a-tetrachord
fiat, duos necesse est statui in medio nervos, qui ad
may-be-made (that)-two necessary it-is to-be-set in the-middle strings which to
se invicem atque ad extremos tres proportiones efficiant.
themselves mutually and to the-outermost three proportions may-effect

Quomodo Aristoxenus vel tonum dividat, vel genera, ejusque divisionis dispositio

V.15 · ARISTOXENUS' DIVISION OF THE TONE AND THE GENERA

Boethius sets out Aristoxenus' division of the tone and the genera. Measuring only intervals, Aristoxenus makes the diatessaron 60 units and the tone 24, and divides each tetrachord by numbers: enharmonic 6–6–48, soft chromatic 8–8–44, hemiolic chromatic 9–9–42, tonic chromatic 12–12–36, soft diatonic 12–18–30, intense diatonic 12–24–24. A genus is dense when its two lower intervals together do not exceed the one left at the top, so the enharmonic and chromatic are dense and the diatonic is not.

Hoc igitur diatessaron Aristoxenus per genera tali ratione
this therefore diatessaron Aristoxenus through the-genera by-such a-method
partitur. Dividit enim tonum in duas partes, atque id
divides he-divides for the-tone into two parts and that
semitonium vocat. Dividit in tres partes, cujus tertiam
a-semitone he-calls he-divides (it)-into three parts of-which the-third
diesin vocat chromatis mollis, dividit in 4, cujus quartam
the-diesis he-calls of-the-chroma soft he-divides (it)-into 4 of-which the-quarter
cum propria medietate, id est cum octava totius toni,
with its-own half that is with the-eighth of-the-whole tone
appellat diesin chromatis hemiolii. Rursus solam quartam
he-names the-diesis of-the-chroma hemiolic again the-sole quarter
ejus vocat diesin enharmonios.
of-it he-calls the-diesis enharmonios
Cum igitur haec ita sint, cumque generum divisio
since therefore these-things so are and-since of-the-genera the-division
secundum eum sit duplex, unum quidem genus est mollius;
according-to him is twofold one indeed genus is softer
aliud vero incitatius. Et mollius quidem est enharmonium,
the-other but more-intense and the-softer indeed is the-enharmonic

incitatius vero diatonicum. Inter haec vero consistit
the-more-intense but the-diatonic between these but stands
chromaticum incitatione mollitieque participans. Sunt igitur
the-chromatic in-intensity and-in-softness sharing there-are therefore
secundum hunc ordinem differentiae permixtorum generum
according-to this order the-differences of-the-mingled genera
sex. Una quidem enharmonii, 3 autem chromatici, id est
six one indeed of-the-enharmonic 3 moreover of-the-chromatic that is
chromatici mollis, et chromatici hemiolii, et chromatici
of-the-chromatic soft and of-the-chromatic hemiolic and of-the-chromatic
toniaei. Duae vero reliquae diatonici, mollis atque incitati.
tonic two but the-remaining of-the-diatonic soft and the-intense
Quorum omnium talis secundum Aristoxenum divisio est.
of-which all such according-to Aristoxenus the-division is
Quoniam enim quarta pars toni diesis enharmonios
since for the-fourth part of-the-tone the-diesis enharmonios
nuncupari praedicta est, quoniamque Aristoxenus non
to-be-named stated-before has-been and-since Aristoxenus not
voces ipsas inter se comparat, sed differentiam vocum
the-notes themselves among themselves compares but the-difference of-notes
intervallumque metitur, et secundum eum tonus est duodecim
and-the-interval measures and according-to him the-tone is of-twelve
unitatum, hujus igitur erit pars quarta diesis enharmonios
units of-this therefore will-be the-part fourth the-diesis enharmonios
tres. Quoniam vero ex duobus tonis ac semitonio diatessaron
three since but from two tones and a-semitone the-diatessaron
consonantia unitur erit tota diatessaron ex his duodecim
consonance is-united will-be the-whole diatessaron from these twelve
ac duodecim ac sex unitatibus constituta.
and twelve and six units composed

Sed quoniam saepe fit ut si usque ad octavas velimus
but since often it-happens that if all-the-way to eighths we-should-wish
deducere partes, non integros numeros, sed in aliquas
to-carry-down the-parts not (upon-)whole numbers but upon some
particulas incurramus, idcirco quidem facienda est tota
small-fractions we-run therefore indeed to-be-made is the-whole
diatessaron consonantia 60. At vero 24 tonus, semitonium
diatessaron consonance 60 but truly 24 the-tone the-semitone
duodecim, pars quarta, quae diesis enharmonios dicitur, sex,
twelve the-part fourth which the-diesis enharmonios is-called six
octava autem tres; juncta vero octava cum quarta, sex scilicet
the-eighth moreover three joined but the-eighth with the-quarter six namely
cum tribus, ut faciat diesin chromatis hemiolii, erunt
with three so-that it-may-make the-diesis of-the-chroma hemiolic they-will-be
novem. His igitur ita constitutis, tria genera, enharmonicum,
nine these therefore thus established the-three genera enharmonic
chromaticum, diatonicum, has Aristoxeno videntur habere
chromatic diatonic these to-Aristoxenus seem to-have
proprietates, ut alia eorum dicantur spissa, alia minime. Spissa
properties that some of-them may-be-called dense others not-at-all dense
sunt, quorum duae graviores proportiones: unam eam quae
are-(those) of-which the-two lower proportions the-one that which
ad acutum apposita est magnitudine non vincunt. Non spissa
at the-high set is in-magnitude not surpass not dense
vero quorum duae proportiones unam reliquam poterunt
but (those)-of-which the-two proportions the-one remaining will-be-able
superare; est autem enharmonium et chromaticum spissum,
to-surpass is moreover the-enharmonic and the-chromatic dense
diatonicum vero non spissum. Itaque enharmonium secundum
the-diatonic but not dense and-so the-enharmonic according-to
Aristoxenum dividitur sic 6, 6, 48, ut inter gravem nervum
Aristoxenus is-divided thus 6 6 48 so-that between the-low string

ac prope gravem sit quarta pars toni, quae dicitur
and the-next-to low there-be the-fourth part of-the-tone which is-called
diesis enharmonios, cum sit tonus 24 unitatibus constitutus.
the-diesis enharmonios since is the-tone 24 units composed
Item secundum intervallum a gravi nervo ad tertium, sit
likewise the-second interval from the-low string to the-third let-be
eadem quarta pars toni. Reliqui vero qui restant
the-same fourth part of-the-tone the-remaining-(ones) but which are-left
ex sexaginta qui totius proportionis sunt inter tertium
out-of sixty which of-the-whole proportion are between the-third
a gravi nervo atque acutissimum quartum ponuntur 48,
from the-low string and the-highest as-the-fourth are-placed 48
et duae proportiones ad gravem positae, id est 6 ac 6,
and the-two proportions at the-low placed that is 6 and 6
unam reliquam ad acutum locatam, id est 48, non vincunt.
the-one remaining at the-high located that is 48 not surpass
Chromatis vero mollis hanc facit divisionem 8, 8, 44, ut
of-the-chroma but soft this he-makes division 8 8 44 so-that
octo atque octo sint tertiae partes tonorum; est enim tonus
eight and eight be third parts of-tones is for the-tone
(ut dictum est) 24 unitatum, et dicitur toni pars tertia
as said it-was 24 of-units and is-called of-the-tone the-part third
diesis chromatis mollis. Item chromatis hemiolii diatessaron
the-diesis of-the-chroma soft likewise of-the-chroma hemiolic the-diatessaron
ita partitur, 9, 9, 42. Est autem diesis chromatis hemiolii
thus he-divides 9 9 42 is moreover the-diesis of-the-chroma hemiolic
pars octava toni cum quarta, id est ex 24 sex cum
the-part eighth of-the-tone with the-quarter that is out-of 24 six with
tribus. Item chromatis toniaei talis secundum Aristoxenum
three likewise of-the-chroma tonic such according-to Aristoxenus

partitio est, 12, 12, 36. Scilicet ut in duobus intervallis
the-division is 12 12 36 namely so-that in the-two intervals
singula semitonia constituat, et quod reliquum est in ultimo.
single semitones he-may-establish and what remaining is in the-last
Atque in his omnibus duae proportiones, quae graviori nervo
and in these all the-two proportions which to-the-lower string
sunt proximae, reliquam quae ad acutum posita est,
are nearest the-remaining-(one) which at the-high placed is
magnitudine minime superant, sunt enim (ut dictum est)
in-magnitude by-no-means surpass they-are for as said it-was
spissorum generum. Spissa quippe genera sunt enharmonium
of-the-dense genera dense indeed genera are the-enharmonic
atque chromaticum. Diatonica vero divisio ipsa quoque
and the-chromatic the-diatonic but division itself also
est duplex. Et mollis quidem diatonici divisio est hoc
is twofold and of-the-soft indeed diatonic the-division is in-this
modo, 12, 18, 30, ut 12 semitonium sit, decem et octo
manner 12 18 30 so-that 12 a-semitone be ten and eight
semitonium, et quarta pars toni, 30 vero quod reliquum
a-semitone and the-fourth part of-the-tone 30 but what remaining
est. Quorum decem et octo et 12 efficiunt 30, nec superantur
is of-which ten and eight and 12 make 30 nor are-surpassed
ab ea parte quae reliqua est. Item diatonici incitati talis
by that part which remaining is likewise of-the-diatonic intense such
partitio est, ut semitonium ac duos habeat integros
the-division is so-that a-semitone and two it-may-have whole
tonos 12, 24, 24, ex quibus 24 et 12, id est 36, non
tones 12 24 24 out-of which 24 and 12 that is 36 not
superantur a reliqua parte quae ad acutum est, sed potius
are-surpassed by the-remaining part which at the-high is but rather

vincunt. Est igitur secundum Aristoxenum tetrachordorum
they-surpass is therefore according-to Aristoxenus of-the-tetrachords
praedicta partitio, quae subjecta descriptione monstratur.
the-aforesaid division which by-the-subjoined diagram is-shown

Quomodo Archytas tetrachorda dividat eorumque descriptio

V.16 · ARCHYTAS' TETRACHORD DIVISIONS

Archytas puts everything in reason, and Boethius reports that he neither achieved what he was after nor produced a reasoning the ear agrees with. He keeps the same gravest and most acute sounds, 2016 and 1512, in all three genera, and sets the string next to the gravest at 1944 in each, a proportion of 28 to 27. The third string he places at 1890 in the enharmonic, 1701 in the diatonic, and 1792 in the chromatic, this last fixed by giving it to 1701 the proportion 256 to 243.

Archytas vero cuncta in ratione constituens non modo sensum
Archytas but all-things in reason establishing not only the-sense
aurium in primis consonantiis observare neglexerit, verum
of-the-ears in the-first consonances to-observe neglected but
etiam maxime in tetrachordorum divisione rationem secutus
even most-of-all in of-the-tetrachords the-division reason followed
est. Sed ita, ut neque eam quam quaerebat efficaciter
has but so that neither that-(aim) which he-sought effectually
expediret, neque sensui proposita ab eo ratio consentiret. Ille
he-achieved nor to-sense the-proposed by him reasoning agreed he
enim tria esse genera arbitratur, enharmonium, chromaticum,
for three to-be genera judges the-enharmonic the-chromatic
diatonicum. In quibus eosdem gravissimos statuit atque
the-diatonic in which the-same lowest he-sets and
acutissimos sonos, in omnibus quidem generibus gravissimos
highest sounds in all indeed the-genera the-lowest

sonos faciens 2016, acutissimos vero 1512; inter hos in tribus
sounds making 2016 the-highest but 1512 between these in the-three
generibus nervum gravissimo proximum collocat, eum
genera the-string to-the-lowest nearest he-places that-(one)
scilicet qui sit 1944, ut ad eum 2016, sesquivicesimam
namely which is 1944 so-that to it 2016 the-sesquitwentieth
septimam obtineant proportionem. Post haec vero infra
(and-)seventh may-hold proportion after these-things but below
acutum nervum, tertium vero a gravissimo eum collocat in
the-high string the-third but from the-lowest it he-places in
enharmonio genere, qui sit 1890, ad quem 1944 sesquitricesima
the-enharmonic genus which is 1890 to which 1944 by-a-sesquithirtieth
quinta proportione jungantur. Idemque 1890 ad acutissimum
(and-)fifth proportion may-be-joined and-the-same 1890 to the-highest
1512 in sesquiquarta proportione sit constitutus. Item
1512 in a-sesquiquartan proportion may-be established likewise
in diatonico genere tertium quidem a gravissimo nervo,
in the-diatonic genus the-third indeed from the-lowest string
secundum vero ab acutissimo eum ponit, qui sit 1701, ad
the-second but from the-highest it he-sets which is 1701 to
quos 1944 sesquiseptima proportione conjuncti sunt.
which 1944 by-a-sesquiseventh proportion joined are
Ipsi autem 1701 ad acutissimum, 1512 sesquioctava. In
themselves moreover 1701 to the-highest 1512 (is)-a-sesquioctave in
chromatico vero genere tertium a gravissimo, et secundum
the-chromatic but genus the-third from the-lowest and the-second
ab acutissimo eum ponit, qui ad 1701, qui est tertius
from the-highest that-(one) he-sets which to 1701 which is the-third
a gravissimo in diatonico genere eam obtineat proportionem
from the-lowest in the-diatonic genus that may-hold proportion
quam obtinent 256 ad 243; hic autem est 1792, qui est
which hold 256 to 243 this-(one) moreover is 1792 which is

secundus ab acutissimo appositus, habetque proportionem
the-second from the-highest set-near and-has the-proportion
secundus ab acutissimo in diatonico genere, id est 1701
the-second from the-highest in the-diatonic genus that is 1701
ad secundum ab acutissimo in chromatico genere, id
to the-second from the-highest in the-chromatic genus that
est 1792 eam quam habent 243 ad 256, eorumque
is 1792 that-(proportion) which have 243 to 256 and-of-these
tetrachordorum, secundum Archytae sententiam, divisorum
tetrachords according-to Archytas' opinion divided
formam monstrat subjecta descriptio.
the-form shows the-subjoined diagram

Quemadmodum Ptolemaeus et Aristoxeni et Archytae tetrachordorum divisiones deprehendat

V.17 · PTOLEMY'S CRITIQUE OF BOTH

Boethius reports Ptolemy's censure of both divisions. Archytas' chromatic string at 1792 holds superparticular proportion to neither neighbor; the ear finds the chromatic's first step greater than he made it; and the enharmonic's first step should be far smaller than the other genera's, yet he sets 28 to 27 in all three. Aristoxenus puts the lower steps of his soft and hemiolic chromatics a twenty-fourth of a tone apart, which no hearing catches, and gives the diatonic only two species.

Sed utrasque tetrachordorum divisiones Ptolemaeus ita
but both of-the-tetrachords divisions Ptolemy thus
reprehendit. Architam quidem primo, quoniam secundus ab
censures Archytas indeed first because the-second from
acutissimo nervus chromatico genere, id est 1792, ita est
the-highest string (in-)the-chromatic genus that is 1792 so is
collocatus, ut nec ad acutissimum 1512, nec ad proximum
placed that neither to the-highest 1512 nor to the-next

graviori 1944, ullam superparticularem efficiat proportionem,
to-the-lower 1944 any superparticular may-effect proportion
cum Architas tantam superparticularibus comparationibus
whereas Archytas so-great to-superparticular comparisons
habuit dignitatem, ut eas etiam in consonantiarum
held esteem that them even in of-the-consonances
ratione susceperit. Dehinc quod primam a gravissimo
the-reckoning he-admitted next (in)-that the-first from the-lowest
nervo proportionem in chromatico quidem majorem sensus
string proportion in the-chromatic indeed greater the-sense
deprehendat quam fecit Architas, hic namque in chromatico
detects than made-(it) Archytas he for in the-chromatic
genere 1944 ad 2016 distare fecit sesquivicesimam septimam
genus 1944 to 2016 to-be-distant made (by-)the-sesquitwentieth (and-)seventh
proportionem, cum secundum consuetam chromatici generis
proportion whereas according-to the-accustomed of-the-chromatic genus
modulationem a sesquivicesima prima esse debuerit. Item
modulation by the-sesquitwentieth (and-)first to-be it-ought likewise
enharmonium genus ea proportio, quam primam a
(as-to)-the-enharmonic genus that proportion which first from
gravissimo secundum Architae retinet divisionem, talis est,
the-lowest according-to Archytas' retains division such is
ut longe minor esse debeat quam in caeteris generibus
that by-far smaller to-be it-ought than in the-other genera
invenitur; hic autem aequam eam caeteris generibus statuit,
it-is-found he however equal it to-the-other genera sets
dum primas a gravi proportiones in tribus generibus
while the-first from the-low proportions in the-three genera
sesquivicesimas septimas ponit.
(as-)sesquitwentieths (and-)sevenths he-sets

Aristoxenum vero culpat, quoniam in chromate molli et
Aristoxenus but he-blames because in the-chroma soft and
in chromate hemiolio tales posuerit primas secundasque
in the-chroma hemiolic such he-set the-first and-second
a gravi nervo proportiones, quae a se minimo
from the-low string proportions which from one-another by-the-least
et quantum sensus non possit internoscere distarent.
and as-much-as the-sense not could to-distinguish were-distant
Est quippe proportio prima in chromatis mollis, prima
is indeed the-proportion first in of-the-chroma soft the-first
divisione secundum Aristoxenum octo, at in chromate
division according-to Aristoxenus eight but in the-chroma
hemiolio novem. Sed octo ad novem unitatis differentia
hemiolic nine but eight to nine of-a-unit by-the-difference
distant. Est autem tonus totus 24 unitatibus secundum
are-distant is moreover the-tone whole 24 (of)-units according-to
positionem vel propositionem quorum unitas 24
the-position or the-proposition of-which the-unit (the-)24th
est. Primae igitur a gravi inter se proportiones
is the-first therefore from the-low among themselves proportions
chromatis mollis, et chromatis hemiolii 24 parte
of-the-chroma soft and of-the-chroma hemiolic (by-the-)24th part
toni distant, quod propter brevitatem differentiae
of-the-tone are-distant which because-of the-shortness of-the-difference
nullo modo sentit auditus. Idem etiam Aristoxenum
in-no way perceives the-hearing the-same-(man) also Aristoxenus
reprehendit cur diatonici generis duas tantum fecerit
censures why of-the-diatonic genus two only he-made
divisiones, ut in molle incitatumque divideret,
divisions so-that into the-soft and-the-intense he-might-divide

cum possint aliae quoque diatonici generis species
although could other also of-the-diatonic genus species
inveniri.
to-be-found

Quemadmodum tetrachordorum divisionem fieri dicat oportere

V.18 · HOW THE TETRACHORD OUGHT TO BE DIVIDED

Boethius begins Ptolemy's own rules for dividing the tetrachord. The middle notes must exceed one another by superparticular proportions, and by unequal ones, since a superparticular cannot be cut in half. The step made against the gravest string must be smaller than the steps toward the acute, and in the dense genera the two lower proportions together must be smaller than the one left at the top. The rule for the non-dense genera breaks off mid-sentence, where the transmission ends.

Ptolemaeus tetrachorda diversa ratione partitur, illud
Ptolemy the-tetrachords by-a-different method divides this
in principio statuens, ut inter duos altrinsecus sonos
in the-beginning establishing that between the-two on-either-side sounds
tales voculae aptentur, quae se superparticularibus
such little-notes be-fitted which one-another by-superparticular
proportionibus excedant, inaequalibus tamen, quoniam
proportions they-may-exceed unequal-(ones) however since
superparticularis proportio non potest in aequa dividi.
a-superparticular proportion not can into equal-(parts) to-be-divided
Dehinc ut omnis comparatio, quae fit ad eum nervum
next that every comparison which is-made to that string
qui est gravissimus, in tribus minor sit caeteris,
which is lowest in the-three-(genera) smaller be than-the-others
quae acutis vocibus conjunguntur. Sed in his ea
which to-the-high notes are-joined but in these those

quae spissa nominamus talia esse debent, ut duae
which dense we-name such to-be ought so-that the-two
proportiones, quae gravitati sunt proximae, minores
proportions which to-the-low-(side) are nearest smaller
sint ea proportione quae relinquitur ad acutum, in
be than-that proportion which is-left at the-high in
non spissis vero ut in diatonicis generibus nusquam
the-not dense-(ones) but as in the-diatonic genera nowhere
una.
one
